Biomathematics

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Lecture No. # 08

Differentiation and its applications (Contd…)

Welcome to this lecture of Biomathematics. We have been discussing differentiation and its application, the first part of the calculus and in the last lecture, we discussed about curvature. So, let us have a look at, quickly have a look at what we mentioned, what we discussed about the curvature in the last lecture. So, the lecture title is differentiation and its application and the thing about curvature we learnt was that, if we have a convex curve, a curve, which, which looks like inverted u, this kind of a curve, so, the slope is going from positive slope, 0 slope to negative slope.

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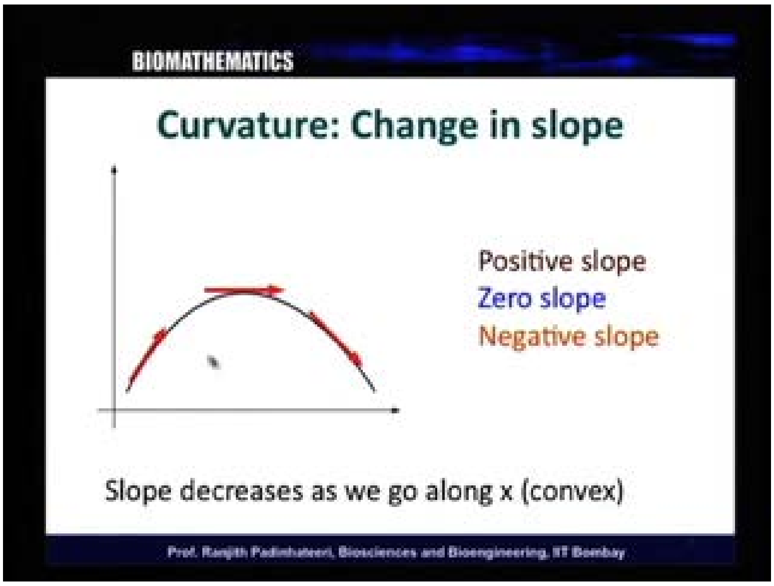

So, the slope is basically, decreasing. So, the derivative is decreasing and one important thing we learnt here is, the slope at this maximum point, here, at this particular, this slope here, is 0. So, if you calculate this slope here, this is 0.

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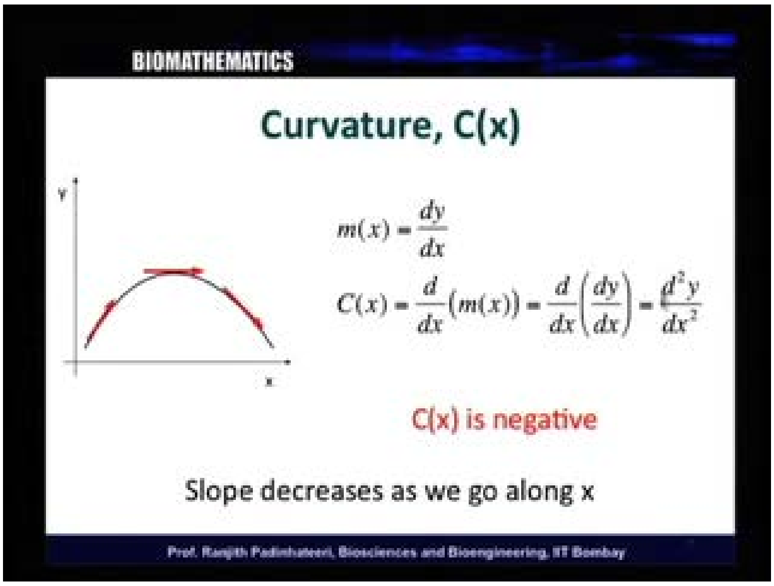

So, this and we found the curvature is nothing, but the first derivative d y by d x is slope. So, you can calculate the slope at different points here, here, here, here, here, here, and you will find that, the slope goes from positive to negative and then, you can calculate how this slope itself changes. So, the change in slope, d by d x of m or m of the slope is the curvature. This second derivative is called curvature; d square y by d x square is d by d x of d y by d x. d y by d x, we calculate and again, you find the derivative, we will get d square y by d x square. We will calculate, take some quantities and calculate d square y by d x square today or in the next lecture.

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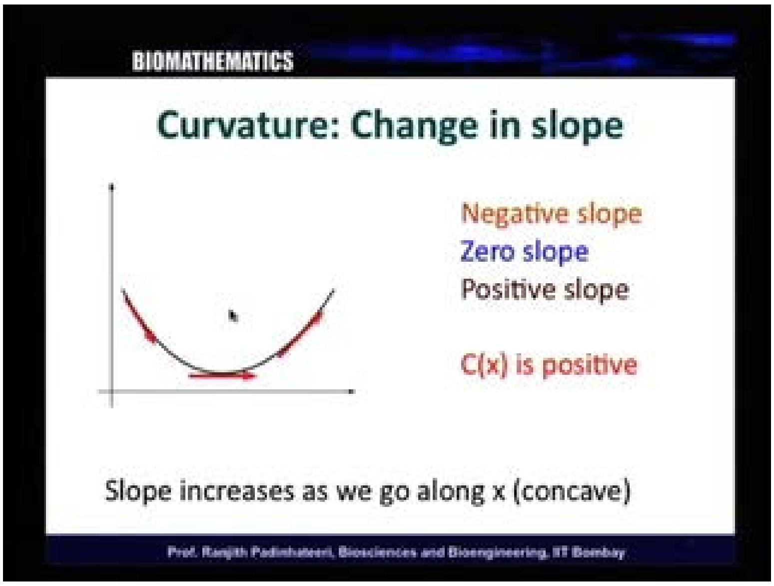

But again, the thing to remember here is, even in this case, where it is like a u, it is like a curve, which, which is con, this is a concave curve, even for this concave curve, at this point of minima, the slope is 0; the derivative is also 0. So, here, the derivative is going from negative to positive; that is, the derivative is increasing and the slope is increasing; in the previous case, in the convex curve, the slope was decreasing. So, combining this two, what (( ))…So, these are the things we learnt that, the slope, for a slope, for a, the thing we learned is, for a curved, for a function that is curved, either, anyway it is curved, if it is not a straight line, the slope will keep changing. So, the derivative will keep changing, if you go from one place to other place. And, this change in derivative has a meaning. So, the change in derivative is the curvature of the function and this is important. We will, we will see what its application is and the point, the maxima or minima, at those extremum points, like, we call it extrema, because, either it is maximum or minimum; at those points, the derivative is 0, the slope is 0. So, now, let us now look at the next slide here.

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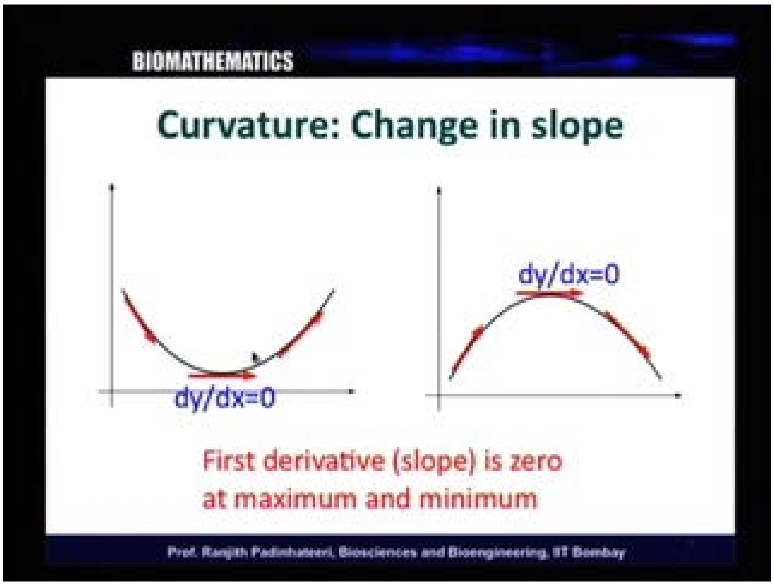

So, this is, we combine what we learnt in the last lecture. So, curvature is change in slope and at this extrema, and the minimum d y by d x is 0 here, at this particular point. So, you calculate the slope; draw a line which is touching this minimum point. So, this is called a tangent. Again, draw a line which just touches this point, only this point, and the slope of that will be 0. Again, here also, you can draw a tangent, which is a line, just touching the maximum point; a straight line, which touches only on the maximum point, and that will be, have a slope equal to 0. So, the slope at this particular point is 0. So, the first derivative is 0 at maximum and minimum, for any curve. So, if a curve has a local maximum or a minimum, you will see that, its derivative is 0.

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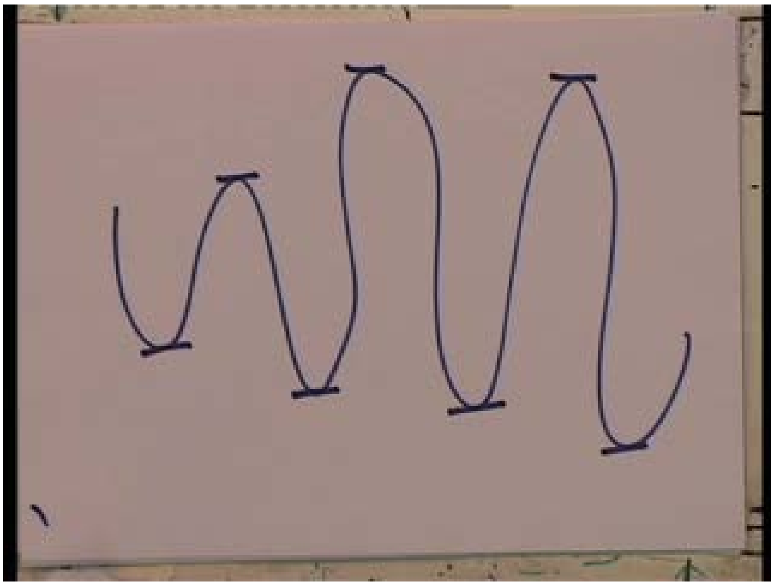

So, let us have a look at this implication, like…So, let us say, you have a, have a look at this, here. Let us say you have a curve, which is like this. So, at the each of this points, this is either maxima or maximum or minimum. So, if you find the derivative at each of these points, the derivative, you will find 0. So, this is one thing, which you should remember that, the derivative at extrema is equal to 0. So, what is its application? Have you...

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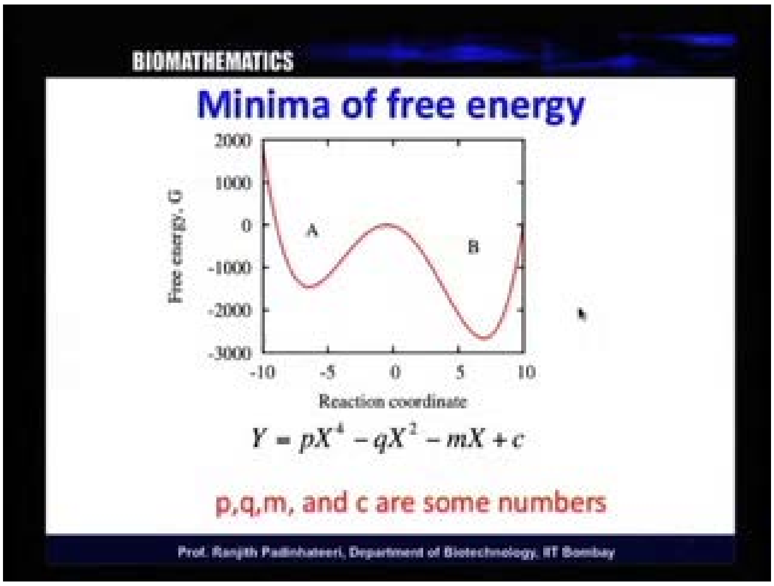

So, let us have a look at the next slide. The immediate application is minima of free energy. So, you have free energy, which is like a… So, one important idea in Biology, as you all know, is energy landscape. So, the form of energy, which is like, has lot of peaks and valleys. So, you can see here, this free energy has a minima here, a maxima here and a minima here. This is two minimum; there is a minimum here, another minimum here, another maximum here. So, two minima and a maximum. So, this is the simplest form of free energy, which you can think of. For example, a reaction, which is a going to b. So, this is, a and b are a protein having two configurations a and b. But in reality, proteins could have many confirmations and depending on that, you could have a complicated free energy surface. So, to understand about this free energy landscape, this idea of minima, maximum, minimum, maximum and minima and maxima are important and the derivative will help us in understanding this.

So, another thing which you should note in this slide here is that, this equation for this particular curve is, some constant time x power 4, minus some other constant times x square, minus some other constant times x plus c. This is the equation which I have plotted here, with some values for p, q, m and c. We will discuss what this values are later. But given some values, you will get, this as a curve. Given such a function, a free energy function, how do we find out this minimum points and maximum points; that is one thing, which we want to discuss today. And, so, let us take the simplest example for this kind of a free energy surface. So, the simplest energy which has the minima, minimum, is the energy of a spring. So, let us have a look at, here. So, you can imagine a protein, let us say, which is a spring like protein, which is like a spring.

And, you all know that, the spring has an energy. So, if you just take any spring, you all know that, you can stretch a spring. The more you stretch, the more difficult it is and you can… So, you cannot, you can also compress the spring, but the more you compress, the more difficult to compress it is. So, the force you needed to stretch a spring, the more you stretch, the more force you need. So, and the more you compress, the more force you need. At a particular, when you apply no force, the spring has a certain distance. So, the two ends of this spring is having a certain distance, when you apply no, when you apply the zero force, when you apply no force. So, if you write this idea in mathematically, the idea we know is that, when you apply 0 forces, when you apply no force on a spring, the distance between two ends of a spring is a constant. So, let us call this constant x 0. So, let us call this constant x 0, which is distance between the two ends of this spring; this is one end and this is another end. When you apply no force, the distance is x 0. Now, you, the more force you apply, the more difficult to stretch is. So, the force, the force you need to stretch is proportional to how much you go away from x 0. So, x minus x 0. So, the more you go away from x minus x 0, this distance, either you increase the distance or decrease the distance, the force will be affected.

So, and the energy for such a spring is proportional to x minus x 0 square, so, which is known that, k by…So, k into x minus x 0 whole square. So, half k x minus x 0 whole square. This is the energy of a spring. So, now, let us take an example of such a spring. So, this is the simplest form of energy you can think of. A protein that can be stretched or compressed, the spring like protein; if we, you can, you can imagine a protein like this. And, we, for the simplicity, we are imagining a protein of this particular nature and if we have this protein and the energy of the protein is given by k by 2 into x minus x 0, where k is some constant, x 0 is the distance when you apply no force and the x is the distance at any time, after any, whenever you stretch it.

So, if this is the energy, we can plot this energy. So, let us, let us a bit more, have a look at, what does exactly this mean. So, I can just take this protein. So, I can take this particular protein and stretch it. So, let us say, you have a protein which is… So, this is x 0, this is the distance x 0. Now, I can stretch this protein, the spring. Now, the distance is x. So, this distance is x. So, this x is bigger than x 0. So, the energy of for stretching this spring, this much, is half into k, which is some spring constant times x minus x 0 whole square, this is what it means. So, the energy needed to stretch a spring of this length to this length, the energy needed to stretch is half into, half k into x minus x 0 whole square. Now, let us look at this example.

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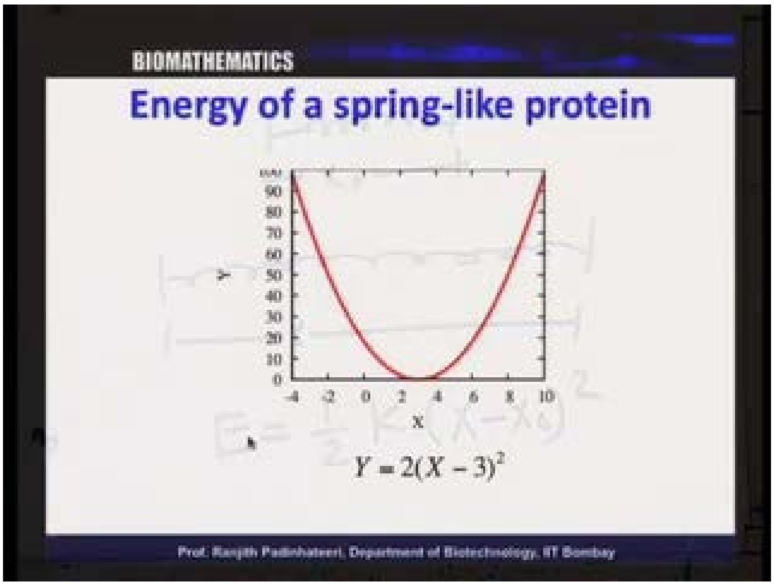

So, let us look at this next slide. So, what we have plotted here, is the energy of a spring like protein and as we said, the expression we plotted here is, the spring constant is 2. So, the k, instead of k we have like…So, half, spring constant k is 4, actually. So, k by 2 is 2; k by 2 is this 2 and x minus, x 0 is 3. So, that is 3 nanometer. So, let us say, 3 nanometer is the, when you apply no force, 3 nanometer is the length of this protein. And, you can stretch or compress this protein and the energy needed to this is x minus 3 whole square times 2, where 2 is the half of the spring constant. Now, we need to find a few thing. We need to find the minima, the position, where the energy is minimum, using the idea of derivative, using calculus, using mathematics, we want to find out the point at which the energy is minimum. So, this is our exercise. So, let us do this. Let us try and do this. So, what we have is, here is, as we have written, k by 2 into x minus x 0 whole square was the energy.

Now, in this example, we have k by 2 equal to 2. So, k is 4. So, 4 by 2 into x minus 3 whole square. This is 2 into x minus 3 whole square. So, this what we plotted, what we saw plotted. This is our energy. Now, we want to do and we know that, the energy looks like this and we want to know this particular point on the x axis. So, this is x axis, and y axis. So, we want to find out this particular point on the x axis, where the energy is minimum. So, the way to find out this is, as you know, as we said, we find the derivative. So, let us have a look at it. So, the derivative, energy is 2 into x minus 3 whole square. The derivative at that particular point, derivative is d E by d x; this is, the derivative is d by d x of this quantity, which is 2 into x minus 3 whole square. So, this is what we want to find out. Now, this is slightly more complicated than what we learnt last time. We just learnt, how to find out the derivative of x square, x cube, x power n; but this is x minus something whole square. So, how do we find the derivative of such a thing? So, the simplest, one idea to do derivative of such thing is that, if we do not know this particular form, you make it to a form that we know.

So, let us look at this again. So, if you take this x minus 3 as some u, this will end up as, this x minus 3 as some u, the energy will end up as 2 u square, where u is x minus 3, where u is x minus 3. So, if you have…Now, what we want to find out is, the derivative of u square. So, if, any case, if at all, if you have such a thing, where u is a, something is a function of x. So, u here, is the function of x; u is x minus 3 and you want to find out the derivative of u. The idea of the, the thing to find, the, the method to find out the derivative is this. So, we will find a new rule, to learn about this derivative. So, the rule is this. So, what, let us have a look at what we have. So, if we have here, y, this is energy, which is 2 times u square, where u is x minus 3. So, energy is a function of u, which is a function of x. So, this is what we have, energy is the function of u; energy is 2 u square and u itself, is a function of x. So, you have y is the function of u, which is the function of x; y is, whatever we plot in the y axis. So, we plot here, energy in the y axis. So, this is y and this is x. Y itself, is a function of u and u is a function of x. So, if we have such a case, the rule to use is this. So, let us have a look at this.

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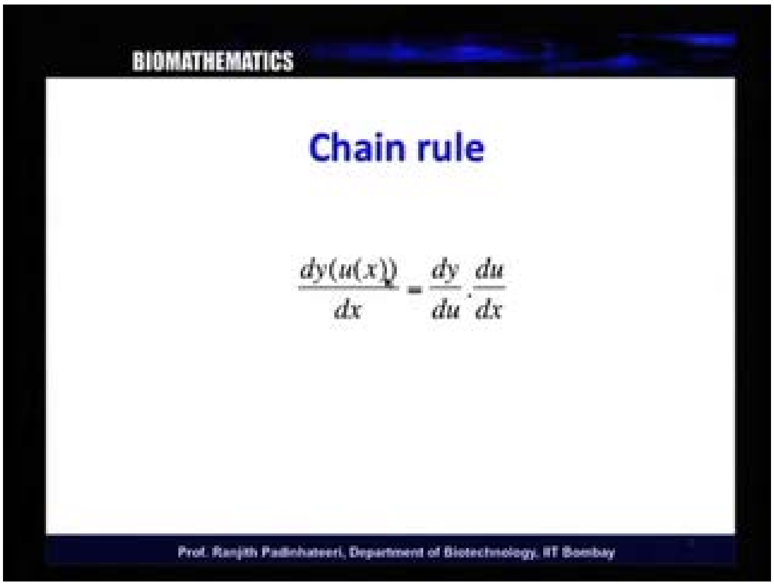

So, we will learn a new rule of differentiation. It is called chain rule and the rule is that, d by d x of y, which is the function of u and a function of x, which is the function of x, so, d by d x of y which is the function of u and eventually, a function of x is that d y by d u into d u by d x. That is, first, you find the derivative with u and then, multiply with the derivative of u with respect to x. Very simple rule. So, let us apply this rule and see what we can get. So, let us apply this rule and let us see what we get. So, what we have here is that, y, which is our energy is 2 u square, where u equal to x minus 3. So, if we want to find out d E by d x, the rule we learnt is that, E is a function actually, of u first and u is a function of x. So, first, you find derivative of E with respect to u and multiply with derivative of u with respect to x.

So, this is the formula we learnt. This is called the chain rule. So, if we apply this particular formula, let us see what we get. So, let us have a look at here. So, let us first find, d E by d u, d E by d u. So, E is 2 u square. So, this is like, we learnt k n power x power n. So, derivative of u square is 2 u. So, 2 into 2, 4. So, let us, let us write this little more carefully. So, let us write it little more carefully. So, what we want is E is 2 u square and u is x minus 3; and d e by d u into d u by d x is what we want to find and d e by d u is d by d u of 2 u square. So, according to the rule we learnt, this is the constant. So, we can just take it out. So, this is 2 into d by d u of u square, which is 2 times, again 2 u. So, this is 4 u. Now, another thing we want to find is, derivative of…So, we, we found that, 4 u is, the d E by d u is 4 u.

Now, what is d u by d x? d u by d x is d by d x of x minus 3. So, x, d by d x of x, this is the sum rule we had; this two terms, x and 3, this is x and minus 3 is the sum of two terms. So, the derivative of x, plus derivative of constant, which is 0. So, this is essentially, d by d x of x minus d by d x of 3, which is 0. So, minus 0. So, this is essentially, 1; d x by d x is 1. So, d u by d x is 1. So, the answer we get is, 4 into u and times d u by d x; d u by d x is 1; d u, d E by d u is 4 u. So, if we rewrite this here, if we rewrite this in this, we found that, this is 4 u and this is 1. So, the answer is 4 u times 1; the answer is 4 u times 1. Now, let us rewrite this, what is 4 u? So, the 4 u was actually, the answer is d E by d x is 4 times u and we knew that, u is x minus 3. So, 4 into x minus 3. So, this is the derivative. And, we learnt that, whenever we have a minimum, at this particular, we want to find out this point, where it is minimum and we learnt that, at this particular point, the derivative has to be 0. So, this, we equate this derivative equal to 0. So, at this particular point d E by d x has to be 0; that means, 4 into x minus 3 is 0. What does it mean? 4 into x minus 3 is 0; 4 is not 0. So, x minus 3 has to be 0. So, x minus 3 equal to 0 means, x is equal to 3. So, the result of this is that, x equal to 3.

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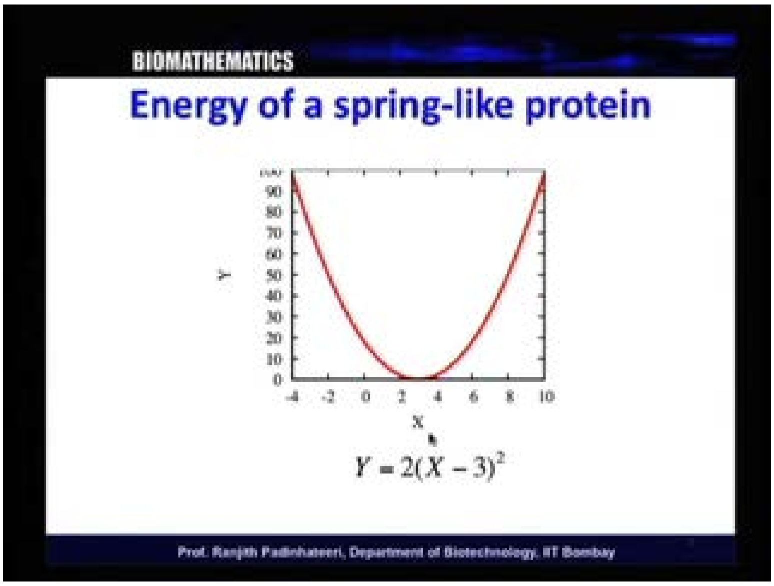

So, this implies that, this point is nothing, but x equal to 3. So, now, let us look at this plot once more. So, have a look at this plot and this, this here. So, at this point…So, look at this point. This is between 2 and 4, this is actually 3. So, we just found that, if you calculate the derivative and equate that to 0, we got 3 and exactly at 3, you have this function at, and minimum of this function. You can also, the minimum of this function is at x equal to 3. So, what did we learn? We learnt that, you take a function, find the derivative, equate the derivative to 0, you will get the maximum or minimum of that function. Now, the question is, how do we know that, this is a maximum or a minimum? Both at maximum and minimum, you will find this derivative equal to 0. So, the next thing which we learn is that, how do we find this is a maximum or minimum. So, we, last lecture, we mentioned this idea of finding this is that…So, let us quickly have a look at here.

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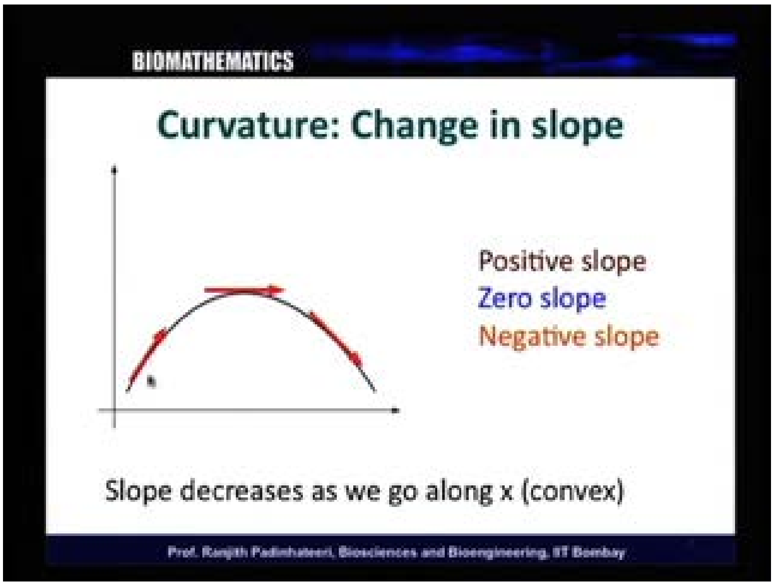

So, if you have maximum, if you have a maximum, the slope is going from a positive value to a negative value; the slope will decrease; or, the second derivative, the curvature…So, we found the second derivative; the second derivative will tell you, whether it is a maximum or a minimum.

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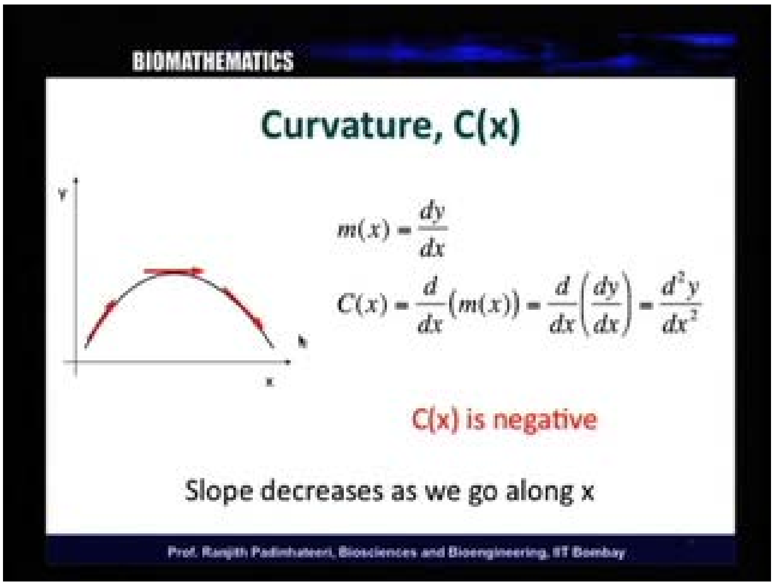

Depending on the second derivative, if it is a positive or negative, we can find out, whether it is a maximum or minimum. So, the idea, the trick to find out whether it is a maximum or minimum, is to find the second derivative. We found the first derivative and found the point at which the function is maximum. And, now, we will find out the second derivative and find out, whether the function is maximum or minimum. So, now, let us quickly do this. So, let us have a look at this function once more, what is the function we learnt. So, the function we learnt was, let us have a look at here, the energy which is 2 u square, where u is x minus 3. So, 2 into x minus 3 whole square was the function we took and we found the first derivative. And, the first derivative was 4 u or 4 into x minus 3. So, the first derivative was 4 into x minus 3. So, let us, the first derivative we found as 4 into x minus 3. Now, what we want is this second derivative.

So, let us go and do the second derivative. So, the first derivative. So, E is 4 into x minus 3 whole square. The first derivative we found that, it was 4 into, sorry, this is 4 by 2 into x 4. The function we had was 2 into x minus 3 whole square; that is 4 by 2 and the derivative we found that, d E by d x was 4 into x minus 3. Now, we, what we want is this second derivative. What does it mean? The derivative of the derivative. So, d E by d x, we already have and you find one more derivative of this; this is the second derivative. So, d by d x of d E by d x. d E by d x is 4 into x minus 3. So, let us write here, 4 into x minus 3. So, 4 is the constant. So, you can take this out of this derivative. 4 does not depend on x. So, you can take this out. So, what you have is 4 into d by d x of x minus 3.

So, this is what we have. By just looking at this itself, we can do, get the answer, without doing any calculation. So, you know that, d by d x of a plus b is d a by d x plus d b by d x. So, this is sum. So, x, derivative of x is 1 and derivative of 3 is 0, because 3 is a constant. So, this is 1 and this is 0; derivative of this is 1 and derivative of this is 0.

So, essentially, the derivative of this is 1. So, the whole thing has a derivative, d by d x of x minus 3 is 1. So, let us just do it and convince, convince you that, the derivative is indeed 1. So, let us…What we want, we want d by d x of x minus 3, which is nothing, but d by d x of x minus d by d x of 3; d by d x of x is 1 and derivative of, 3 is the constant, which is 0. So, 1 minus 0 equal to 1. So, the answer is actually, 1, right; that is what it is. So, the second derivative, d square E by d x square is d by d x of d E by d x and we found that, d E by d x is 4 into x minus 3. So, d by d x of 4 into x minus 3 and derivative of x minus 3 is 1. So, this is, the answer is just 4. So, the second derivative is 4, which is a positive quantity. So, whenever the second derivative is positive, what does it mean is that, it is a minimum. So, have a look at this slide here.

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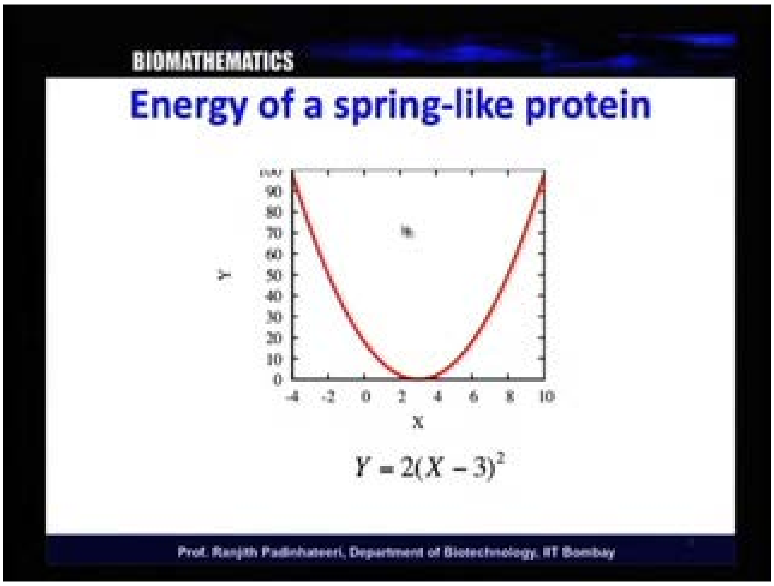

So, whenever you have a thing that is minimum, the second derivative, you will get the second derivative as positive. And, if it was a maximum, the second derivative would be negative. So, let us quickly have a look at this one example, where you have maximum. So, you can have a function which is energy, you could have, let us say, minus k x square. So, have a look at this paper here. So, the second derivative of minus k x square.

So, E is equal to minus k x square and this will look like, if you plot this, this function will look like this. So, like an inverted k x square. So, this is your x and y axis, x axis and y axis. So, this is the function will look like. So, at this, if you find second derivative of this…So, let us find the first derivative, d E by d x. The first derivative of k x square is 2 times k into x. So, this is minus 2 k x; this is the first derivative.

The second derivative d square E by d x square is d by d x of minus 2 k x; so, is d by d x of minus 2 k x. So, k is a constant; 2 is a constant. So, you can take them out. So, d by d x of minus 2 k x. So, d by d x of minus 2 k x, where k and 2 are constants is nothing, but minus 2 k, minus 2 k times d x by d x, which is 1. So, this is minus 2 k. So, what does it say? It says that, whenever you have a curve which is inverted, which has this shape, the maximum, at this maximum, the derivative has a negative value. So, whenever the curve is like this, the derivative has positive value. So, the curvature, the second derivative is greater than 0; for this, second derivative, this is less than, C, which is the curvature, which is the second derivative, is greater than 0 and less than 0 for this two kinds of shapes. And, this is important for free energy and many other things in Biology.

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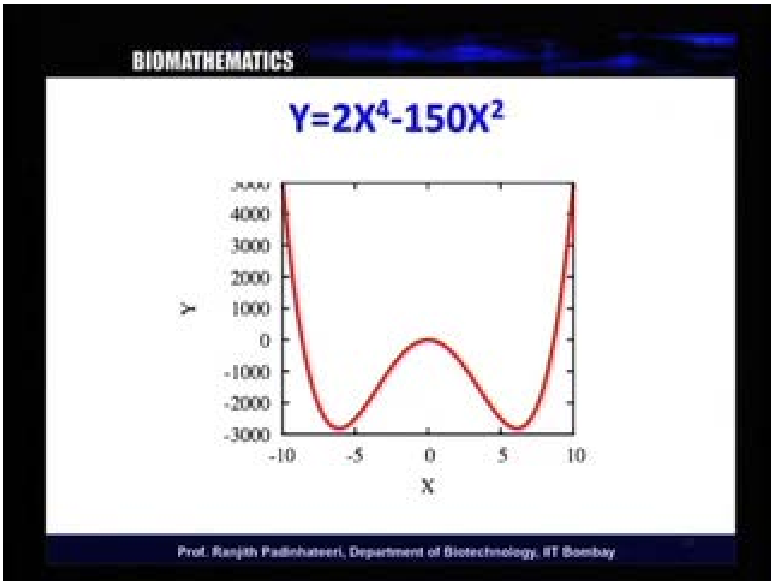

So, that is why, we basically, want to discuss this in detail. So, let us go ahead and now understand a bit more about the free energies, the shapes of free energies. So, what we learnt so far is, we took the simplest of all energies, half k x square, half k spring energy and if we found that, if you find the first derivative and equate that to 0, we will get the point, where the energy is minimum and the second derivative is positive; that means, it has a bell shape, with, with, which has a concave shape; that is what it means. Now, let us have a look at the free energy which we were…So, let us have a look at this slide. Now, the shape here, is another function; I have written the mathematical function here. So, what is plotted here is, 2 into x power 4 minus 150 times x square. This is a function. If I plot this particular function, I get this particular shape, which has two minima, one minimum here, one minimum here and a maximum, maximum here. So, now, according to the idea we learnt, according to the things we learnt, if we have such an energy landscape, the derivatives of this function will tell me, where is this minimum; where the energy is minimum and where the energy is maximum.

So, let us just quickly calculate the derivative of this particular function and then, see, these points are indeed the right points. So, have a look at here. So, this is 5. So, this is just above 5 something, close to 6, 6 something. So, this is just above 5; here also, this is just above 5 and this maximum is at 0. So, this is, from this curve, we already know the answer, but let us verify this, by finding the derivative. So, the idea we are using is that, the derivative will give you, wherever the derivative is 0, at that point, you have maximum or minimum of function; maxima and minima of the function. So, let us find the derivative of this particular function. So, the function we have is that, y is 2 x power 4 minus 150 x square. So, this is the function we have. Now, the derivative d y by d x is d by d x of this quantity. So, 2 x power 4 minus 150 x square. So, 2 x power 4 is the first, is d by d x of 2 x power 4 minus d by d x of 150 x square.

So, this is what we have to find. So, 2 is a constant, we can take it outside. So, then, what we have left is, just the derivative of x power 4. We can see, the derivative of x power 4 is…So, x power n is n x power n minus 1. So, derivative of x power 4 is…So, 2 you can take outside; derivative of x power 4 is 4 x cube; 4, 2 into 4 x cube, x power n minus 1. So, this is the first part; minus 150, you can take out and derivative of x square is 2 into x power 1. So, 2 x. So, this is the, this is the thing. So, this nothing, but 2 into 4 is 8, 8 x cube minus, 150 into 2 is 3 hundred, so, 300 x.

So, 8 x cube minus 300 x. So, that is the derivative of this, derivative of this free energy function. And, if we equate this to 0, we, what we will get essentially, is the minimum and maximum, minima and maxima of this function. So, let us quickly equate this to 0. So, let us equate this to 0. So, the function, the derivative d y by d x we got was, 8 x cube minus 300 x. This is what we got. If we equate this to 0, what we get? 8 x cube minus 300 x equal to 0. Now, the question is, what is x? So, I can take an x common, I can rewrite this left hand side as, x times 8 x square minus 300 equal to 0. So, I take 1 x from here, 1 x from here. So, if you take an x from x cube, it is, divide it, basically I divided throughout; I can divide it throughout by x, is what essentially happened. So, x times 8 x square minus 300 x, 300 is basically, this. So, now, if this is equal to 0, this means, either x equal to 0 or 8 x square minus 300 equal to 0. So, one solution of this is, x equal to 0. So, one, you get x equal to 0; when x is 0, the whole thing, this thing is 0. So, you can look at here. If you put x equal to 0, 8 into 0 is 0 and 300 into 0 is 0. So, the d y by d x is 0, when x equal to 0. Now, what is this second thing? Second thing is, 8 x square minus 300 can also be 0. If this is 0, the whole thing is 0. What does this imply? 8 x square minus 300 equal to 0 means, 8 x square equal to 300; that is what it means. 8 x square minus 300 equal to 0 means, implies that, 8 x square equal to 300. This implies that, x square is 300 by 8; this is 37.5. So, x square is 37.5. So, x is square root of 37.5. So, this implies that, x can be, this gives x is equal to plus or minus square root of 37.5.

So, you know the square root of 36 is either plus 6 or minus 6. So, 37.5 will have a square root, which is just above 6. So, you can find out the square root of 37.5. So, we found three things here. So, what did we find? We found that, we have three; one possibility is x equal to 0; the second possibility is that, plus root of 37.5; third possibility is that minus root of 37.5. So, it turns out that, the root of 37.5 is 6.12. So, this can be plus 6.12 and this is minus 6.12. So, we have three numbers here, that when we equated the derivative to 0 and equated the first derivative to 0, we got three numbers. One is 0; another one is plus 6.12 and another one is minus 6.12.

Now, let us look at this plot here. If you look at this plot, the 3, 0, this is the maximum; plus 6.12, one minimum here; minus 6.12, another minimum here. So, just by equating the derivative to 0, we found three points, three extremas, three extrema - 0, plus 6.12, minus 6.12. So, this is the mathematical, this is the idea, which I want to convey you that, if you find the derivative and equate this to 0, what you will get is basically, the maximum or minimum point, and the minimum point of energy, the minimum of free energy has some meaning in Biology. The minimum of energy is equilibrium state. So, the state of, if we have any system, which is in equilibrium, a protein in equilibrium, it has to have the free energy minimum; it has to be in the free energy minimum. So, to find out which, the which state has the free energy minimum, you can map the energy and find the derivative and see, where all you have the derivative equal to 0, and see, if it is the minimum, by finding the second derivative. So, let us, in this example, we found three points, 1 extrema,1 maxima and 2 minima. So, now, we will find the second derivative and see, whether they are maximum or minimum. So, let us have a look at here, once more. So, three points we found as x equal to 0, 6, plus 6.12 and minus 6.12.

And, the derivative we got was 8 x cube minus 300 x; that was the d y by d x. So, let us go ahead from here and then, find out the second derivative. So…So, the second derivatives are, what we have now is that d y by d x, the first derivative is 8 x cube minus 300 x; this is what we have. Now, we want to find out the second derivative. The second derivative is d by d x of d y by d x; one more derivative of this, which is d by d x of 8 x cube minus 300 x. Now, you can do this yourself. Let me quickly do this. If I do this, 8 into...So, 8 is a constant, and derivative of x cube is 3 x square. So, 8 into 3 x square minus, 300 is a constant and derivative of x is 1. So, this is basically, 24 x square minus 300. So, this is the second derivative, d square y by d x square. So, the second derivative is 24 x square minus 300. So, let us write this, the point of second derivative, a little more carefully. So, what we have here is, second derivative, which is d square y by d x square. This is equal to 24 x square minus 300. Now, the three points, where you have maximum or minimum are x equal to 0, x equal to plus 6.12, x equal to minus 6.12. Now, let us first put x equal to 0 here. So, if you put x equal to 0, what do you get? d square y by d x square, when x equal to 0. So, this is what it means. This means d square y by d x square evaluated at x equal to 0. So, we are going to evaluate this, at x equal to 0. You put x equal to 0 in this, you will get 24 into 0 minus 300. So, we have the answer minus 300; this is a negative. So, the curvature, the second derivative at x equal to 0 is negative. So, when you have the second derivative negative at a particular point, what does it mean is that, it is the, we learnt that, it is having a convex shape.

So, have a look at here, this slide. The second derivative at this x equal to 0 has negative, what does it mean is that, it has maximum. Whenever, whenever the second derivative is negative, it is maximum. Now, substitute…Let us have a look at, back here, the second derivative. Let us evaluate this x at 6.12. So, let us say, x is 6.12 and x square is 37.5 as we learnt. So, now, we calculate the second derivative at x equal to 6.12. So, we will have 24 into 6.12 whole square minus 300. If you evaluate this, do it yourself, you will find that, this is greater than 0. What does it mean? The second derivative is greater than 0 means, it is a minimum. So, at 6.12, plus and minus, square of plus 6.12 and minus 6.12 are same. So, you have this, both the cases, you have this particular value; the value of second derivative is greater than 0. What does it mean here? It means, it is a minimum. So, here and here, you have second derivative greater than 0. So, essentially, what we learnt here today is that, whenever you have a curve, which has extrema, maximum and minimum, or many maxima and minima, you find the derivatives and equate it to 0, you will get the points of extrema, the points of maxima and minima; and you find the second derivative and evaluate the second derivative at those points and you will find out, you can say, without looking at, without looking at the function, whether this is, x maximum or minimum.

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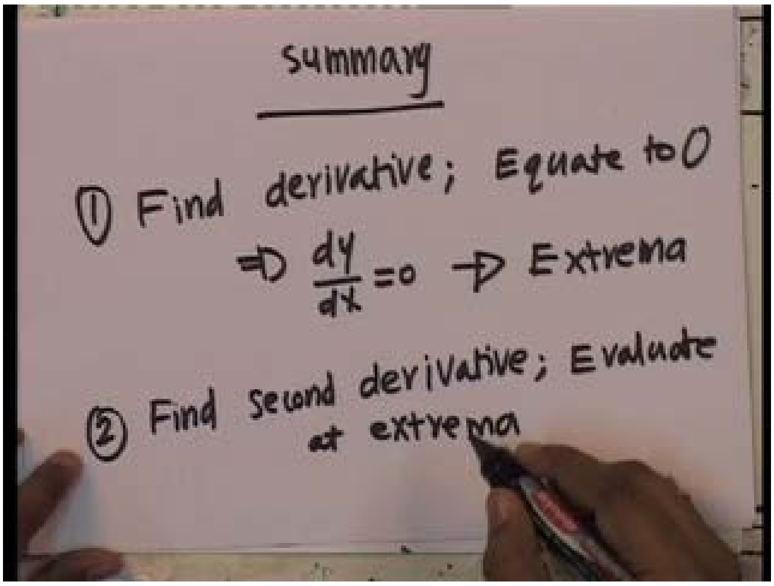

So, what we learnt is that…So, let me summarize this, what we learnt, let me write here and summarize. So, the summary. So, two things we learnt; one, find derivative; equate it to 0; equate it to 0; that is, d y by d x equal to 0. This will give you extrema, maxima or minima; that is what extrema means, this extreme points, maxima or minima. Now, the second thing we learnt is that, find second derivative; evaluate at extrema; that is, evaluate the second derivative at this points. If we evaluate at this maxima and minima, if the second derivative is greater than 0...So, if the second derivative is greater than 0...So, if d square y by d x square is greater than 0, this means, this is a minimum.

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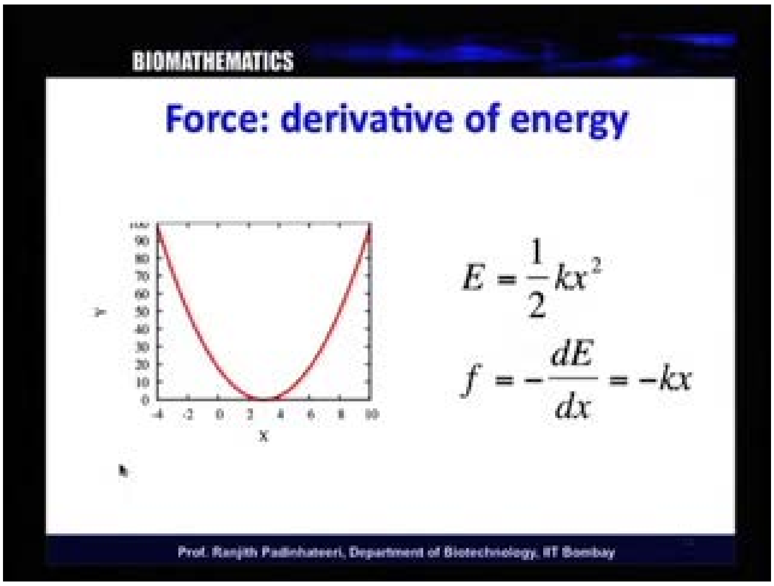

If d square y by d x square is less than 0, that implies, it is a maximum. So, this is what we learnt. So, find first derivative, equate to 0, get the extrema points, extrema. Find the second derivative, evaluate and you get, by looking at whether it is positive or negative, you get to know, whether it is maximum or minimum. Now, this derivative has lot of physical meaning. So, this is the one thing, which we will discuss in next, coming lectures. So, let me just give you a glimpse of what physically this derivative means, in some cases. So, let us have a look at this slide. If you have derivative of energy, it is nothing but force. So, we all know that, for example, this spring energy is half k x square. And, if you calculate the derivative, d E by d x of this function, you do it yourself, you will find, it is minus k x, which is the force. So, the derivative of the energy function is nothing, but the force. So, what does it mean this, this spring energy. At each point here, if you calculate the derivative or the slope, you will get the force; that is what it essentially means.

So, the derivative of energy has a meaning, which is force. So, this is the physical meaning of derivative. In, it has lot of implication in Thermodynamics. The derivative of free energy has different, different, of different meanings, depending on the context. So, the chemical potential is a derivative of free energy; the number of molecules is a derivative of free energy; pressure is a derivative of free energy. So, temperature is a derivative of free energy. So, derivatives of free energy, pressure, volume, force, distance, chemical potential, all of them can be written as derivatives of free energy.

Once we know the free energy, we can calculate the derivative and get all this quantities. So, all measureable quantities in Thermodynamics or in Biology and biological thermodynamics, can be expressed as, either the first derivative or the second derivative of free energy. So, we will discuss this in the next class. So, in the next lecture, we will discuss, how, what we can understand physically from the derivative and some derivatives of free energy and few more examples of derivatives, where this is applied, useful in Biology. So, at this point, we will stop today's lecture and we will continue in the next lecture. Thank you.