Biomathematics

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Lecture No. # 04
Functions and its derivatives

Hi. Welcome to this lecture of Biomathematics. We have been discussing the idea of functions. In the last few lectures, we got introduced to many different kinds of functions that are generally used in different contexts in Biology. In this lecture, we will discuss functions and its derivatives. We will just learn one or two functions more, and then go to this idea of derivatives. So, the idea that you will learn in this lecture is, the idea of derivative. We will discuss what a derivative is, as we, when we reach that point. But, before discussing the derivative, we have to just see one more last part, very short section about functions. So, we have seen many functions like Y equal to X, Y equal to X square, Y equal to X cube and all that; and, we also said that, function is nothing, but a relation between Y and X.

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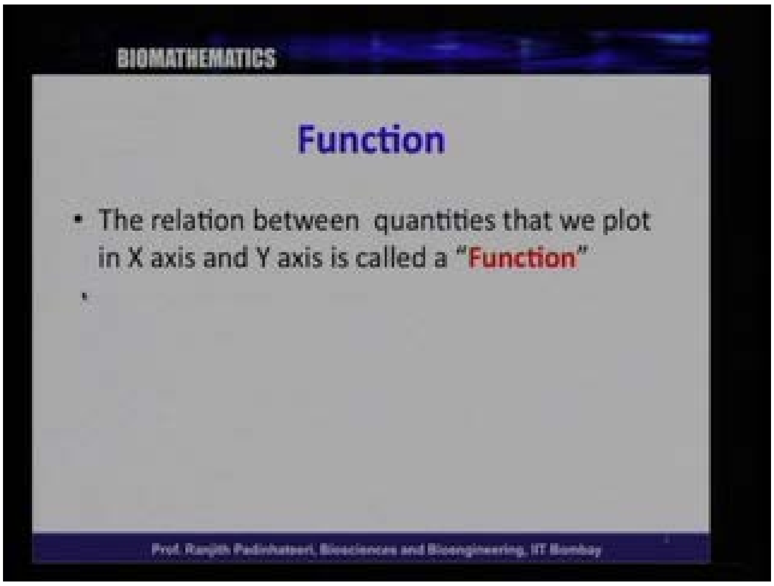

So, that is what we defined as function and we saw many functions like Y equal to m X plus C, which was a straight line; Y is equal to k X square, which is the quadratic function, which is a slope like function; Y is equal to e power x, exponential function; Y is equal to sin x and cos x, which are periodic functions, which repeat and we said that, many physical quantities can be represented by this functions.

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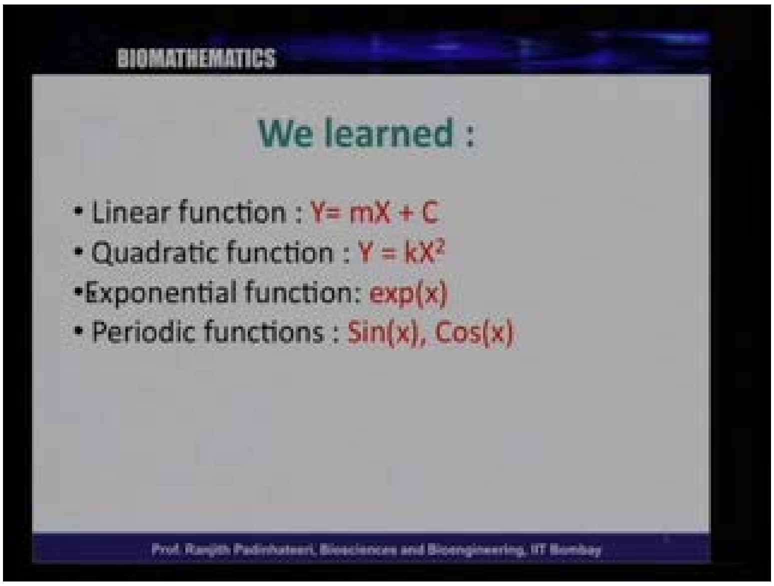

And, like, those things like, which periodically repeat, like seasons, for example. Or anything, that periodically repeat, can be written as some combination of sin x and cos x. And, we saw that, the bacterial growth is some sort of an exponential function, which is 2 power x. So, all these mathematical functions we learned so far, are useful in understanding different biological, different experimental situations or different natural phenomena. When we learn about all these natural phenomena, and if you want to express, if you want to tell somebody this, the things that you observe in this phenomena or in this experiments, as we learned in the first, we said in the first class, all these mathematical functions is some way of precisely, quantitatively, telling someone, what is happening. If something is periodically repeating, if you say of something, this is repeating like sin x, or repeating like cos x, or repeating like a combination of sin x and cos x, it is very precisely telling, how exactly it is repeating.

And, when you say that, something is moving like, like a linear function Y is equal to k X, we are very precisely telling, the way things change; like, we had seen some examples of molecular motors walking along microtubule and the, and the position, how does the position changes with time and so on. So, all this examples, we learnt so far. The examples of different functions, all can be applied in different context. We will come back to that, but the immediate aim of this course is, to get you familiarized, to with all this functions and then, go and apply in this, wherever it, it is needed. So, we will keep discussing examples, but we will also learn the technical information, the technique, the mathematical techniques, that is needed for you, to understand a physical phenomena in a much better way.

So, we learnt many functions like e power x and 2 power x. So, like, let me just draw here, that; we had e power x which look like this and similarly, 2 power x which also look, something like this. So, this is e power x and 2 power x, is a bacterial growth, sorry, this is no...So, just 2 power x. So, all this functions, which basically we learned so far are, are examples of… We represent many natural phenomena; for example, 2 power x represent the bacterial growth. Now, let us just learn another function.

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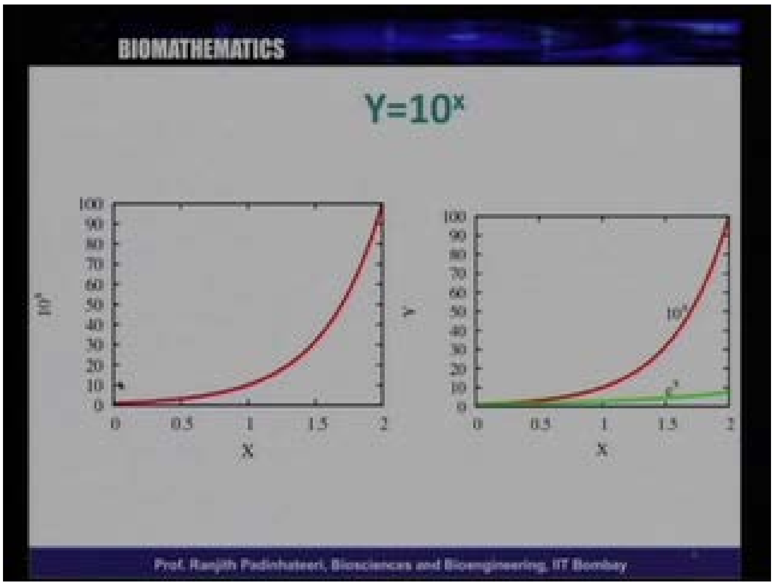

So, let us have a look at this graph. So, this function is Y is equal to 10 power x. There is a reason, why I say this function; it is not randomly. I will come to this why we learn this function. So, the function is Y equal to 10 power x and as you see in the graph, the, the it is a, it is a 10 power x. So, this also an exponential function; this is, x is in the exponent. So, 10 power something. So, this increases very rapidly, as you can see here; as this, whenever this is increased by 1 or 2, when it, this x went from 1 to 2, the Y went from 10 to 100. So, this is increasing rapidly. And, this is increasing fast, much faster than e power x. So, let, what is provided in the right hand side, is a comparison of 10 power x and e power x.

If you look this, you can see that, e power x grows slower compared to 10 power x; 10 power x grows faster than e power x. So, this is, this will be, this information will be useful at some point, but just to understand that, if you provide, 10 power x is an exponential function, it increases very fast; and, we will come back and see why this is important. So, one more function is 10 power x. And, now, we learnt many functions, like Y is equal to some functions, we learnt from the beginning, Y is equal to X square, Y is equal to X cube and so on. So, we had something like a Y equal to X square and for every X value, we had a Y value. So, now, if we have such a function, you can ask this question. So, what we had, if we know the X value, the corresponding Y value is given by square of it. So, if this was 3, this would be 3 square, equal to 9. Now, if you know the Y value, how will we calculate the corresponding X value? Let us say, you know this value, how will you, calculate the corresponding X value?

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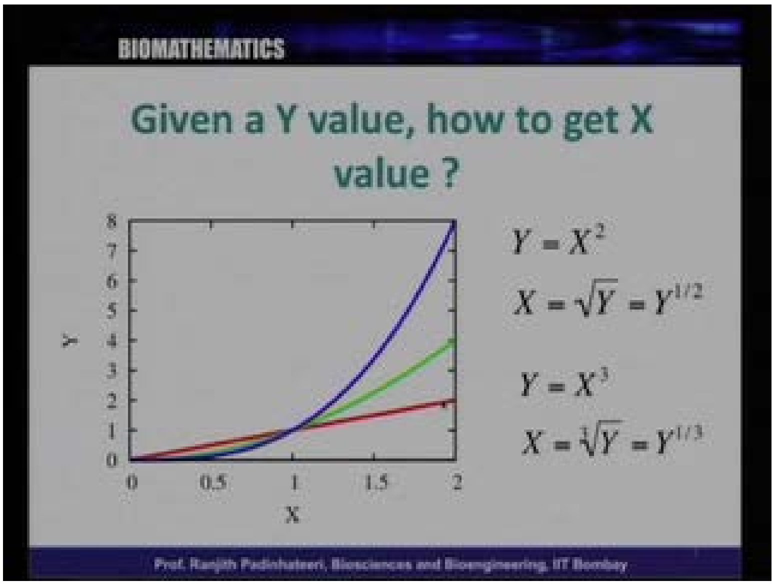

So, you could ask this question that, given a Y value, how to get the X value. So, let us have a this, look at here, some examples, red, green and blue curves are Y equal to X is red, Y equal to X square is green, Y equal to X cube is blue. We have seen this plot in the previous lectures. So, if, let us say, in any of this case, you know the Y value and how will you calculate the X value. So, let us take the example of Y equal to X Square.

To calculate X, we will find the square root of Y. This is square root and it is also written as Y power half. So, we saw this square root function last time. So, if we know the Y value, let us say, the Y value is 4, the corresponding X value is square root of 4, which is 2.

So, similarly, if we know that, the, our function is Y equal to X cube, and if you know the Y value, to calculate the corresponding X value, we will find the cube root. So, this symbol is cube root, with a root with 3 here; this is cube root or Y power 1 by 3. For example, let us look at this blue curve. When the Y value is 8, the corresponding X value, if you draw a line from here and down, the corresponding Y value is 2. So, we know that…So, if Y is equal to X cube, and we know that, Y is equal to 8 and then, X is, to calculate X, we have to find cubic root of 8, which is 8 power 1 by 3, which is 2. So, this is how we calculate, given a Y value, this is how we calculate X value. For simple functions, like X square, X cube, X power 4, etcetera, you can calculate the root; you can calculate as a square root, cubic root. So, the root of a function is basically, given a Y value, how do we calculate the X value? Now, you could ask this question, if it is an exponential function or a 10 power x function or any exponential function, how do we do this?

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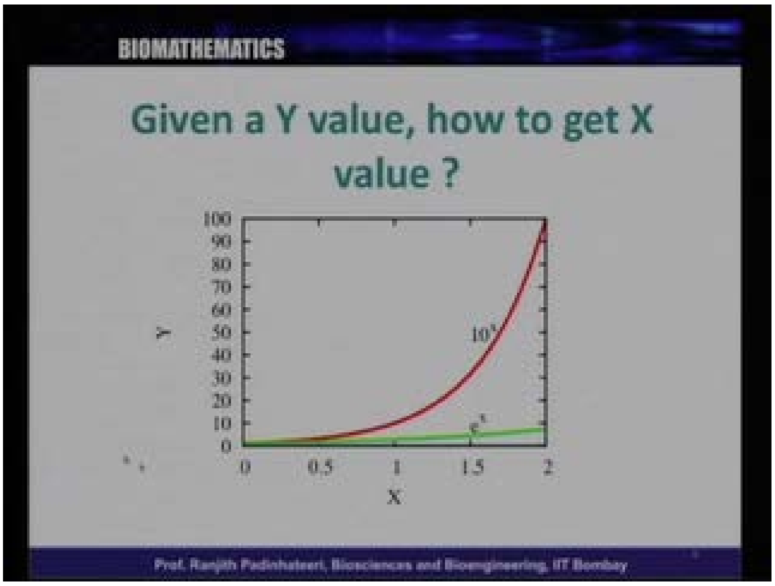

So, let us look the next slide and ask this, slide, ask this question, if you have a curve which is 10 power x, which is shown in red here, given a Y value, which is 80, how do we know what is the X value? The corresponding X value for this 10 power x? So, or, if you know the Y value, what will be the corresponding X value for e power x? This is not square root. So, this function, where, which is used to calculate the X value, given the Y value, is called log or logarithmic function, which you all have heard of, logarithmic functions and it is used very often in different contexts.

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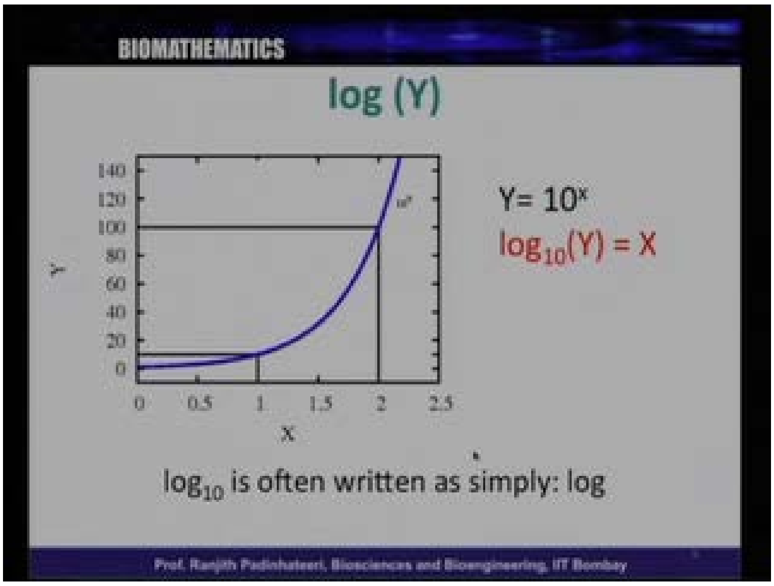

So, let us look at this slide. The next slide shows Y equal to 10 power x in the blue, as a blue curve here. So, the blue curve is Y equal to 10 power x. And, let us take the Y value a 100 and let us ask the question, what is a corresponding X value. So, to find the corresponding X value, you draw a line to the curve. You draw a line to the 10 power x curve, line parallel to the X axis and then, draw a perpendicular line to touch the X axis. So, this says that, for 100, the corresponding X value is 2. When the Y value is 100, the corresponding X value is 2. Similarly, this point is 10, which is the middle of, between 0 and 20, this is 10. When the Y value is 10, the corresponding X value is 1, which we actually know very well; we know that, Y is equal to 10 power x; when, if X equal to 1, Y is 10 power 1 equal to 10, so, which is as we seen in the graph. Similarly, if X equal to 2, Y is 10 power 2 equal to 100. So, if the Y value is 100, the corresponding X value is 2.

If the Y value is 10, the corresponding X value is 1. So, this is what basically, we know and, but, to calculate this, the function which inverts this graph, this is called inverting, is essentially, log of Y. So, if we calculate the log of Y, we will get X. So, there is some, there is a 10 here. So, this is called log to the base 10. If you use this 10 power x curve as a reference curve, then, we call it log to the base 10. So, log to the base 10 of Y, will give you the X. If Y is equal to 10 power x, if you calculate the log of, log of Y, we will get X. So, let us see this. So, log of, to the base 10 of 100 is 2. This is something that you know. Log of 100 is 2. So, if you take 100 and draw a curve and calculate the corresponding X value, this is 2. So, you might have seen, this log, the logarithm in different context in Biology.

What does it convey essentially, is this; that, if you had a 10 power x curve and if you know the Y value, log, essentially, calculates the X value. So, any log value, if you want to calculate the log of something, the right, one way to do is that, you plot Y equal to 10 power x graph, find that value on the Y axis, draw lines and wherever it hits the X axis, you get the value, which is log of that. Let us, for example, like, just like we saw here, if this is Y equal to 10 power x, any value, let us say, this is 23, if you want to calculate the log of 23…So, this is X, this is Y; and, this curve is Y equal to 10 power x; and, if you want to calculate the log of 23, draw a line, which and wherever it hits 10 power x curve, bring it down and this value is log of 23.

Log of 23, to the base 10. So, log 10, (( )), as it is written in the slide here, log 10, is often written as simply log. So, when you see log of something, it is most often, it is log to the base 10; that is, you are calculating, what is the X value, corresponding to the Y value, if you take this 10 power x curve as a base curve and then, draw these lines. This is what logarithm gives. Similarly, you could ask this question, instead of this 10 power x as the base curve, if you had e power x as the base curve, what will be the corresponding Y value. So, let us say instead of 10 power x, you had…So, this is X, this is Y; this is Y equal to e power x, and if you take a Y value and calculate the corresponding X value, this is also a logarithm, but you call it log to the base e. So, let us say this is 51. So, this value is 51 here.

And, if you draw such a line and calculate the corresponding X value, you call it log of 51 to the base e. So, just like, log of…You can take any curve you want, as your base curve, 2 power x, 3 power x, 4 power x, 5 power x, and you can calculate, take a Y value and calculate the corresponding X value, and you can call it a log to the base, whatever curve you take. So, it could have log to the base 2, log to the base 3, log to the base 4. So, most often used things are log to the base e and log to the base 10.

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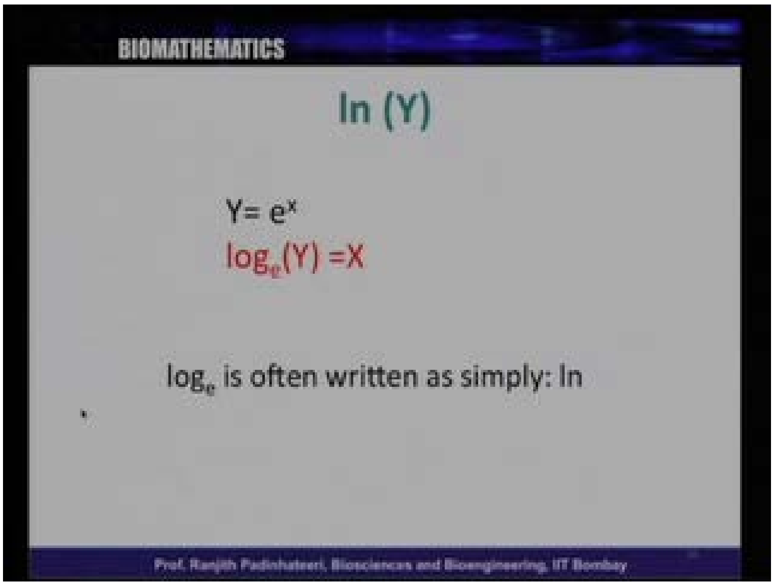

So, let us look the next slide here. So, if you take Y equal to e power x, the X value is obtained by calculating the log of this Y value, to the base e and log e is often written as lon, l n; this is read as lon. So, ln e, ln Y, when you say, somebody says ln Y, what they are saying is basically, they have a Y value and they will want to calculate the corresponding X value, if you draw a line to the e power x curve, you want to calculate the corresponding X value. Now, if we have many numbers 1, 2, 3, 4, 5, 6, 7 and you want to calculate the log of each this quantity, each of this quantities and plot them.

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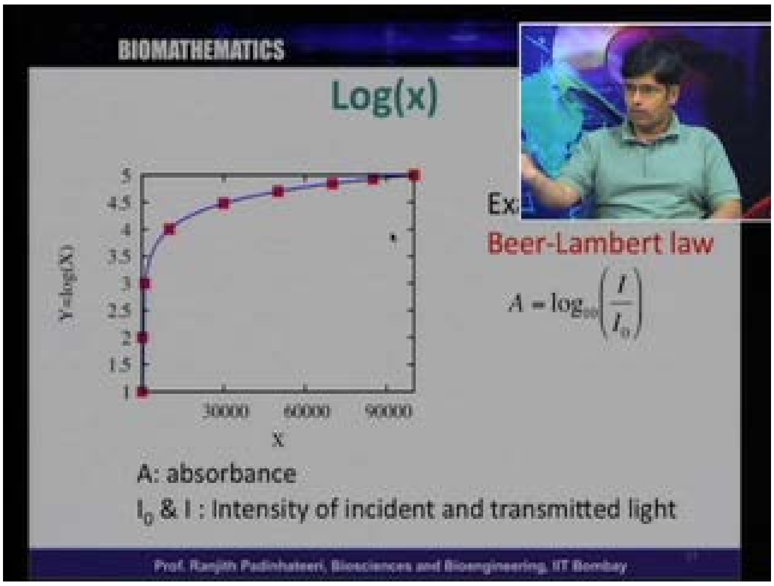

So, you get a function. That function is called log, logarithmic function and let us see, how does it look like. So, what is plotted here is Y equal to log x. So, for each X value, you have a corresponding Y value here. So, one interesting thing you should note here is that, the X axis is goes to, if you look the values in the X axis, it is like 1, it is like 30000, 60000, 90000; so, huge values. But the corresponding Y value is 3, 4, 5. So, if you calculate the log of huge values, it appears that, you get a small value, which is true. So, if you, the log of big number is actually a small number. This is log of 90000 is somewhere very close to 5; just between 4.5 and 5. You know that, similar, as a…This is one thing you should keep in mind that, if you take a huge number and calculate the log, you will get a small number. This idea, this idea that, log of big number is a small number is very often used. This, this, this property is so often used to plot graphs, because it is very convenient to write small numbers. So, very often, you will see log-log plots.

So, the reason, why we have log-log plots, is something related to this, which we will come, when we see a, we will discuss a log-log plot. Just keep in mind that, log of a huge, a big number is a small number. Log of 100 is just 2. Log of, let us write log of some huge numbers. So, log of 10 is 1; log of hundred is 2; log of 1000 is 3; log of 10000 is 4; you all know this. Log of 10…So, 10000 is 10 power 4 and 10 power 5 is 5; log of 10 power 6, it is a huge number, is a million, is just 6. So, log of a million is 6. So, just remember that, log of huge number is a small number and we will be using this property, at some point in this course.

So, now, what is the use of this log? Like, as you see here in this slide, if you remember, many of you might have learnt this Beer-Lambert law, which is basically, something, some, a law that relates absorbance of…So, if you have, if you have a liquid and you shine light on this liquid, the amount of light absorbed is basically, related to the incident, the intensity if incident light and transmitted light. So, if you have I and I 0, which is intensity of incident light and transmitted light, the absorbance is basically, the logarithm of this ratio. So, this is the natural property. You, you all might of learnt that, absorbance, like, how much the liquid absorbs the light, how much a, given a liquid, how much of light it can absorb, the absorbance is essentially, log has, it is logarithmically related to the ratio of intensity of transmitted and incident lights.

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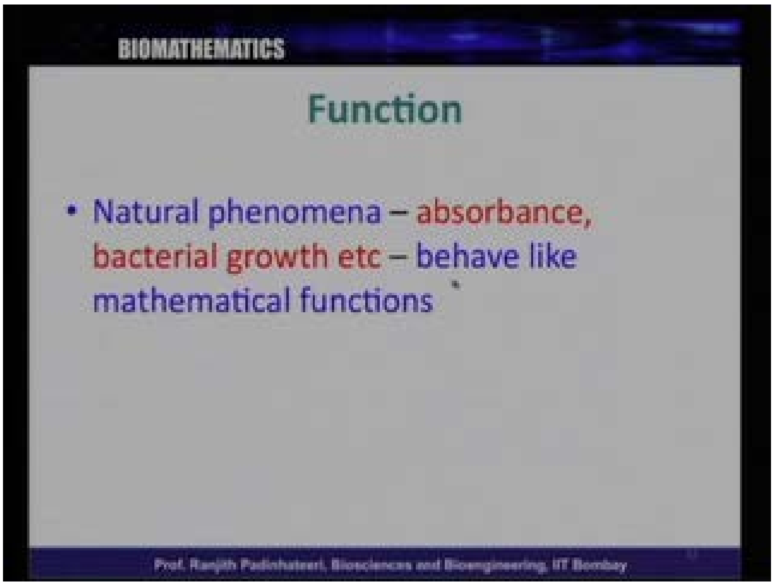

So, here is an example, a natural phenomena which is absorbance, which is a mathematical, behaves like a mathematical function. So, this is intriguing, this is interesting, is it not. Natural phenomena like absorbance, behaves like a mathematical function. In the last lecture, we saw that, bacterial growth, the number of bacteria after a time T, is given by a function 2 power x. So, the bacterial colony, behaves as if, it is like a mathematical function 2 power x. Similarly, here, you have a natural phenomena, which is absorbance of light, which behaves, as if it is a mathematical function. Think about it. This is very, now, a very very intriguing and then, interesting. So, this is why, we learn mathematics and function for a, various functions, because all of this represent some natural phenomena or the other. So, to summarize this function part, let us say that, many natural phenomena like absorbance and bacterial growth, behave like mathematical functions.

And, we learnt many, many functions so far and we will go to a new section now, which is called the derivatives. So, what we will discuss now, is the idea of derivatives. So, we will discuss, what is derivative. This is a part of Mathematics called Calculus, which is very useful in understanding many things, that we see in nature. As you all might know, like, Isaac Newton was one of the pioneer who, early persons, who actually developed the, the calculus, as we know it today. Even many people in India, it appear that, knew some idea about calculus at very early stage. However, as the mathematics that, the calculus that we learn today, is basically derived by Isaac Newton and many other scientists. So, we have learnt the idea of derivatives and how it is useful and we will go into the calculus. The later, as we go into other parts of calculus, we will, we will discuss more about this. And, so, let us start with an example. So, we had take this, mentioned about this example of molecular motor walking along microtubule. So, from the, in cells, from the center of the cell to the periphery, there are microtubules and molecular motors like kinesin work along this microtubule and at some point, at some, in some regime, if you look at the position of kinesin with time, it may look like what we discussed.

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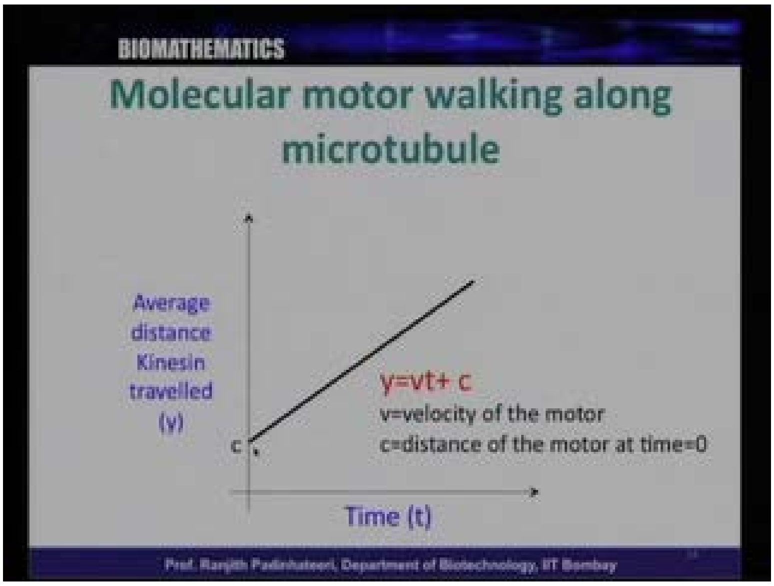

It may look like, something like this. At some particular regime, it is not always it will look like this, but at some regime, it could look like this, where, if, if you plot the position on the Y axis and the time on the X axis, it could look like a straight line. So, and we said that, Y is equal to v t plus c, some equation for this and v is the velocity. So, in other words, when we set all this, unknowingly, we use the idea of velocity.

So, what is the idea of velocity? If you know the position, which, as a function of time, for different value of time, if you know the position, we can calculate the velocity or speed. Think about, instead of molecular motor, talk about, think about you walking somewhere; you walking along a road and you know the position, like, at t equal to, when it is 3 o clock, you start from your home and when it is 3 10, you reach your college. So, in that 10 minutes of time, you know the, you know the distance you travelled. So, if you know the distance you travelled, and the time taken for traveling, you know how to calculate the speed or the velocity. So, this idea, is related to the function, which is, the function here, is the position. Like the position, in the position of molecule motor or the position you were travelling. So, this, if you know the position and time, we can calculate velocity. So, let us look at this slide, where the simple equations of velocity is, change in distance by change in time.

How much distance you moved in a time, in a, when the time changed by a certain amount? If you just say that, the distance you moved is d Y, in a time d t, the velocity or speed is nothing, but d Y by d t. You all know this. This is learnt, you learnt in school that, the velocity is d Y by d t; or, the change in distance by change in time, gives you the speed or the velocity. So, using this, we can calculate the velocity of molecular motors or velocity of you talking, you walking and known, if it is a simple graph, like linear graph, the calculating velocity is easy; if it is not so simple graph, it is slightly complicated. So, we will come to each of this, but by calculating this d Y by d t, or this change in distance by change in time, what we are calculating is nothing, but some quantity called slope.

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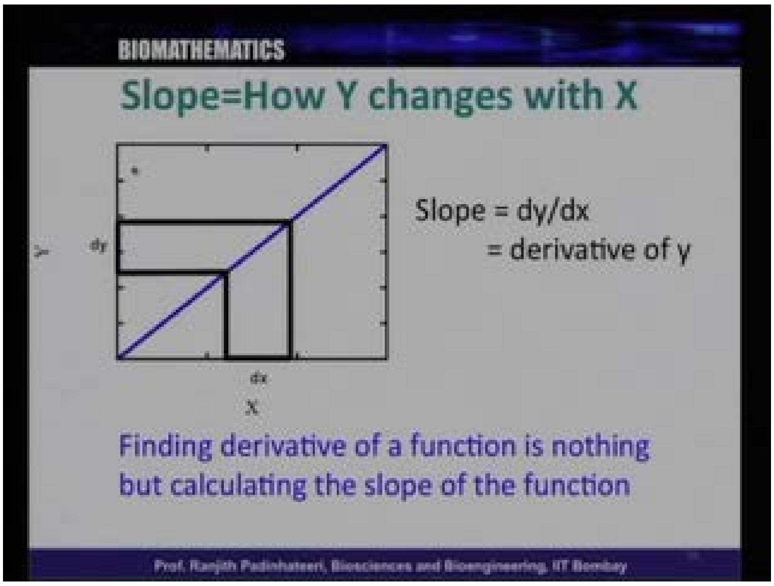

So, let us look at what is slope. So, the slope is nothing, but how Y changes with X. If we have a small change in X, d X, how much the Y will change? So, let us look this graph carefully. Let us look, this is, you start from this value of X, and the corresponding Y value is this. You move along the X axis, certain small distance d X, and see, how much the Y value changed. The corresponding Y value is this. So, if you moved a distance along the X axis, the corresponding Y value also changes by certain amount d Y, and this ratio d Y by d X was called slope; it is also called derivative of Y. So, we will come to this, we will discuss this derivative name later. But, just understand for the moment that, finding derivative of function is nothing, but calculating the slope. So, in the, as you see in this, here, calculating d Y by d X, how much the Y changes when you change X, is calculating the slope. So, here, you calculate the slope of this straight line. So, finding derivative of a function is nothing, but calculating the slope of the function and this is very useful. So, if you had calculated, if you calculate the slope of this position-time curve that we saw in the previous graph, you will get the velocity. So, now, let us understand this little more carefully, by calculating slope of some known curves.

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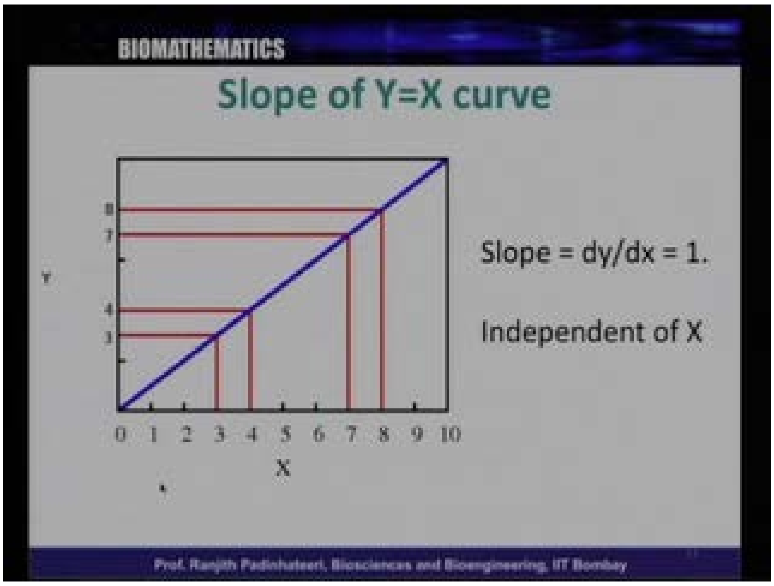

So, we know many functions. We have, we know, Y equal to X curve, V I equal to X square. So, let us calculate the slope of certain curves that we know, and let us see, what we learn from this. And, later, all this what we learn, can be used in the context of Biology. So, let us take this Y equal to X curve. So, see this graph. The blue line is nothing, but Y equal to X. Now, if you want to calculate the slope of Y equal to X curve, draw a line like this and starting from the X axis, calculating the corresponding value of Y and another value at X axis. So, if you start two lines from 3 and 4, it ends up in 3 and 4 here; that is, if you, if you change the value from 3 to Y, the X value 3 to 4, the Y, corresponding Y value also changes from 3 to 4. So, the d X. So, let us look at this. If you change the X value, so, the X was 3 here; the second X value is 4; the Y value corresponding to this X value, that is Y 1, which is the corresponding to the X 1 value, Y 1 is also 3 and Y 2 is also 4.

So, as you saw in this curve, here, so, this is X and this is Y; this is d X and this is d Y; if this is 3 and this is also 3; for this is, if this is Y equal to X curve, this is 3 and this is also 3; this is 4; this is also 4; this point is 4; this point is 4; this is 3; this is 3. So, the d Y here is 1, 4 minus 3. So, d Y is Y. So, this is Y 2, d Y is Y 2 minus Y 1, is 1. d X is X 2 minus X 1 is 1. So, let us have a look at this graph once more. So, this d Y by d X, which is, how much Y changes, when you change X by a certain amount, d X, is called the slope and if you calculate for this curve, it is, you will get that, d Y by d X is 1. So, if you take the Y equal to X curve and calculate the d Y by d X, you will find that, d Y by d X is 1.

So, take it as an assignment, as to calculate d Y by d X; plot yourself for Y equal to X curve on a graph sheet. Take some small interval d X here, and see what is the corresponding d Y interval and you will find that, they are equal to, d Y equal to d X. In other words, d Y by d X equal to 1. So, you will find, if Y equal to X, you will find that, d Y equal to d X.

In other words, this implies that, d Y by d X is 1. This is simple plotting; just plot it and see. There is nothing more than plotting here. Just plot and look at this interval d X and look at this interval d Y. This is 3 and 4; this is also 3 and 4. So, this interval is 1; this interval is 1. So, d Y by d X is just 1. Now, instead of taking 3 and 4, let us take 7 and 8, as you see here. If you plot a line from 7, you end up in 7. You plot a line from 8, the corresponding Y values is 8. So, 8 minus 7 is 1; here also, 8 minus 7 is 1. So, d X, the change in X here is 1; the change in Y here, is also 1. So, d Y by d X is 1. Whatever be the X value, two X values you take, if you just take any interval on this curve and calculate the corresponding Y interval, you will always get the same; that is d Y by d X is 1 and it is independent of whatever X value you take. So, this is how we calculate the slope. So, we, you all know how to calculate the slope. So, this thing, we are calculating the slope, is called calculating the derivative. So, we can say that, derivative of Y equal to X is 1. So, what we are saying here is that, derivative of Y equal to X curve is 1. Or, derivative of Y equal to X function is 1; it is a number. It is independent of X.

We will see some quantities, which is dependent on X, and we will see, how this is useful soon. But, just understand that, you take Y equal to X curve and calculate this slope, you will get 1. Now, let us take the Y equal to 2 X curve. Have a look at this. So, similar, just like what we did before, let us start from 3 and calculate the corresponding Y value. So, the curve for, the function here is Y equal to 2 X. So, since the Y is twice the X, if we have 3, you get 6; and, if you have 4 as X value, the corresponding Y value is 8. Now, you calculate the d X, the change in X. If you go from 3 to 4, the change is 1. If you go along the X axis, and you change this X value by 1, the corresponding Y value will be changed by 2. So, 6 to 8, the Y value was 6 here; it became 8 here. So, 3 to 4, 6 to 8. So, d Y is 2, d X is 1. So, let us understand this. So, for Y equal to 2 X, Y equal to 2 X, so, for Y equal to 2 X, if you plot, this is 3 and 4, and the corresponding values are 6 and 8.

So, this quantity is 1, the difference 4 minus 3 equal to 1. Here, this difference is 8 minus 6 equal to 2; this difference is 2. So, the change in...So, let me call this d X, which is the change in X, which is 4 minus 3 and this is d Y, which is 8 minus 6, and d Y by d X is 2 by 1; 8 minus 6 by 4 minus 3, which is 2 by 1, which is nothing, but 2. So, the derivative of Y equal to 2 X is 2. We did not learn any fancy technique. We just used something that we know. We just know how to plot functions. If you want to calculate Y equal to 2 X derivative, what you have to do is very simple. Just plot it on a graph, calculate the slope, take a X interval and calculate what is the corresponding Y interval; if you can plot and do this much, you get some number and that is, basically, the derivative here.

So, now, let us see, instead of, we took 3 and 4, as we see here; 3 and 4 and you got 6 and 8; let us take 7 and 8. Just like we did in the previous case, if we take 7 as X value, the corresponding Y value is twice 7, is 14. If you take 8 as X value, the corresponding Y value is 16. So, the change in X is 8 minus 7, which is 1; the change in Y is 16 minus 14, which is 2. So, the d Y by d X is again, 2 here.

So, whether you calculate it here, or you calculate it here, you always get the answer 2, which says that, the slope or the derivative, is independent of where on X axis you start from. So, this is independent of X. So, 2, the number 2 is surely independent of, X is the constant; does not matter, where you start on the X axis. Wherever you calculate, you can take any two points on the X axis and calculate the corresponding points on the Y axis, you will always get 2, for this Y equal to 2 X curve. So, if the function is Y equal to 2 X, the derivative is 2. So, this is, we learn this just by plotting. We have not yet learnt the formula for derivative or the technique of calculating the derivative, which we will come. But, so far, we know that, by just calculating, just by plotting it on a graph, we can calculate the slope, and the slope essentially, gives you, slope is nothing, but the derivative; what we call derivative is nothing, but the slope. So, we call slope as the, slope as derivative. So, let us, now, let us have a look at a small movie.

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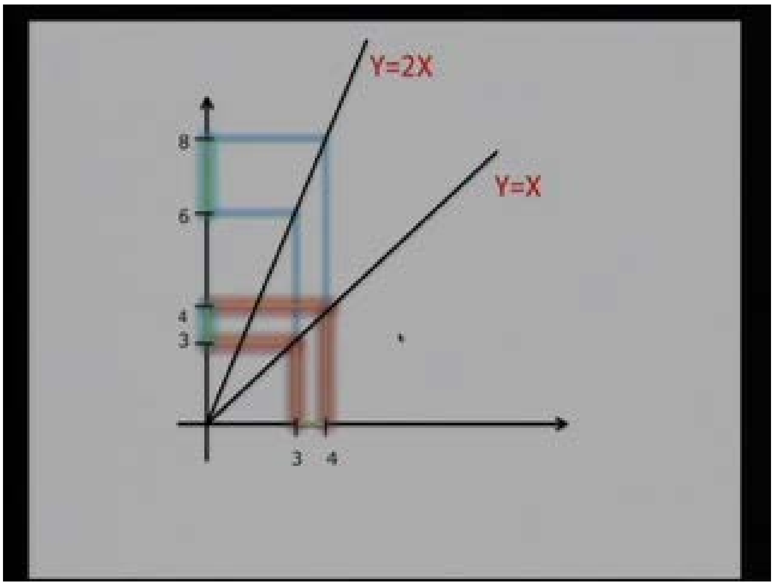

So, there are two curves here. Y equal to X and Y equal to 2 X. And, let us start from 3 and 4 and draw two lines and see, what is the corresponding X values. So, if the X values are 3 and 4, let us calculate, what is the corresponding Y values. So, if you draw two lines to Y equal to 2 X curve, the corresponding Y value will be 6 and 8. So, 3 and 4, 6 and 8. So, 2 times 3 is 6; 2 times 4 is 8. So, if you take Y equal to 2 X curve, and I ask the question that, if the d X is 4 minus 3, the change in X is 1, see, if the X changes from 3 to 4, the corresponding Y will change from 6 to 8. On the other hand, if you take, if you draw this line only up to this X axis, if you ask this question, if you take this Y equal to X curve, how much will the Y change? You will see that, Y only changes by a small amount. So, you can see that, this is bigger than this. If you start, if the X is changed by the same quantity, the change in Y is more for this curve, compared to this curve. So, the slope of Y equal to 2 X is bigger than Y equal to X.

As we saw, the slope of Y equal to 2 X is 2; or, the derivative of Y equal to 2 X is 2 and the slope of Y equal to X is 1. So, that is, you will see that, 3, 4 here, 3, 4 here. So, the d Y is 1, d X is 1; d Y by d X equal to 1 for this curve. d Y is 1, and the corresponding, sorry, d X is 1, the corresponding d Y is 2 here. So, d Y by d X is 2, for this curve. So, the slope for, the larger the slope, the larger the derivative, and this is some…Let me play this movie once more. Let us see this. I start, I am starting from 3, 4 and I am drawing lines, perpendicular lines to Y equal to the 2, to the Y equal to 2 X curve and ask the question, what is the corresponding Y value? For Y equal to 2 X, the corresponding Y value for 3 and 4, is 6 and 8. If you do this, for Y equal to X, and the corresponding Y value is 3, 4 itself.

So, now, if you calculate the slope or the derivative d Y by d X, you will get a bigger number here, because, see, even though the X, change in X is same, the change in Y is different. This has a bigger change here, compared to this. So, this idea, that the change in Y will be bigger, if the slope is bigger; and this idea is useful. This, this is useful in many contexts, we will learn this. The speed could be bigger, many other quantities could be bigger; we will, we will come back to the examples, where this is relevant. But, so far, when we learnt Y equal to X and Y equal to 2 X, we saw that, whatever value, wherever you take this, start from X axis, the slope was the same. Now, let us take an example, where this is not true. So, let us look at this next case, which is Y equal to X square.

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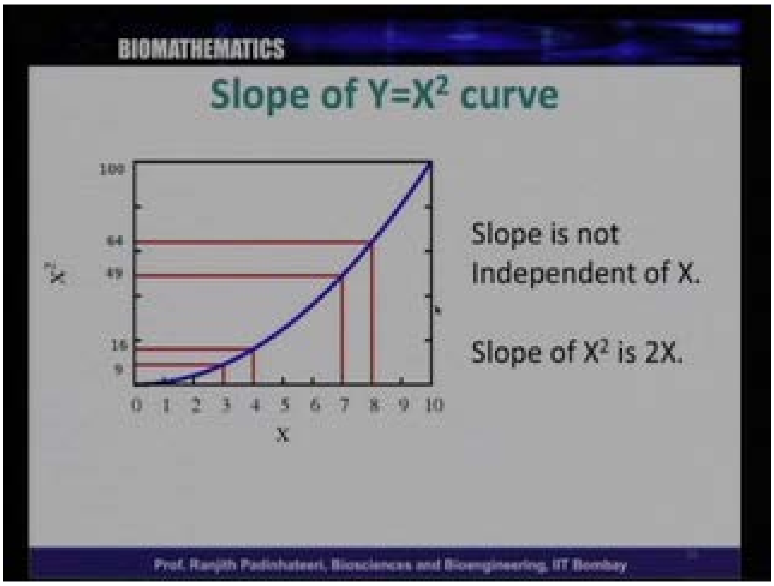

So, if you take this Y equal to X square curve, with a blue curve here, is Y equal to X square. In this blue curve, let us start from 3, 4 and draw two lines, perpendicular lines and calculate the corresponding Y values. So, since this is, Y values are a square of X value, the square of 3 is 9; the square of 4 is 16. So, let us try and understand this. So, have a look at this. So, you have a curve, which is Y equal to X square. So, we have 3 here; the corresponding value is 9, 3 square; and, if we have 4 here, and the corresponding value is 16. Now, let us calculate d Y by d X. So, what is d Y by d X? What is d X? So, d X is nothing, but this, this amount. How much is change in…

This is d X. This is nothing, but 4 minus 3, which is 1. So, the d X is 1. Now, what is the d Y? The d Y is this amount, this much, this much is the d Y. So, this much is the d Y. d Y is 16 minus 9, which is, which is 7. So...So, the value we got, 7. So, the d Y by d X is, d Y by d x. So, let us calculate here, d Y by d X, which is 7, which is d Y; this is d Y is 7 and d X is 1; this is 7. So, we found that, the d Y by d X is 7 here, when we calculated.

So, now, let us go to another place and you have a look at this. Let us now take, start from 7 and 8. So, the square of 7 is 49. So, the square of 8 is 64. So, now, let, in this case, let us calculate the d Y by d x. So, again the d X is 1. You varied, went from 7 to 8; the X is only changed by 1, the X is increased by 1. But the corresponding Y value was increased by 15, 49 to 64. Let us have a look at, here. So, if we had this curve, little more here, if we start from some value 7, we reached 49. If we start from 8, we reached 64. So, this d X was 1, the d Y was 15. So, this is 1. So, the d Y by d X, the d Y which is 15, by d X, which is 1. So, the d Y by d X is 15 by 1, is 15. So, d Y by d X is 15. So, you got 7 in the previous case and 15 here. So, depending on the X value, wherever you calculate, we got different answers.

So, we got, when we took the, sorry, when we took the X value between 3 and 4, we got the corresponding Y values between 9 and 16. When we took the X values between 7 and 8, we got 49 and 64, which is a 15 difference. So, what we should learn here is that, the slope of this Y equal to X square curve varies, as we go along the X axis. The slope here is small compared to the slope here. In other words, slope here is larger compared the slope here. The slope here is 15, as we saw, and the slope here is just 7. So, the slope, as we go along the X axis, as we calculate the slope here, it will be larger than the slope at some other point, at the bottom. So, the slope of Y equal to X square curve increases with X. So, this is something which we learn from, just by drawing, we learnt some idea. We learnt that, slope of Y equal to X square curve increases with X.

In other words, slope is proportional to X. Slope, the X increases, the slope increases. And, we will, we will learn this and so, that means, the derivative of Y equal to X square is proportional to X. So, it just tells out, the slope of X square is 2 X. In other words, derivative of X square is 2 X, as we see here. We will see, how we understand this. So, we will see, what does it mean, in the next class. So, but, at this point, just understand that, the slope here is not independent of X; it depends on the value of X; it is unlike the Y equal to X and Y equal to 2 X.

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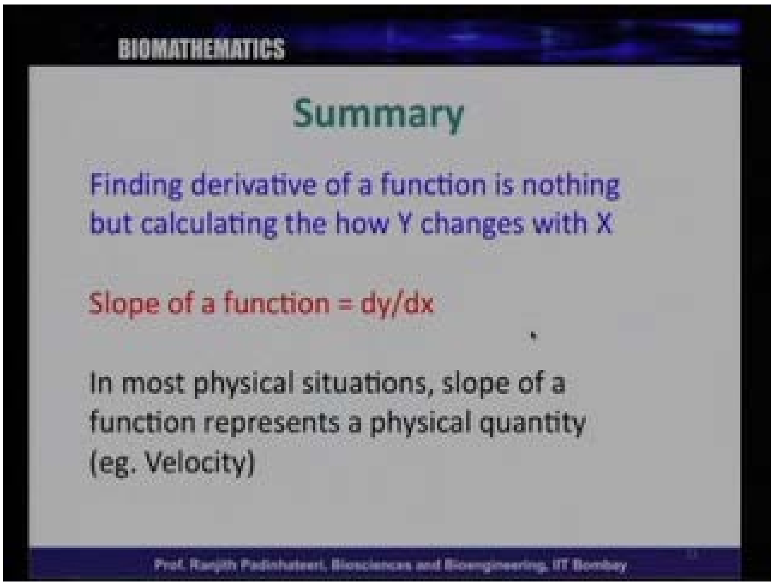

So, now, let us summarize what we learnt today. So, we, we learnt that, finding derivative of the function is nothing, but calculating how the Y changes with X; and the slope of the function is nothing, but d Y by d X. How much the change in Y, if the change in X is d x. So, if the change in X is d X and the change is, if the corresponding change is d Y, their ratio gives the slope or the derivative. And, in most physical situations, the slope of a function represents a physical quantity. For example, the velocity, if you just, when you are walking or moving with some, along a line, or, if you are, if you know the position versus time, if you know the position as the function of time, the slope of it gives the velocity or the speed. Now, we will, we will come and learn many more examples and the idea of derivatives in a much more better way, in the next class.

But the aim of this class, is just to introduce you the idea of derivative and the way to calculate the derivative. The way to calculate the derivative, as we saw, is nothing, but plot the function, calculate a d X and calculate the corresponding d Y, and calculate the d Y by d X. In other words, calculate the slope. Slope equal to derivative is the thing that we learnt today. And, in the next class, we will take more examples and get into detail and learn, how to calculate derivatives of different functions and why we should calculate the derivative at all. And, what we, should we learn by calculating the derivatives. The use of this derivatives will also, we will discuss as we go along. So, let us stop here today. Thank you.