Biomathematics
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Lecture No. # 06
Differentiation and its application in Biology

Hi. Welcome to this lecture of Biomathematics. We have been discussing differential calculus, basically, how to calculate derivatives of a, of different kinds of functions. So, we learnt the idea of derivative and calculating them, for a few functions. We will continue with this today and learn a couple of examples from Biology. So, today, we will briefly discuss, a bit more about derivatives and the continuation of what we, where we stopped last time and other end, we will discuss two examples from Biology, where the idea of derivatives is directly applied.

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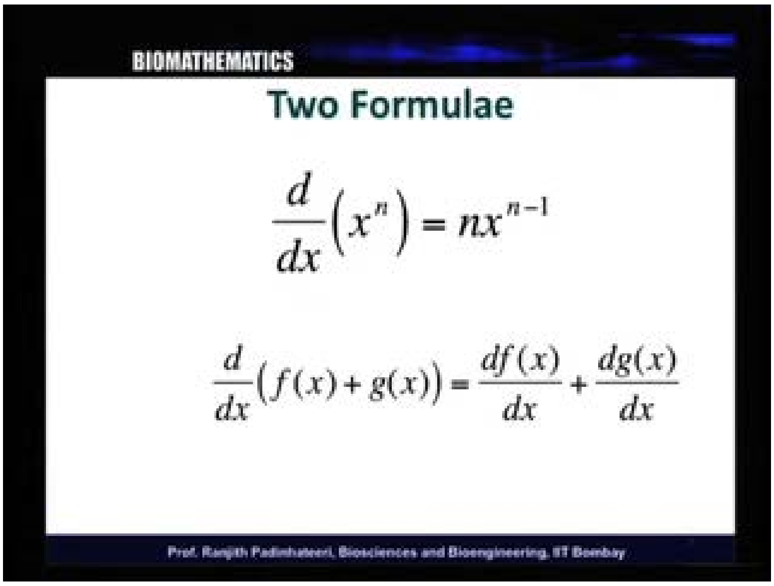

So, let us start. So, the title of today's lecture is Differentiation and its application in Biology. And, last time, we learnt a formula, actually, two formulae, as you see in the screen here that, d by d x of x power n is equal to n x power n minus 1; this is the first formula that we learnt. Then, we learnt the second formula which is, when you calculate the derivative of sum of two functions; f of x and g of x are two functions of x and if we have sum of them, f of x plus g of x and find the derivative of the, derivative of this sum, the answer is basically, you can find the derivative of this individual function and sum them. So, this is the second formula that we learnt and there is one more important formula that one should understand, one should learn, so that, you can do the derivative or you can differentiate any function, pretty much any functions. So, this is the third formula, the, that we are going to learn, and this is very interesting formula.

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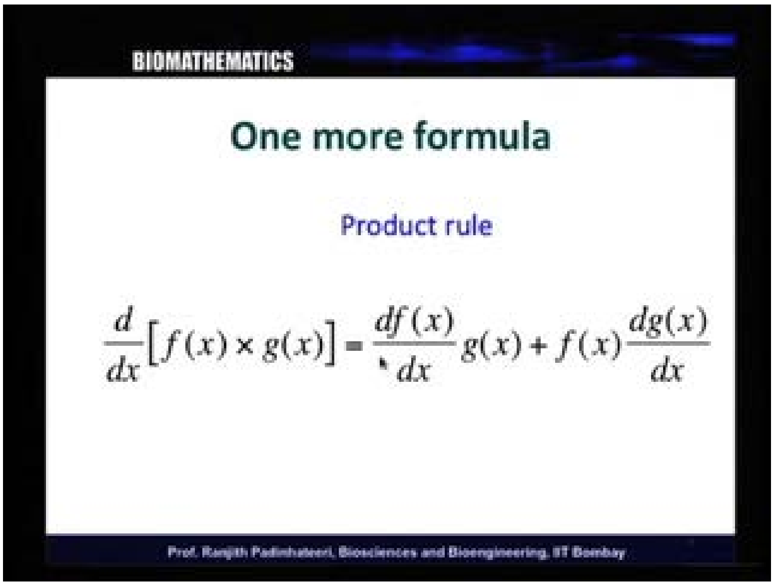

So, have a look at the screen. And, so, this is called product rule. (( )) is a product rule, it is like a product of two functions. F of x and g of x are two functions and the product of this two function is f of x times g of x; so, this into mark, this is times. So, f of x times g of x. So, the product of two functions and if you want to find the derivative of this product d by d x of f of x times g of x. (( )) The idea is very simple. First, find the derivative of f of x and multiply with g of x and find the, sum it with, finding the derivative of g of x and multiplying with the f of x. So, this is simple. First, find derivative of f of x, d f by d x; multiply it by g of x, plus, f of x into d g by d x. So, this is the product rule of differentiation. If you understand this, you can learn a lot of things. Just by extending this, just by using this, just one formula, if you remember this, just one formula, you can keep using this and learn or find out derivatives of many, many functions actually. So, let us take the simplest examples for this formula.

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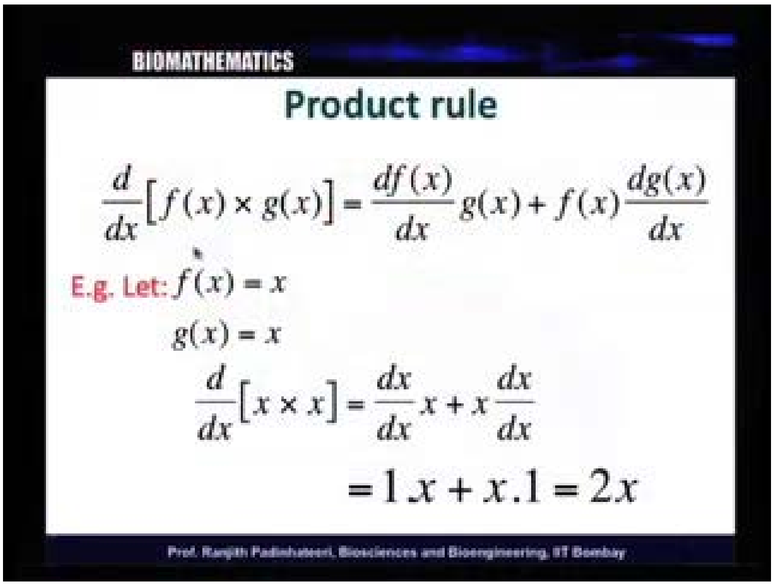

So, let us look the formula once more. The formula says f of x times g of x is a product and its derivative d by d x of f of x times g of x is equal to derivative of the first function d f by d x times the second function g of x, plus, the first function f of x times multiplied with the derivative of the second function, which is d g by d x. So, this is the formula, and now, let us take the simplest example here. So, the simplest example is f of x is x itself; g of x is x itself. So, x times x is x square. So, last time, if you remember, we learnt the derivatives of x square. So, we are going to calculate d by d x of x times x; that is, we are going to get…That is, basically, nothing, but d by d x of x square. This is what we are going to calculate. D by d x of x square, this is what exactly we are…And, we learnt, from the last time, last time, we had this rule of, this is a rule, which we just said that, d by d x of x power n is n x power n minus 1.

Using this rule for n equal to 2, it is clear that, the derivative of this is, d by d x of x square is 2 x; n is 2, x power n minus 1, 2 minus 1 is 1; this is 2 x. So, from that, what we learnt last time, we already know the answer that, the derivative of x square is 2 x. Now, let us apply today’s rule. The rule we learnt is, d by d x of f of x times g of x is d f by d x into g of x plus f into d g by d x. So, this is what we learnt. Now, let us apply this. f is...So, in our example, in the today’s example, the first example we took is, f of x is x and g of x is x. So, this is x into x. So, now, let us say...So, d is the…

So, basically, we are going to find out d by d x of x into x. Now, d f by d x, f is x. So, d, this is, f is x. So, d x by d x is a first term and g is also x, plus f is x, g is also x. This is what we get. So, now, what is d x by d x? So, we all know, what is d x by d x. This is nothing, but 1. So, we know d x by d x is 1 and this is 1. So, this is 1 times x plus 1 times x. So, let us look at the screen. So, d by d x of x times x is d x by d x into x plus x into d x by x. There is no magic here. I just applied this formula. Once you apply this formula here, once…I took this formula and applied this to this. So, you get this, if I apply this formula here, I get d x by x into x plus x into d x by d x; I am noting that d x by d x is 1.

We get 1 times x plus x times 1 which is 2 x. So, we get the same answer. So, basically, this is basically, x plus x equal to 2 x. This is, this is something. 2 x is the answer; same answer we got here. So, basically, by just doing this, we get the answer, we know. You can continue doing this for any other function, in any, any function that we learnt and do it many, many times and you will, you will see that, you can calculate, just by knowing d x by d x is 1 and this rule, you can calculate derivative of any function. And, the maximum you will have to apply the other the sum rule we learnt, in some particular cases. So, if we know the sum rule and the product rule, and the first, derivative of x power n, you can calculate derivative of any function.

So, let us apply this product rule to some other cases. So, let us apply the product rule to some cases. So, now, let us take the derivative of a constant times x. So, according to the product rule, you have two functions A and, two things A and x, the product of A and x. So, the derivative of this will be, d A by d x times x plus A times d x by d x. So, take the first one; derivative of the first one, multiply it with the second, the x, plus, keep the second one, A constant and find the derivative of this, which is d x by d x. Now, A is a constant.

So, in our case, A is a constant. It is like example 2. Let us say, A is just 2. Then, d, 2 does not change with x; 2 is just a constant. If you, since…So, d A by, the…If A is a constant, change in A is basically zero; A cannot change, because it is just a constant. So, d A by d x is, change in A looks 0. So, if this is 0, this term becomes 0. Then, your answer is just A times d x by d x, which is 1. So, the answer is A. So, d by d x of A x is just A. So, this is something which we can learn, just by applying our product rule. You can extend this. You can apply something else. You can say d by d x of A is another, A is constant, x square.

This is nothing, but you can, as you know is, you can write d A by d x into x square plus A times d by d x of x square. As I said, A is a constant. So, derivative of A is 0. So, this term is 0; this is, 0 times anything is 0. So, the first one is 0, plus, A d by d x of x square is 2 x. So, this is A times 2 x. So, this is basically, 2 A x. So, d by d x of A x square is 2 times A times x. You can keep applying this formula to any function you like and you will see that, you can, you can learn interesting things.

So, let us just take one more example, some other, some more examples. We will come back to this. But another thing that we learnt last time, is e power x. So, let us quickly remember what we learnt. will We know that, e power x can be written as, the function e power x can be written as 1 plus x plus x square by 2 plus x cube by 6 and so on and so forth and the derivative of e power x is e power x itself. So, this is something that we learnt last time, the derivative of e power x is e power x itself. Just revisit what we learnt last time, using this product rule, like…So, just go back, as an exercise, just go back to the lecture of, slides of last lectures, the last lecture, in fact, when we found the derivative of e power x and try using the product rule. It will help you.

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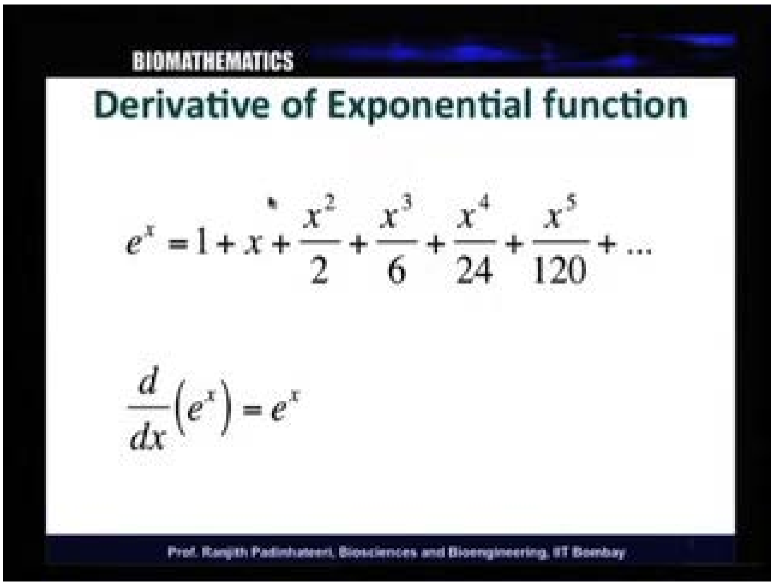

So, let us, let us think about it. Say, if you want, let us say, the function is 1 plus…So, let us just quickly, redo this. So, we said e power x is 1 plus x plus x square by 2 plus x cube by 6. So, now, d by d x of this is, d by d x of e power x is nothing, but d by d x of this whole thing, which is 1 plus x plus x square by 2 plus x cube by 6. Now, as the sum rules says that, if you want to find the derivative of sum, you can take the derivative of each of this individual terms and sum them. So, d by d x of the…So, the first one is, d 1 by d x which is 0; second one is d x by d x; third one is d by d x of x square by 2 and this one is, d by d x of x cube by 6.

This is what it is. Now, the first, this here, it is 1; this is 1. So, d 1 by d x is 0, because 1 is a constant. Just like we did A, 1 is a constant; change in 1 is 0, because there is change in, 1 cannot change. Now, the second function, let us look at here, d x by d x. d x by d x is 1. So, this is 1. Derivative of d x x is 1, plus, now, we want to calculate derivative of x square by 2. So, now, here, we can use the product rule. So, x square by 2, x square by 2 is basically, d by d x of half into x square.

So, half is like A, we had; half as a constant times x square. Then, you can use again d by d x of 1 by 6 times x cube, 1 by 6 is the constant. So, 1 by 6 times x cube. So, you can use this product rule like d by d x of A times x square and d by d x of A times x cube and you will see that, you will, you will see that, finally, you will get the answer, e power x itself. So, each of the...By using the idea of infinite series, you will get that, e power x itself. So, just, just as an exercise, just redo this, what we learned last time. So, we also learnt last time, we also mentioned that, derivative of sin x and cos x.

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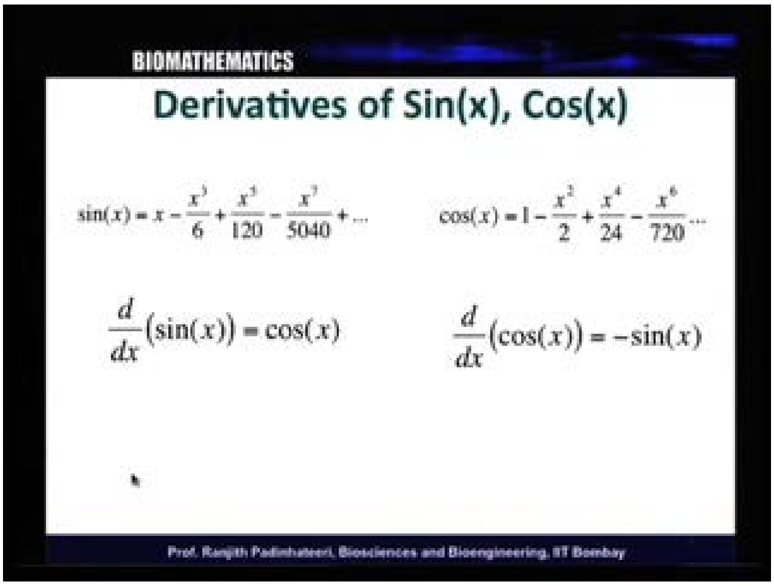

So, what is derivative of sin x and cos x? So, we learnt that, look at this, we learnt that, sin x is x minus x cube by 6 plus x power 5 by 120 minus x power 7 by 5040 and all that, and cos x is 1 minus x square by 2 plus x power 4 by 24 minus x power 6 by 720 and so on and so forth. So, derivative of sin x, if you use the, say this three rules, which we learnt, the d by d x of x power n, the first formula that we learnt, use the second formula, which is d by d x of sum of few functions and use the third one, d by d x of products of functions, if we use this formula to this sin x and calculate derivative, you will get cos x. Just try doing this. And, if you do that for cos x, you will get this minus sin x. So, I will not do this in this, but just like we did e power x last time, you could just continue doing this, just try doing this and you will get the answer. So, now, let us today, try and understand what does this mean. What does it mean to say that, derivative of sin x is cos x? What does this mean?

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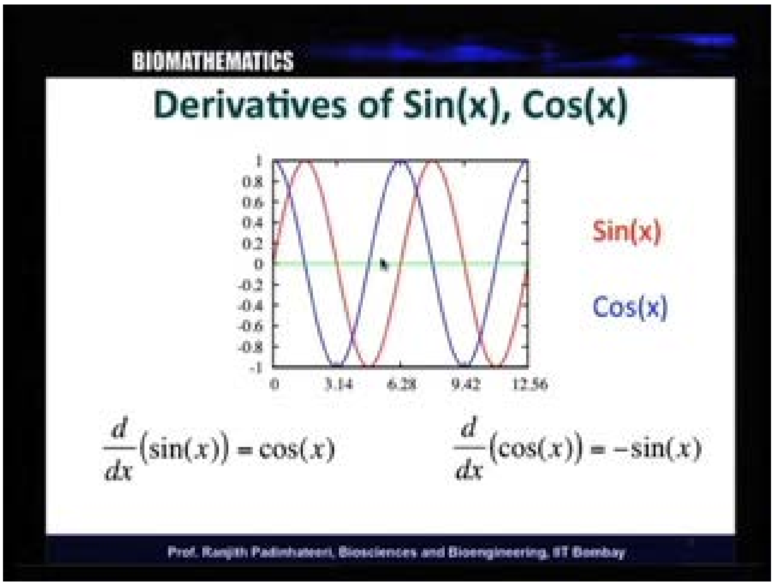

So, for this, first, let us look at this graphically. So, let us, in the next slide, we will look at the plot of sin x and cos x. So, have a look at here. So, what is plotted in red is sin x. At x equal to 0, it is 0 and when x equal to pi by 2, 3.14 is pi. So, this is around 3.14. This is, this is pi; this is pi by 2, which is 90 degrees. So, when x is 90 degree, d by d x the sin x is basically, 1; sin 90 is 1 and sin 180, which is pi by, sorry, sin, this is 3.14, which is pi. So, sin pi, which is 180 is 0; sin 3 pi by 2, which is between pi and 2 pi, is 3 pi by 2, which is minus 1 and again sin 2 pi is 0. And, you can see this, the red curve represents sin and the blue curve represent cos, cos of x. So, look at it. For 0, cos 0 is 1.

So, starting from 1, cos pi is 0; again, cos 2 pi is 1. So, sorry, cos pi is minus 1; cos 2 p is is plus 1 and cos pi by 2 is 0. So, you can see that, the blue curve represent the cos of x and the red curve represents sin of x. Now, what our claim is that, we claim that, derivative of sin x is cos x, we claim that. We also claim that, derivative means slope. We learnt that derivative means slope. So, let us think about this. So, this. What does it mean that, if you calculate the slope here, at this local point here, you will get the slope is 1. And, you can see that, this is, look at, concentrate only on the red curve. So, just look at only the red curve. So, when you look at the red curve, you can see that, this is a positive slope. What does it mean?

So, if you look at the red curve, if you just look at, the red curve has something like this. So, this is x and this is y. As x increases, y is also increasing. So, you can see this, red curve is something like this. So, the, in this part, here, this part, as x increases, y increases. So, d y by d x, which is, as x changes, does the y change or not; if y also increases as the x changes, d y d x is greater than 0; it is positive. What does it mean? There is a positive slope. This slope is greater than 0. So, that is what it means. So, now, try, have a look at, just plot this and see, whenever x and y are increasing, the slope is positive.

So, this is a rule, for even for a…You might have seen that, for a straight line, which has, looks like this, the slope is positive. The slope is positive slope. You might have also seen that, if the straight line looks like this, here, this is x and this is y; as the x increases y is decreasing; for this value of x, for this particular value of x, this is y value; for this particular value x, this is y value. So, as x increases from here to here, but the y decreases from here to here. So, as you can see, this has a negative slope. So, let me just make this once more clear. Positive slope means, as x increases, y also increases; negative slope means, as x increases, y decreases.

Now, if you look at this part of this sin x curve, as x increases, y also increases; the slope is positive. So, now, let us look at this curve here. For the red curve, before this particular value of pi by 2, the slope is positive; that means, the cos x value, which is the slope or which is the derivative is positive. And, at here, if you look at this part, this part, just, that is after pi by 2 and before pi, the red curve has a negative slope. It is decreasing, as the x increases, the function is decreasing. So, it has the negative slope. That means, its derivative has to be negative. So, see here, the cosine, the cos is negative here. So, the cos was positive here, negative here, and then, it is negative. Similarly, you will see that…

So, whenever you are having positive slope and negative slope, the function will increase and decrease. And, you should also note that, the peak, the point which changes from negative slope to positive slope, sorry, positive slope to negative slope, the slope is 0. At this peak, the slope turns out to be 0. Even at this peak, slope is 0. At this peak, slope is 0. So, this is an interesting phenomenon, which you will soon learn that, whenever there is a peak of any function, if you find this slope or the derivative of that peak, you will get that value 0. This is something that you will quickly learn, we will discuss soon.

But just, just note that, when you plot sin x and find the derivative, you will get 0 at this peaks, where the slope is changing. Or, and, also now, it has positive slopes and negative slopes. So, this is the graphical representation of sin x and cos x. We will come back to this, but just plot it in your computer, play with it and just also, if you have a graph sheet, just plot sin x and cos x and just play with it and see, when the x is increasing, where is y increasing; when x is decreasing, where is, x is increasing, where is y decreasing and what is the slope at each point, so on and so forth.

If you just play around, you will get a better feeling. So, once you have a feeling for functions, once you understand this function in a graphical representation, once you understand the graphical representation of the function and you get some idea about the slope, where is slope increasing, where is slope decreasing, where is slope changing where is slope is 0, you, that will help you a lot in understanding the calculus. So, if you want to understand mathematics very well, it is important that, you play with graphs. So, you take paper, pen and paper or graph sheet and just keep plotting. That is one way where you can, one way by which you can learn lot of mathematics.

So, we now learnt a few functions. We learnt all polynomial like x, x square, x cube, x power 4 functions. We also learnt the derivative of this functions and the sum of their functions like e power x, which is sum of x, x square, x cube, x to the power 4, etcetera. Even sin x and cos x can be written as some linear combinations of x power ns. So, we learnt the derivative of sin x and cos x also. Using this technique, you can learn derivative of any, pretty much any function, that you, we discussed so far. So, as an exercise, you could go ahead and learn the derivative of all the function that we discussed in the previous lectures, and then, see what you get.

And, you will see that, you will see that, it is, things that you thought was very difficult, turns out to be very easy, by just learning this, just three formulas. This three formulas that we discussed, that derivative of x power n, the sum rule and the product rule. So, now, see, once now, that we understand simple functions and their derivatives, let us learn two applications in Biology, which will be basically, two simple applications and using which, we will use, whatever we learnt, some things that we learned about derivatives and functions, and that will help us to understand a few things in this two example, in the biological example that we are going to discuss.

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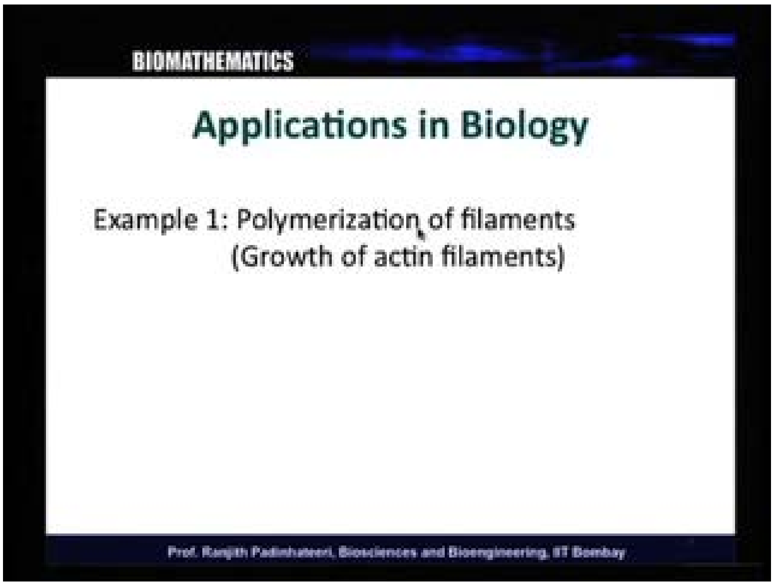

So, the first application in Biology, the first example I am going to take, polymerization of filaments. For example, growth of actin filaments. So, as you might know, there are many filaments in Biology; actin is just one example. So, for those who are not familiar with actins, let me quickly tell what actin filaments are. Actin is a cyto-skeletal polymer. It polymerizes in the cyto-skeleton of a polymer. It is a protein that polymerizes and becomes long filaments. So, just like any polymer, it is made of many monomer units. So, let us quickly represent it here. So, for simplicity, I will represent monomer by these blocks. So, this is monomer of actin. So, monomer of actin, actin is a…So, monomer of actin is called G-actin. So, G-actin, it is called globule actin, sometime.

So, the monomer of actin is G-actin and this monomer polymerizes and becomes a long polymer. Polymerizes means what, they join together. So, one after another, many monomers join together and form a long polymer. So, we are going to understand this process of polymerization, filament, those, this G-actin becomes F-actin. So, this is called F-actin, because this is filamentous actin; it is a filament. So, this G-actin becomes F-actin. We are going to learn something about…We are going to use the idea of derivative, the idea of calculus and functions we learnt, to understand something about this process. So, it turns out that, lot of things in Biology, like cell motility, the way cells crawl, the first generation for cells to crawl.

Lot of things depends on this polymerization reaction and it is rate and so on and so forth. Not just actin, there are other filaments like microtubules. Microtubules are another important cyto-skeletal filament. They are also, the monomers join together and form long polymers, long filaments. So, the monomer of microtubule is called tubulin. So, whatever we are going to discuss now, is in general polymerization of any polymer; for example, actin, or it could be microtubule, or it could be any other filament. There are examples in bacteria also, like FTSG, ParM; these are some examples of filaments seen in bacteria, a bacterial cytoskeleton. So, this filaments, they polymerize and we want to learn something about the polymerization and how fast they polymer and so on and so forth.

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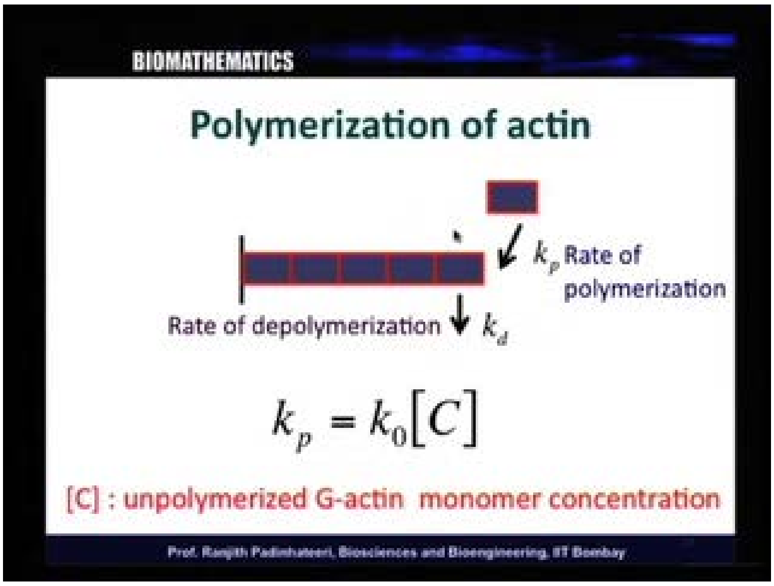

So, let us, let us look, quickly look at this slide here, this diagram here. So, in this cartoon here, as I discussed, this single box here, represents G-actin, which is the monomer of actin and this is a filament. And, with some rate k p, this monomer binds on to this filament and elongates. So, the k p is the rate of polymerization. k p is typically, something per second, let us say, 10 per second, 20 per second. So, that is k p. k p is the rate of polymerization. What does it mean to say k p is equal to 10 per second? What does…It means that, in 1 second, this, 10 of this, will be added; that is the meaning of saying k p equal to 10 per second. And, k d, which is the de-polymerization rate, which is also something per second, let us say, 2 per second. 2 per second means, in 1 second, 2 of them will de-polymerize, two monomers.

This one and this one can be de-polymerized, on an average, in every second. That is what the meaning of k d equal to 2 per second. So, k p and k d are two rates. k p is the polymerization rate. It is the rate with which this filaments polymer. So, these monomers, they just keep joining together and becoming longer and longer filament. And, k d is a rate of de-polymerization. If we have this monomer, they de-polymerize and come off. So, this rate is k d. So, the net polymerization, obviously, is what we want to calculate. But, before that, when you think about this, when you think about this polymerization, for this polymerization to happen, all this monomers, the G-actin monomers has to come and bind here. So, it is clear that, the more the G-actin monomers, the more likely that, they will bind; because, there will be, all around, there will be monomers; these monomers will come and hit this polymer and likely that, they will bind.

So, the rate of binding is directly proportional to C, which is the concentration of un-polymerized G monomer actin concentration in the solution. So, if you think about it, if you have, if you think about this...So, if you think about an experimental set-up, you have a container, let us say, ((skewette)). In this ((skewette)), you will have some small seed of polymer and there will be many monomers, like, there will be a concentration. So, there will be many monomers floating all around. So, C is basically, the concentration of this monomers; concentration, this is, concentration means, number by volume. In this volume, how many monomers are there, that is the concentration.

So, the more they are, the more likely that, they will go and hit and then, form this bind, binding. So, the k p, the rate of polymerization is likely to be more. The more this, the more this monomers are, more likely that, they will bind. So, this is proportional to this C, directly proportional to the C and this proportionality constant is some rate k 0. So, the rate k 0 here, is the proportionality constant. So, the k p, the polymerization rate is k 0 times the concentration C. So, once we realize this, we can think about, what is the net speed with which this will polymerize; the net speed of filament growth, because the growth of filament is important in Biology.

Because, the faster the filament grows, the more force this actin can apply on the cell walls and cell membranes, so that, the cells can crawl. Cell motility depends crucially on the rate of polymerization and so on. So, now, let us look at here. Once we understand that, k p is k 0 times C, the rate of polymerization k p is k 0 C, we can write down the growth speed is k p minus k d, which is commonsense again.

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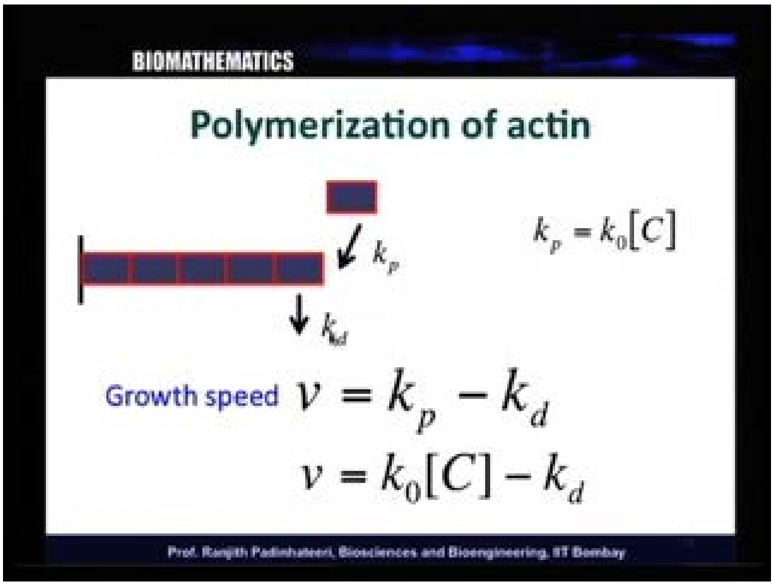

The net speed is, whatever polymerized minus whatever de-polymerized. So, if you have 10 monomers polymerizing per second, and 2 monomers de-polymerizing in second, the net speed of the filament will be 8 monomers increasing, polymerizing. The length of this filament will increase by a factor, 8 monomers. So, the growth speed is nothing, but k p minus k d, the polymerization rate minus the de-polymerization rate. And, we saw that, the polymerization rate is k 0 times C. So, I can substitute this here. So, the v is nothing, but k 0 times C minus k d. So, we get an equation for the speed, growth speed or the growth velocity, I should say. So, v represents for the speed or velocity. So, the growth velocity is k 0 times C minus k d. So, this is the function. So, we have a function.

So, let, let us look at v, which is the function of C, is k 0 times C minus k d. In, in other words, you can write, this is your f of x, the function, velocity is the function of concentration. So, the function we are interested in here, is velocity. So, this is like, f of x equal to m x plus c. You might have seen this equation. We, as we learnt, this is a straight line. We can plot this. So, let us imagine, some experiment that you are doing. So, let us say, you want to calculate the velocity. So, let us say, you are doing an experiment, where you have a set-up, you have a (( )). Using florescent microscope, we can look at the length of the actin, by varying the concentration. That is, you fix the monomer concentration of actin to a particular value, let us say, 1 micromolar, 2 micromolar, 0.5 micromolar. So, you keep a constant value of the free G-actin monomer concentration and look at, how fast this filament is growing. And, if you do that, you can get a plot, you can make a graph, which is like this. So, have a look at this graph.

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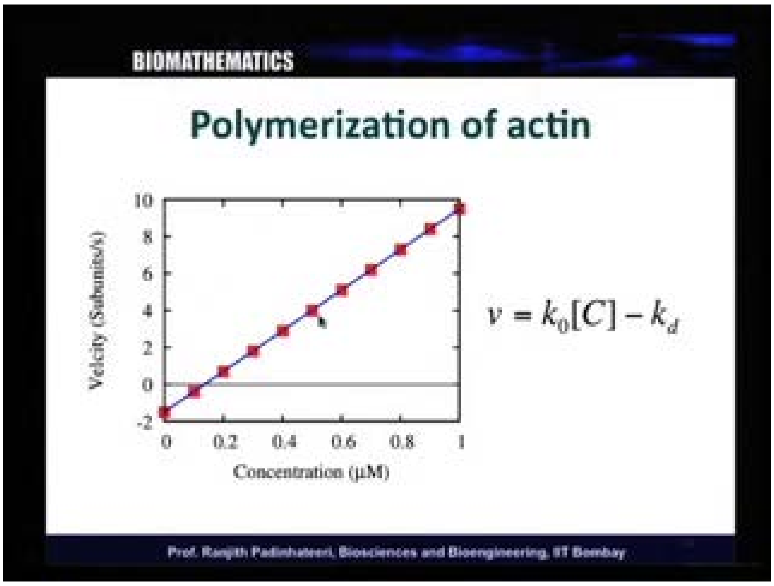

So, these points, the red points here, is for different values of concentration. 0 concentration; so, basically, let us start from the right end. 1 micromolar; the x axis is concentration, y axis, the velocity. x axis is in micromolar; concentration unit is micromolar; molar is the unit of concentration. So, micromolar means 10 power 6 molar. This is the typical range, with which one can do experiments on actin monomer grow, actin polymerization. So, and then, you can look at the velocity. So, velocity is plotted here, in sub unit per second. How many sub units grew per second? In other words, monomer per second, I should say. So, later, when the concentration is 1 micromolar, the velocity is just below 10; its actually 9. 5. When it is 0.8, some value here.

Again, 0.7, 0.6, 0.5. 0.4, 0.3, 0.2, somewhere between 0.2 and 0.1 this is crossing the value of 0. So, you can extend this, if you plot this, this much, you can join them and draw a straight line and extend this till that, till you hit the x y axis. So, this is one experiment, this is one curve, you can plot by doing the experiment. By varying the concentration and measuring the velocity, one can get the data points and plot this curve. So, this is a very interesting experiment done in 1980s by two biologists. So, one person who did is, M. F. Carlier. So, you can look at the papers of M. F. Carlier, or you can look at the papers of T. Pollard. These are the two scientists, who did experiments on actins polymerization. So, look at the papers of Carlier and Pollard. So, go to Google’s Scholar, which is, which is basically, scholar dot google dot com; then, look, search for M. F. Carlier or T. Pollard and look at the papers of 1980s, late 70s and 80. So, late 1970s, early 1980s.

So, roughly around this time, if you look at the papers, you can see that, they have done experiments studying actin polymerization; exactly the same experiment that we discussed. And, they wanted to find out, what is the rate of polymerization, what is the rate of de-polymerization, how fast they polymerize and how fast they de-polymerize. So, they basically, did this. They varied the concentration of free G-actin monomers and measured, how fast the polymer grows.

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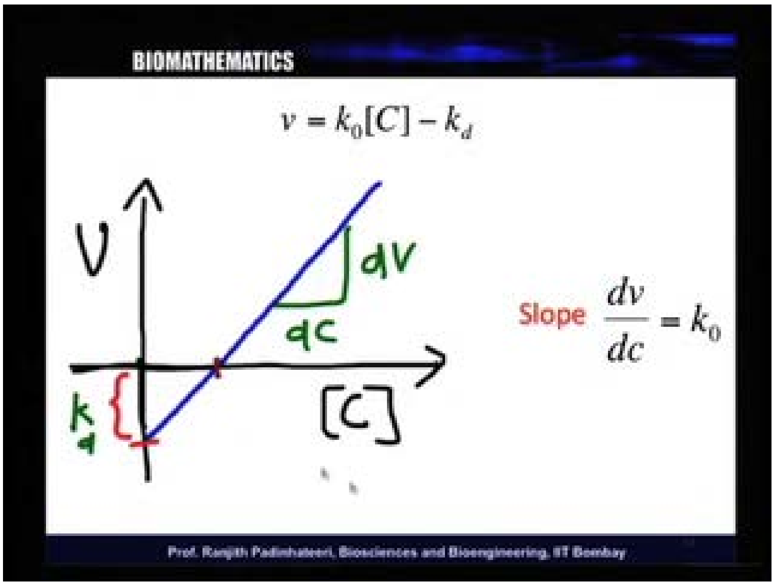

So, they could look at it, plot velocity versus concentration, and let us look, what can they do. So, if they get this curve, from this, we know that, the slope of such a curve, that the y axis, if you just calculate the slope here, this slope is d v by d C, change in concentration; when you change the concentration by some amount, how much the velocity changes.

So, the d v by d C, and that will give you the k 0, which is the intrinsic rate of polymerization. k 0 is a intrinsic rate of polymerization and you multiply with the concentration, you get the rate of k p, rate of polymerization. So, this is the rate constant. And, what happens when C equal to 0? When C equal to 0, v is, this term is 0; so, it is just minus k d. So, the value at which, at 0 concentration, whatever will be the value, that will be the minus k d. But you cannot do experiment at 0 concentration, right.

So, what can you do? You can do experiments at large concentrations, in this positive, this part and extend this and extrapolate it, and plot here, and then, look at the value, at which that hits the in y axis; that is called y intercept and you will see that, this value is k d, the value at 0 concentration. The value at 0 concentration is k d. So, you can see that, v is minus k d, negative, below 0. So, that is this value and you will also see that, this is the value, the concentration at which polymerization equal to de-polymerization. So, this is the critical concentration. So, you might know that, the critical concentration is the concentration at which polymerization equal to de-polymerization; that is k p equal to k

d. So, k p equal to k d, it is polymerization rate equal to de-polymerization rate. So, if you plot a curve, which is C verses v, at some particular point, velocity will be 0.

So, this is 0. So, the point, the concentration at which the velocity is 0; the velocity 0 means, k p equal to k d and we know that, k p, we learnt, we learnt that, k p is nothing, but k 0 C, is equal to k d. So, C is the concentration. So, the concentration at which this are equal, this would imply that, the concentration at which they are equal is k d by k 0. So, this is known as the critical concentration. K d by k 0 will give you the critical concentration. So, you can do two things; either, you can find out k d and critical concentration and then, from that, you can calculate k 0; or, you can calculate the slope and calculate k 0, and you can calculate k d, and this you know, k d and k 0 and you can also calculate the critical concentration.

So, just by plotting and understanding that, the slope or the derivative is nothing, but the polymerization rate constant, we are essentially, using calculus, differential calculus, the simplest form of differential calculus, to learn something about the experiment. Now, once you understand this much, so, what did we learn? What are the bottom lines? So, we learnt one thing. So, we learnt, let us, the bottom line is, velocity is k 0 times C minus k d and d v by d C, that is, the change in velocity, when you change concentration is nothing, but the slope of this curve. This is nothing, but the slope of this curve and slope is nothing, but k 0. So, this is C and this is v, slope is k 0. So, we learn that, slope, slope, slope is basically, the tilt. So, this, this tilt is called the slope.

So, this tilt, how much this is tilting, this is called the slope. So, the slope is k 0; that is, the polymerization rate constant. So, once we understand this, that polymerization rate constant is k 0, what can we say about other filaments. Without doing any experiment, we can roughly draw, how will the curve look like, for some other filament. We do for actin. We can see how would it look like another filament. So, let us quickly understand, for actin, from whatever we learnt.

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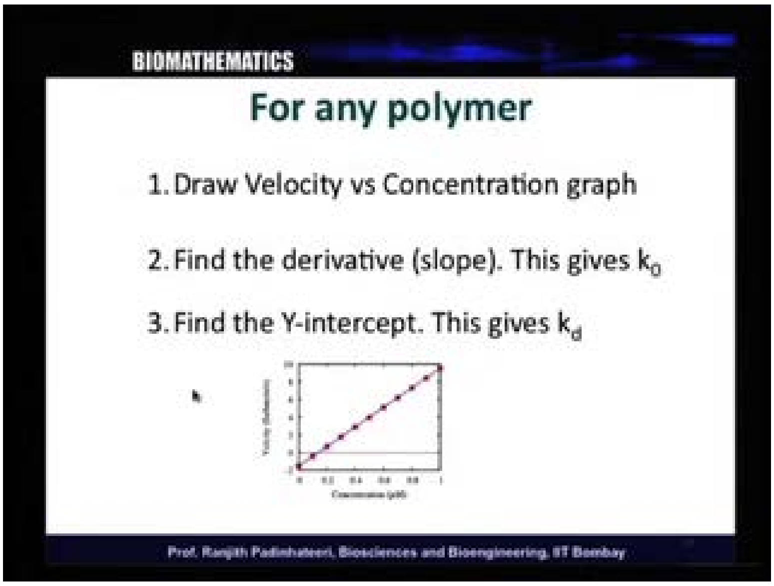

Let us have a look at values of k 0 and k d for actin. So, what we, for any polymer, what we can do experimentally is, draw velocity versus concentration graph; find the derivative and this derivatives gives you k 0; find the y intercept, this gives you the k d. So, this is the recipe essentially, for calculating k 0 and k d. And, what we are using here, is the simplest form of differential calculus. We are using Mathematics essentially, to learn or to get k 0 and k d. Many people unknowingly have used this, but you basically, are knowingly used. You have learnt about slope, derivative, everything, and now, you can understand the whole thing about this and does little more. So, let us look at the actin values.

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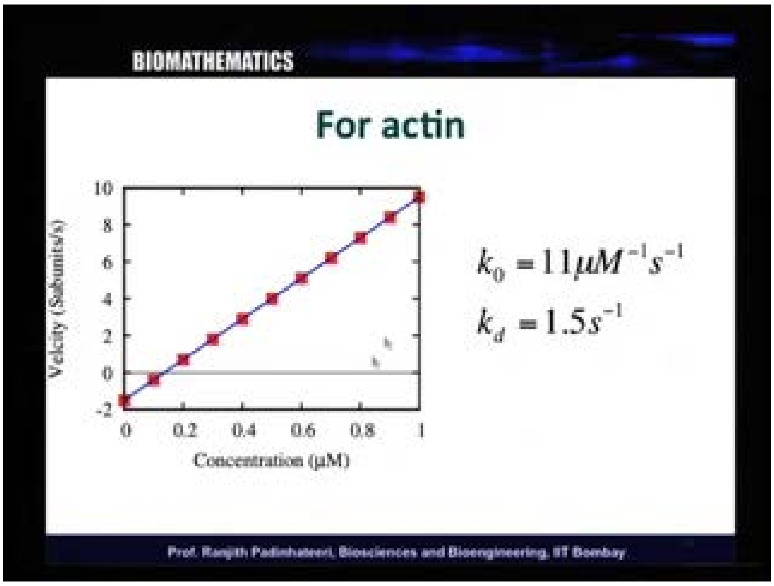

So, now if you look at this, for actin, if you look at this curve, what is k d? k d is the y intercept; this is k d. So, this, this y intercept here, this gives you k d and this gives you k 0. So, now, let us look at the k d here. For actin, k d is just below, between 1 and 2. So, it turns out that, if you look at this point, the point at which this line hits the y axis, what is called y intercept, k d is turns out to be 1.5 per second; 1.5 per second and k 0, which is the slope of this, is 11, 11 per second, 11 micromolar per second. So, now, you can easily look at this. When you, how do we calculate this slope without doing too much of calculation? Since this is a straight line, calculating slope is very easy. Look at the change in x here; x changes from 0 to 1, 1 whole 1 micro molar. And, how much will be the y change? y change from 1.5 to this value, which is just below 10. It is actually 9.5, sorry, yes 9.5.

So, 1.5 to, minus 1.5 to plus 9.5. So, that is about 11. So, this is, if you count, minus 1.5 to this particular point, which is plus 9.5, this turns out to be 11. So, when…So, you can see, the slope is d v by d c. So, d c is 1 and d v is 9.5 minus minus 1.5, which is 11. So, you can see that, that is how you get the answer 11. So, look at, take this value of 11 and 1.5 and draw yourself a graph, which is, which is, which will look like this, and you can learn. Now, from this, the next question you can ask is, what else can you learn from this. From, just by knowing this much, what you can say, what can you...So, the one thing about learning theory or mathematics is that, you should be able to predict something; without doing any experiments, just about thinking about situation, doing some mathematical calculations, you must be able to predict something. So, just by learning, what we learnt today, what can we predict? We can predict one thing.

So, we can say, by just listening to this lecture, you, you can predict one thing. So, the prediction is this. So, we had, we had this curve, which is concentration versus velocity, for actin, which…So, for this particular curve for actin, looked something like this. Now, we predict that, if you do an experiment for some other polymer, let us say, you do an experiment for microtubule, how will the curve look like?

So, as you learnt here, the k 0, the depends, is basically, the slope of the curve, is the polymerization rate and the y intercept is the de-polymerization rate. And, you can imagine that, for microtubule, the rate of polymerization, rate of de-polymerization will be some other value. It could be some other value. So, k 0 could be something else for microtubule and k d could be something else. So, if you do an experiment for microtubule, you can predict that, you will get some other curve, which looks like, may be different. So, let us say, you could get some curve, which looks like this, which has the different y intercept and a different slope. If you do for some other curve, let us say (( )) filament, you might get some curve like this; for some other polymer, you might get a curve like this.

So, what we are predicting here, what we can understand, what we are saying here is that, depending on the polymer that you choose, depending on the filament that you choose, you might get curves with different slopes and different y intercepts. This is something, which you could predict, just by understanding that, the slope is rate of polymerization and the y intercept is rate of de-polymerization. And, you can guess that, different polymers will have different rate of polymerization and different rate of de-polymerization. So, different slope and different y intercept. So, this much, you can say confidently.

So, with this, we will stop here, basically, to summarize. We learnt a few functions and its derivative. We learnt that derivative is slope and we learnt three formulae. We learnt the derivative of x power n, the sum of two functions and its derivative, product of two functions, its derivative. Use that formulae to calculate, use those formulae to calculate derivatives of many functions and one example we discussed now, which is basically, the polymerization of actin. And, using the ideas of calculus, we learnt, how to get rate of polymerization and rate of de-polymerization. And, using that, we could say something about, something about how the rate and velocity versus concentration curve will look like, for some other filament. So, this much we could understand. So, with learning this, discussing, we will stop here and we will continue with more examples in the coming classes. We will take some other examples and see, what we can learn from that example, that we did not know. So, the mathematics we learn, will help us to understand something more that and predict something, which we did not know. So, let us stop here. Thank you.