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Lecture No. #05
Functions and its Derivatives

Hi. Welcome to this lecture of Biomathematics. In the last lecture, we have been discussing the idea of derivatives. We discussed functions and we also learnt, how to calculate, what is the definition of derivative. In this lecture, we will go ahead and learn more about derivatives, and how to calculate derivatives of certain functions. So, it is very simple, nothing complicated. So, we will, we will learn in the simplest way, how to calculate derivatives of many functions that we already learned. So, the lecture name is functions and its derivatives, and in this lecture, we will discuss how to calculate derivatives of various functions.

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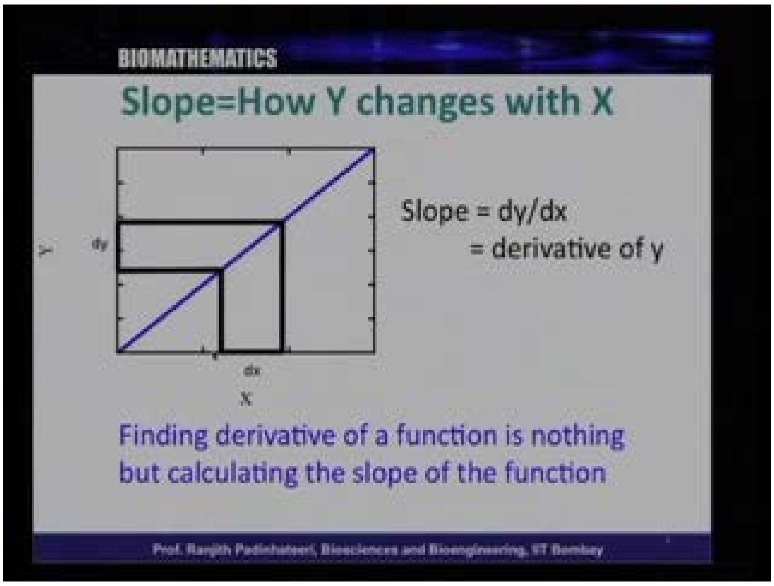

So, to briefly, before going to the details of how to calculate derivatives, we will quickly revisit, main, important points of last lecture, which is, first, we learnt this, that slope, the derivative is d y by d x, is nothing, but how this function Y, changes with respect to X. So, as you can see here, the, you have Y and X, and if you change X by a quantity d x, how much the y changes; that is measured by this ratio d y by d x and we call it derivative of y. So, and we also learnt that, the finding derivative is like finding the slope of the function. So, let us understand this a bit more carefully.

So, we had some functions. So, let us plot it here. So, we had some function, which was linear and we wanted to get the derivative. So, we took some point, x1 on X axis. So, this is XY axis and another point x 2 and we calculated, what is the corresponding y points; this is y 1 and y 2. So, for x 1, there is a corresponding y point, y 1; for x 2, there is a corresponding y point, y 2. So, and we said that, the derivative of y, with respect to x. So, we call it d y by d x. That is, how much y will change, if we change x by a quantity d x. So, what is the d x? d x here, is nothing, but change in x, which is x 2 minus x 1.

And, the corresponding y is y 2 minus y 1. So, this is d y by d x, y 2 minus y 1 divided by x 2 minus x 1. So, this is the derivative and this is the, it is also the slope of this, this straight line. So, this is what we mean by derivative. We can also express this in another way. This is also, so, in some books, you will see this treated in a different way and that is y at this point x 2. So, this y, corresponding to x 2 is y 2, minus y corresponding to x 1, divided by x 2 minus x 1. So, what does it mean? Let us understand this a bit more. So, y 2 is nothing, but y corresponding to x 2. For x 2, for a value x 2, what is the corresponding y value? Thus y of x 2 means, the y value corresponding to the x value x 2. And, y, if you subtract from that, the y corresponding to x 1 and divide it by the change in x 2 minus x 1, you get d y by d x. So, this is all the same thing, but it can be written in different ways. So, in some books, you will see it in a different way.

So, one of the important rules while doing derivatives that you should remember is that, if you look at here, this quantity x 2 minus x 1, so, this is d…So, the smaller this x 1 in, the closer the x 1 and x 2, the better this definition of derivative. So, the in strict...So, a strictly speaking, the definition of d y by d x, so, let me write it here. So, d y by d x is equal to y of x 2 minus y of x 1 divided by x 2 minus x 1. So, now, x 2 minus x 1, let me call it, let me call x 2 minus x 1 equal to d x; this is change in x. So, this is d y by d x. So, you can also write this. So, if you just look at this curve we had here. So, we had a straight line. So, if you look at these two points, two points on x axis and two points on y axis. So, I can call this x and this x plus d x. So, because this, and you can call this y at x, y value corresponding to the x value.

And, I can call this y corresponding to x plus d x. So, you can also write this definition in a different way. So, I can write d y by d x equal to the y of x 2 minus y of x 1, which is like y of x plus d x, which is this point minus this point, y of x. Again, this is nothing, but this, this distance. Divided by x plus d x, which is this minus x. In other words, y of x plus d x minus y of x divided by d x. So, this is another way of writing the derivative. Now, this is, strictly speaking, we would like to have the d x very small. That is, the more closer these two points are, the better this definition is. So, the correct, the accurate, the mathematically accurate definition is d y by d x is, when d x is very small, when d x is very small. So, we want, we want this, the more, the closer, the more accurate this will be. So, we want this d x to be very small, so that, the definition is proper, but for a straight line, it does not matter, the value of d x; but for complicated curves, the better, the closer you take these two values in the x axis, the, the proper, correct derivative one would get. So, strictly speaking, the definition of d y by d x is, mathematically, they call limit d x going to 0.

That means, the d x gets smaller and smaller, like as small as, very close to 0. So, this d x arrow 0 means, when the d x is very small, very small, close to 0, y of x plus d x minus y of x divided by d x. So, this is the definition of derivative. Essentially, what we learnt now, this is the same thing that we have been discussing, but just that, we should take the d x very small; that means, if you want to find the derivative of this curve at any point…So, let us say, you want to find this derivative or slope of this curve around this point. We should take two x values, very close to each other, like, very close to each other; this d x, the smaller, the better it is.

And, the corresponding y values you calculate, and if you calculate the d y by d x, you get the slope and this is the derivative. So, as, since we are trying to understand more about the biological significance and how this can be applied in Biology, we would not really go in detail into this technicalities of this definitions, but just keep in mind that, whenever you find the slope, whenever you find d y by d x, take the x values as close as you can. So, this is just, once you keep this in mind, I really, as we go, I will keep this in mind and whenever we calculate the derivative, we will keep this x values as close as we can.

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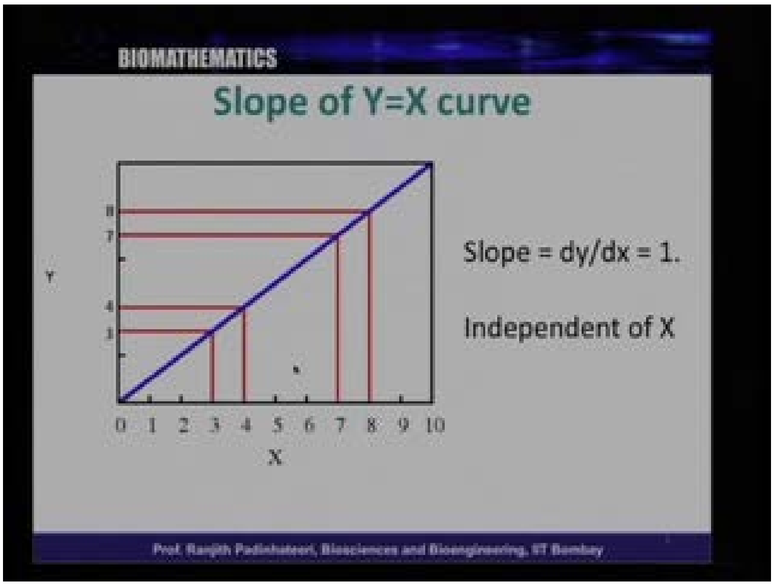

Ok. So, now, let us again look a little more to this, what we have…So, as we see here, d y by d x, you can calculate by, you know, putting down the d y and the d x and we saw last time that, whether you take d x here or here, for a straight line, you always get the same slope. So, the d y by d x, if you take this d x around this 3, between 3 and 4, or some other point 7 and 8, you will always get slope equal to 1; that means, the wherever the values, where you take along the x axis does not matter, as long as, when you find the slope of a y equal to x curve, or when you find the slope of a straight line, it does not matter, where you take the slope; it is always a constant.

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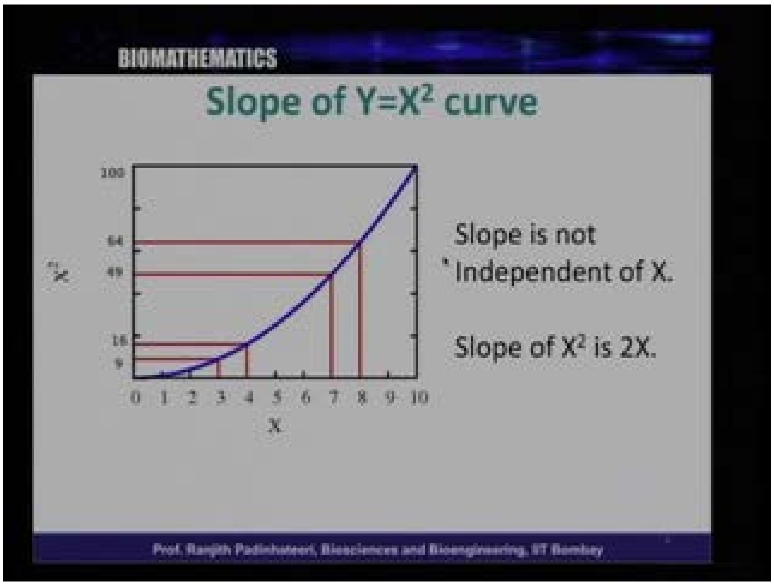

This is a property, only for a straight line. Thus you, you can see this by trying it your, yourself. But if you take another curve, like y equal to x square, which is not a straight line, which is a curved function, you will see that, whether if you take between 3 and 4, you will get one answer and you will get, when you take between 7 and 8, you will get different slope. So, the slope here, is different from the slope, the slope here, is different from the slope here. So, what does it mean is that, the slope changes as you go along the curve. So, as you go along the x axis, the slope of a curved line, so, slope of a quadratic function, y equal to x square function, changes. In other words, it is a, slope itself depends on the x value. So, now, the slope of x square is 2 times x. This is something which we learnt by just trying out, trying out this, by drawing this, we tried out. We, last time, we took 3, the corresponding value was 9; 4, the corresponding value was 16. So, we found that, 16 by 9. So, we, similarly, we could, we calculated all this values and we found that slope of x square is 2 x.

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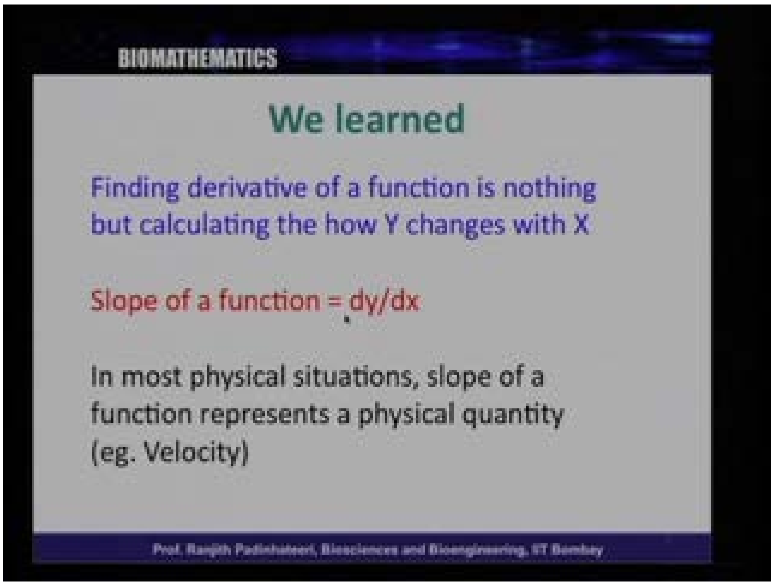

And, we also learned that, finding derivative of a function is nothing, but calculating the how y changes with x and in most physical situation, slope of a function represents some physical quantity. For example, velocity was one quantity, we discussed last time. We will discuss more and more quantities, but today, we will learn how to calculate the derivative of various functions. So, the only function or two functions we learned so far, how to calculate derivatives are, y equal to x, which we found as a constant slope, and y equal to x square. Now, we have, we know many functions and it is all very simple to calculate the slope of all these functions. We have to remember just two things. If you remember just two things, we can calculate derivatives of many, many functions.

So, we, when we started to study functions, we just learnt couple of functions. We learnt x, x square, x cube, etcetera and we wrote all of the functions as some combinations of this. Similarly, we will do a trick. We have to just learn the derivative of one or two functions and then, we are done. We can just use these derivative definitions and then, see, how other functions, calculate the derivatives of other functions. So, let us learn a simple rule of derivative and then, in fact, two rules we will learn; so, and we will use those two rules and then, calculate derivatives of many, many functions. So, this is so simple, much simpler than many other things you would learn in Biology. So, just the first thing you should learn is, derivatives of x power n.

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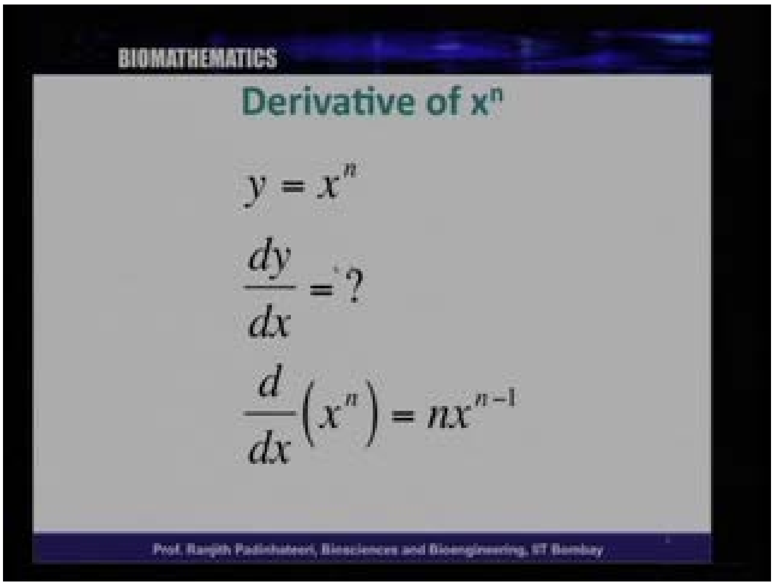

So, you had x, x square, x cube. So, n is any number. So, x could be 0, 1, 2, 3, 4. So, whenever you have a function y equal to x power n, what is the derivative? What is the d y by d x? So, this is the first thing you should learn and the rule is that, the derivative of x power n is n x power n minus 1. So, let us take some examples. So, we already saw some examples. So, we already saw the first one. So, when n equal to 1, x power n is x. So, the derivative of x is 1. So, let us see, this is, we, we know that, x power n, when a, for n equal, n equal to 1, x power n is x and d x by d x is 1. This much we know; it is a simple idea. Write the d x by d x. So, instead of y, so, y is x power n.

And, is equal to x, for n equal to 1. So, y equal to x power n, this gives you x, for x, n equal to 1. When n equal to 1, y is x and we want to calculate d y by d x, which is equal to d x by d x and this is 1. So, this is essentially, the slope of a line y equal to x. So, this is y equal to x and the slope of this line is 1, any point you calculate. So, now, let us see, if this applies to our rules. The rule we learnt today is that, the rule we learnt is that, d by d x of x power n is n x power n minus 1. So, when n equal to 1, what is this? What is n x power n minus 1? n x power n minus 1is 1. n is 1; x power n minus 1is 1 minus 1, which is 1 x power 0. X power 0 is 1. So, this is just 1.

So, as, according to the rule we learnt, the simplest function, which is y equal to x has a derivative, which is a constant. Now, let us look at this another function, the next function, y equal to x square. So, now, the rule is that, d by, d y by d x is n x power n minus 1. Here n is 2, because x power n. So, n is 2; so, that means, d y by d x is n, which is 2, x power 2 minus 1, which is 2 into x power 1,which is 2 x. So, derivative of x square is nothing, but 2 times x. So, this is the application of this rule, when n equal to 2. Let us apply this rule when n is equal to 3. So, now let us say the next case, n is equal to 3; that means, y is x power 3. d y by d x, according to the rule we learnt, d y by d x is n x power n minus 1. So, n is 3. So, 3 x powers 3 minus 1, which is 3 times x square. So, d by d x of x cube is 3 times x square. So, we learnt this much. So, once we understand this rule, we can use this many, at different places and then, learn derivatives of many other functions, because as we know, many functions are combinations of x, x square, x cube, etcetera.

So, now, according to this rule, let us also see what happens, when n equal to 0. So, let us look at this. So, let us look at what happens, when n equal to 0. What is the n equal to 0 means? n equal to 0 means, y is x power 0. X power 0 is 1. So, when y equal to 1, what is d y by d x? This is n x power n minus 1. So, this is n is 0. So, 0 times x power n minus 1; 0 times anything is 0. So, the answer is 0. So, derivative of 1 is 0. In, it can be generalized that, derivative of any constant is 0. So, derivative of any constant k is 0, where k is any constant, any number. So, derivative of any number is 0. This is can be, again the, we are not learning anything new. We just applied the rule we learnt. So, the rule we learnt, as we see here is that, d by d x of x power n is n x power n minus 1. This is the golden rule that we should remember.

Once we remember this, whatever we said so far, all follows from this just one line. So, if we just remember this one line, everything we saw so far in this lecture, all follows from this just one rule. Now, let us also, just learn a bit more, slight variation of this. So, let us say y is equal to 3 x. Then, what happens, when y equal to 3 x? So, we have to find out d y by d x. So, according to this rule we can, so, it, it turns out that, we will tell this details of this rule later, but it turns out that, the derivative of, we can take keep this constant there and 3 d x by d x. So, this is 3.

So, you can keep the constant there and calculate the derivative of this function and then, that is the answer. So, in other words, we can extend this to many other things. Let us say, y is, in general, k x power n, where k is a constant and n is the number, which is 1, 2, 3, 4, that we saw so far and x is the x variable that we took. Then, the d y by d x and it turns out that, this is equal to, you keep the k there, and then, d by d x of x power n. And, we know that, what is d by d x of x power n is. So, we know what this is. This is nothing, but k times n x power n minus 1. According to what we learnt so far, d by d x of x power n is n x power n minus 1. So, the d y by d x of k x power n is k times n x power n minus 1.

So, this is a simple extension of the rule, what we learnt, that, whether you multiply, when you multiply with a constant, it does not matter; that constant just stands there and you find the derivative of the function. So, this is a constant times any function. So, the constant stays there and you find the derivative of the function; that is the rule in mathematics. Now, let us look at the next rule. So, it is one more thing which we should learn. So, the first rule we...So, let me reiterate the rule, the first rule we learnt that, the generalized d y by d x, when y is k times x power n, d y by d x is k times n x power n minus 1. This is what we learnt now. Now, let us look some other simple functions like, what happens if, if y is sum of two functions, x square plus x power 7.

Because, we learnt many functions which are some combinations of x square, x cube, x power everything. So, what happens when these two functions, when y is a sum of two functions, right. So, what happens, when y is sum of two functions? It turns out that, the derivative is equal to, you can find this derivative of this and this separately and sum them. So, the first, the derivative of the first quantity. So, this is the derivative of this function, plus the derivative of the second function and this is the answer. So, when you have derivatives, when you want to find out derivatives of two functions, derivative of sum of two functions, you can find the derivative of each function and sum them. So, in general, we can write this, like, what I wrote here. So, if you want to find out the derivative of two function f of x and g of x. So, this is, let, we can call this f of x. So, this x power 7, I call this f of x, some function f of x. x power 2 and this I call, some other function g of x and this is, f of x plus g of x is this, sum of this, these two things and the derivative of this is equal to the derivative of the first function plus the derivative of the second function.

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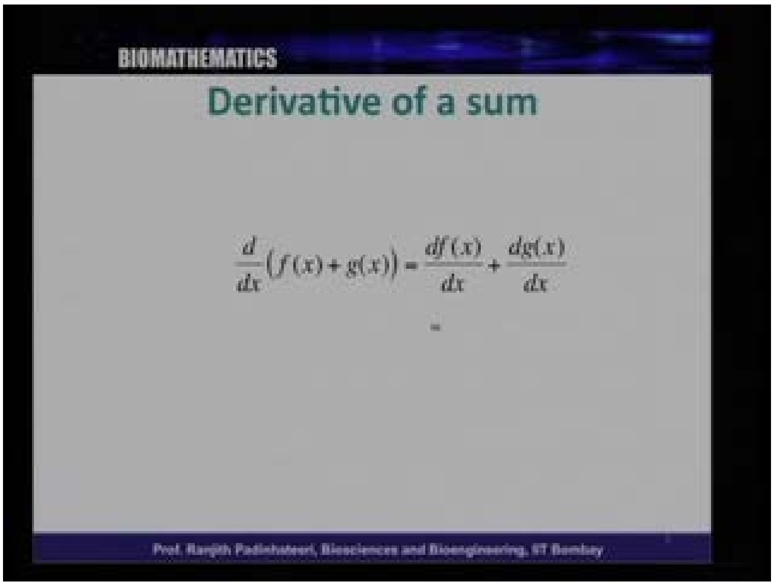

So, this is the second rule that you should learn. So, there are two rules, which you should learn. Derivative of x power n and derivatives of sum of two functions. If you know this much, this is enough to learn many, many things. So, let me just write down these two things once more. So, let me just write down these two things, once more. The two rules we should learn. Rule 1is that, derivative of x power n, d by d x of x power n is n x power n minus 1. The second one you should learn is that, derivative of sum of two functions, if you want to find out derivative of sum of two functions, so, x power 7 plus x power 9, this is derivative of x power 7 plus derivative of x power 9.

So, this is the second rule, derivative of…So, this rule, if I write in words, this says derivative of sum of two functions is equal to sum of their derivatives. So, this is what we saw. So, derivative of two functions, sum of two functions x power 7 and x power 9, those are the two functions. And, if we want to find the derivative of this sum, if you want to calculate that, we can take the derivatives of this individual functions separately and sum them. So, derivative of sum of two functions equal to the sum of their derivatives. So, these are the sum of their derivatives and then, this is the first derivative, second derivative, sum of these two, you get the answer. So, if we know these two things, this is enough for us to learn many things. So, we will go ahead and see a few things. So, as we saw, if we know these two rules, as we, this, this rules written here, if we know this two rules, rule 1 and rule 2, we can calculate derivatives of many functions and let us take the example of the exponential function.

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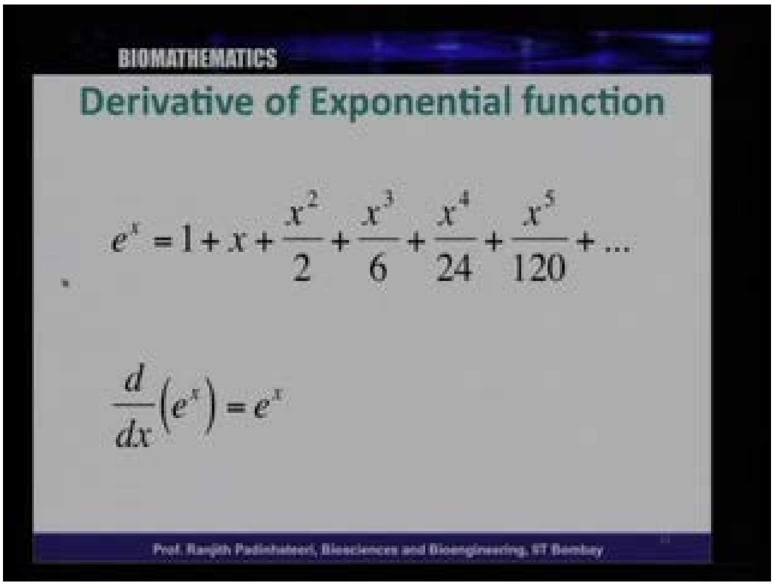

So, let us look at here. So, the exponential function e power x is nothing, but 1plus x plus x square by 2 plus x cube by 6 plus x power 4 by 24 plus x power 5 by 25 plus so on and so forth up to infinity. So, this is sum of infinite terms and let us say, you want to find out the derivative of this. So, now, why I am reaching this e power x is the, this is important. If you want to calculate derivatives of some functions in Biology, like growth processors or d k processors, many of these processors are represented by exponential functions and one should understand this exponential function. We saw 2 power x as examples of bacterial growth. May, we saw 10 power x related to log. So, many of these functions are very much used in Biology and if we want to calculate the physical quantities from that, one should understand how to calculate the slope of these functions.

So, that is why, I am reaching to this e power x; that is why I want to teach you, I want to slowly take you to this e power x and functions, which is used very often in Biology, like sin x, cos x. They are periodic functions, which is used to represent many phenomena, biological phenomena, like, we have many functions that repeat, many things that repeat periodically, in our day to day life, as well as in Biology. So, that is why, I want to take you to e power x. So, as we know, e power x is nothing, but so…

So, let us, let me write this here, e power x is 1 plus x plus x square by 2 plus x cube by 6; 6 can be written as 2 into 3, plus x power 4 by 24; 24 can be written as 2 into 3 into 4, this is 24, plus so on and so forth. This is the definition of e power x. Now, let us calculate the derivative of e power x; that means, d by d x of e power x. This is what we want to calculate; that means, we want to calculate derivatives of…So, now, what is e power x is sum of all these functions. So, one minute ago, we learn this important rule that, derivative of sum of two functions is nothing, but sum of their derivatives.

So, let us just go and learn this rule quickly. So, what we learned here is that, derivative of sum of two functions is equal to sum of their derivatives. So, instead of, if we want to find out derivatives of this sum, what we need, should do is that, we should find the derivatives of each of them and sum them. Simple. So, we know, what is the derivative of 1; we know what is the derivative of x. We can find out, what is the derivative of x square. We can find out, what is the derivative of x cube. We can find out what is derivative of x power 4 and so on and so forth and we can sum them and we will get the derivative of e power x. So, let us try and do this. So, first, let us find the derivative of 1. So, to do this, we will…So, just few minutes ago, we saw…So, the next rule, the first rule we learnt, we will be using here. The rule we saw is that, d by d x of x power n is n x power n minus 1. So, we will write this rule here, in the side here. So, d by d x of x power n is equal to n x power n minus 1or let me write this rule here at the bottom, so that, it is clear. So, d by, rule is that, d by d x of x power n is n x power n minus 1.

So, when n is equal to 0, you get x power 0, which is 1 and derivatives of 1 is 0. So, derivative of 1 is 0. So, this is the derivative corresponding to 1. Now, the derivative of x. So, what is derivative of x? x is nothing, but when n equal to 1. When n equal to 1, this 1 x power 1 minus 1, 0. So, derivative of x is nothing, but just 1. Now, what is the derivative of x square by 2? Derivative of x square by 2. So, derivative of x square is...So, 1 by 2 is a constant. So, we will keep this constant as it is, 1 by 2, and derivative of x square, according to this rule is 2 x. So, this is 2 x. Now, what is the derivative of x cube? Derivative of x cube is 3 x square and whatever the denominator remains, 2 times 3. What is derivative of x power 4? It is 4 x power, x cube, 3, according to this rule, it is 4 x power 3 and whatever in denominator remains that way, plus so on and so forth. So, let us rewrite this. So, 0 is, 0 plus anything is that itself. So, we need not write 0. So, we can write 1 here. So, this is essentially 1plus, this 2 and this 2 cancels. So, essentially you get x.

This 3 and this 3 cancels. So, you get plus x square by 2; this 4 and this 4 cancels. So, you get x cube by 2 into 3, plus so on and so forth. So, you just, you can keep doing the sum. So, what did we get finally? So, let us compare what we got here and what we had here. So, we got 1 plus x plus x square plus x cube by 2 into 3 and so on and what do we have here, 1 plus x plus x square by 2 plus x cube by 2 into 3 and so on. So, this and this, are exactly the same. If we just take the next term, x power 5 term we had and find the derivative, we will get this. So, since this is an infinite term, you will get exactly this as this. What does this mean? This means is that, derivatives of e power x is e power x itself.

So, derivate...So, this is one thing we learnt. We learnt that, derivative of e power x is e power x itself. So, this is an exercise for you that, what you should do, whatever I wrote here, you should take a paper and take many terms, up to 20, up to x power 20, and try out. Take any, any text book of mathematics, or even the logarithm, the tables we have and find out the exponential function, for many terms, 5, 6, 7, 8, 9 terms, up to 10 terms maybe, and then, calculate the derivatives according to this two rules we know, and see what you get; and, you, you will get that, d by d x of e power x is e power x itself.

So, you will get that, whatever you start with, is the same thing you end up with. So, you should take this and use these two rules we learned and convince yourself that, derivative of e power x is e power x. So, the only way you can learn this is that, take a paper and do it yourself. Then only, you will be convinced. Once you are convinced, it is very simple, like, you get much better feeling. So, what is this graphically means? Derivative of e power x is e power x, what does this graphically means? So, slope, derivative is slope. So, slope of e power x is e power x itself. So, what does this mean? This means is that…So, we all know that, e power x, if you plot, looks like some function which increases very fast. So, now, if you find the slope at any point, at any point x, if you find the slope, the slope is e power x itself.

You calculate e time's e power that power, slope at any point is same as that function itself; that is what it means. So, this is also, you can try. You can draw an exponential curve, calculate the slope at different points. So, take, take a graph sheet, draw e power x yourself. Take, go to a point and calculate the slope there; calculate d y by d x. So, calculate the slope there, calculate, this is d y and this is d x and calculate the d y by d x and it will be e power x. So, take some other point, calculate d y by d x, you will get the value e power x. So, you should try this graphically and convince yourself also that, derivative of e power x is e power x itself. Now, the next thing we should learn is derivative e power minus x.

So, once we know e power x, we can easily learn e power minus x. So, let us look here. e power minus x is nothing, but 1 minus x plus x square by 2 minus x cube by 6 plus x power 4 by 24 by, minus x power 5 by 120 and so on and so forth. So, if we know this sum, just like what we did before, we can calculate the derivative. So, we can calculate the derivative of each of these term and then, sum up all of them and you will get the derivative of e power x. So, let us quickly try doing this also. So…So, let us just, quickly do this. E power minus x is 1 minus x plus x square by 2 minus x cube by 6, 6 is 2 times 3, plus x power 4 by 24; 24 is 2 times 3 times 4, minus x power 5 divided by 120; 120 turns out to be 2 times 3 times 4 times 5. So, now, if you find out the derivative of this…So, you want to find out d by d x of e power minus x. So, you can find out the derivative of each of this term. So, derivative of 1 is 0. This is the derivative of 1 and you have minus derivative of x is 1; derivative of x square is 2 x. So, you have plus derivative of x square is 2 x.

We have divided by 2. Derivative of x cube, so, minus sign, I repeat that, derivative of x cube is 3 x power 2; n x power n minus 1, where n equal to 3, divided by 2 into 3, plus x power 4 is, derivative of x power 4 is 4 x power 4 minus 3, which, 4 minus 1, which is 3, divided by 2 into 3 into 4 plus so on and so forth. That is minus here, the next term is minus. So, the plus and minus will keep repeating in it, and the terms will continue. So, now, if you rewrite this, what do we get? We get first term is minus 1; the second term is, this 2 and this 2 cancels. So, it is just plus x and the third term is, this 3 and this 3 cancels. So, you have minus x square by 2 and the next term, this 4 and this 4 cancels and what you have is plus x cube by 2 into 3 and so on and so forth.

So, if we compare this and this, what do we get? You have 1 here, minus 1 here; minus x here, plus x here; plus x square by 2 here, minus x square by 2 here; minus x cube divided by 2 into 3 here, plus x cube divided by 2 into 3 here. So, what you have at the bottom is nothing, but whatever here, multiplied with a negative sign. So, it tells out that, this is nothing, but minus e power minus x. Whatever was there, multiply with a minus 1. So, you multiply all these term with a minus 1, you get it minus e power minus x. So, it turns out that, derivative of e power minus x is minus e power minus x. So, this is another thing which we can learn, just by learning the two rules we said that, derivative of x power n is n x power n minus 1, and that the derivative of sum of many terms is nothing, but sums of their derivatives. Simple. So, now, we, one can generalize this and write little more general thing that, derivative of e power k x. So, if y is e power k x, derivative of y, which is d y by d x and it turns out that, it is k e power k x.

So, you will learn this. So, you should convince yourself that, this is true, but the, this is the derivative of e power k x, where k is any number and the derivative you get is k e power k x, where k could be minus 1; that is what we saw, when k is minus 1, e power k x is e power minus 1. When k is plus 1, e power k x is e power x. You, when k is 2, e power 2 x, and the derivative is 2 e power 2 x. So, derivative of e power 2 x is 2 e power 2 x, that is what this means. So, this is the general rule. So, using this idea, now, we will learn derivatives of few other, the few more other functions, like the periodic functions we learned in Biology, like sin x and cos x and we will stop today's lecture after that. So, so, let us learn derivative of two more functions quickly, sin x and cos x. So, we will quickly just, just mention the derivatives sin x and cos x and then, we, in the next class we will, in detail, we will show that, the derivative, how exactly one can get this derivatives and what does this mean.

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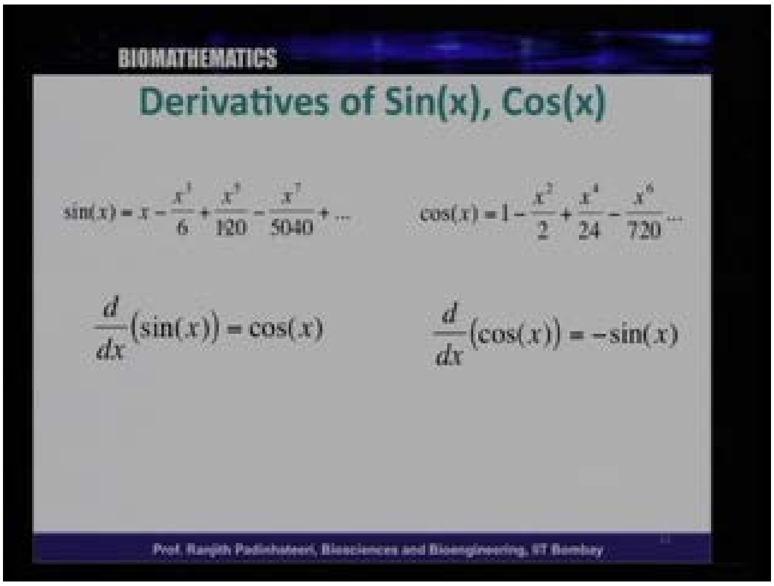

So, so, derivative. So, we, as you see here, last time, we have seen this, the function sin x and cos x, they are periodic functions and they can also be written as sum of, power, sum of series of x, x square, x cube, etcetera, etcetera. So, basically, sin x is x minus x cube by 6 plus x power 5 by 120 minus x power 7 by 5 0 4 0 plus, so on and so forth. And, the cos x is 1 minus x square by 2 plus x power 4 by 24 minus x power 6 by 720 and so on and so forth. And, it turns out that, the derivative of sin x is nothing, but cos x and derivative of cos x is minus sin x. So, this derivative of this function is this function and derivative of this function is negative of this function. So, this, you can convince yourself; you can just take these and just like what we did now, take this sum, take this series and find derivative of each terms using these two rules we learnt, get. We have to just use this two rules that, if a, first that, you have to find individual derivatives and sum them; and the other rule that, derivative of x power n is n x power n minus 1. If we use these two rules, we can find the derivative of sin x and cos x.

And you will see, you will see that, sin x derivative is nothing, but cos x and the derivative of cos x is minus of sin x. Now, you should do this and convince yourself and in the next lecture, we will discuss, what is the class? What is the significance of this and graphically representing the sin x and cos x and this derivative, what does it mean? Does it have some other meaning, physical meaning, etcetera, we will discuss. But, for the moment, we should learn how to calculate this mathematically.

And, how to get (( )) of this derivatives and once you know this, we will learn the significance of this, in the next lecture. So, to summarize today's lecture, we should just revisit those two rules which we saw. So, these are the two rules which we learnt and we can raise up these two rules as a summary of today's lecture. The take home message, the things that you should take home from today's lecture is that, the derivative of x power n is n x power n minus 1 and derivative of sum of functions x is basically, sums of their derivatives. So, if we just know these two rules, rule 1and rule 2, you can calculate the derivatives of many known functions and we will, we will use this in the next class, to learn many more functions. So, we will stop here today.