# Forest Biometry

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## Lecture - 17
## Methods of similar triangles: Shadow & stick

[FL] in this lecture we shall be looking at the methods of measuring heights using similar triangles, also known as the shadow and stick method of measurement of heights.

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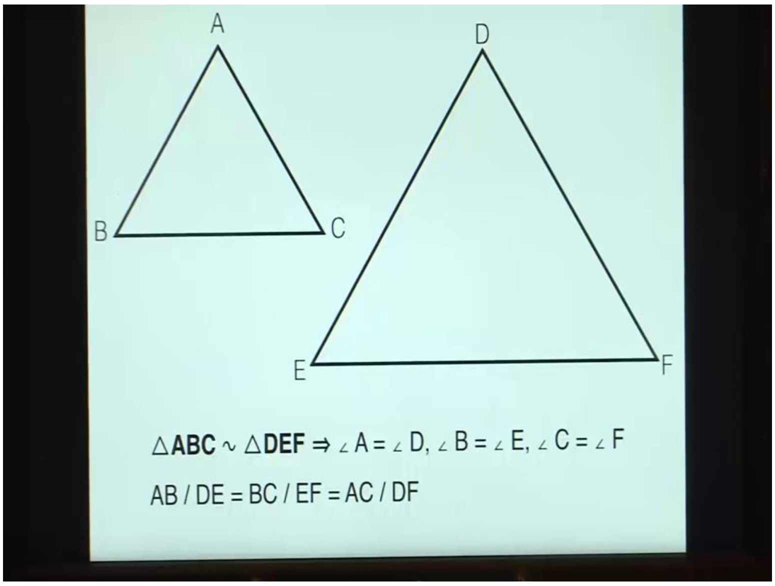


So, to begin with consider two triangles A B C and D E F.

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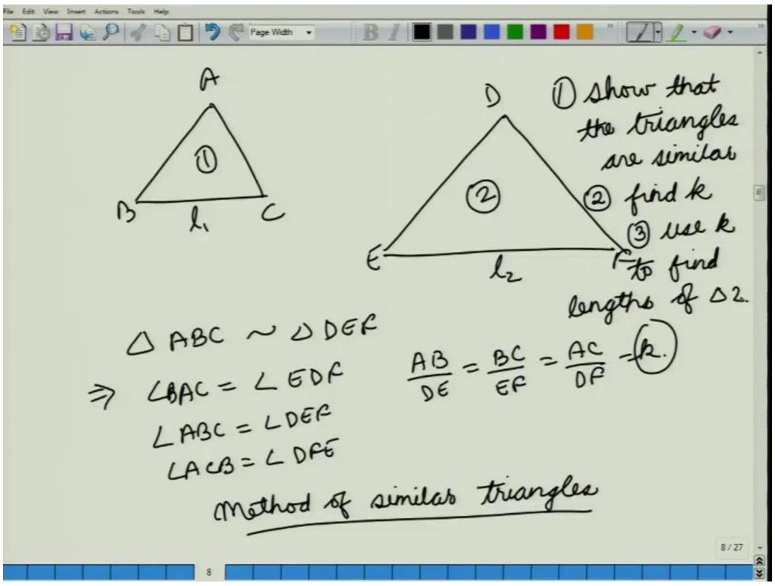


Now, if we see the triangle A B C is similar to triangle D E F, it would imply that we are saying that angle A or angle B A C is equal to angle E D F, angle A B C is equal to angle D E F, angle A C B is equal to angle D F E, and also that the sides of the triangle are in a constant ratio or A B by D E is equal to B C by E F is equal to A C by D F, which would be equal to some constant k.

So, by using this method of similar triangles, so we will have a method of similar triangles, if we can show that 2 triangles are similar and if we were able to measure 2 lengths say l 1 and l 2, then because all the lengths of the first triangle and all the lengths of the second triangle are in a constant ratio k. So, we would be able to get the measurements of the second triangle just by taking the measurements of the first triangle.

So, the steps would be to show that the triangles are similar second find k. So, we can find k by taking 1 measurement of both the triangles the corresponding measurements of both the triangles and then use k to find length of triangle 2. So, how do we use this method in the field.

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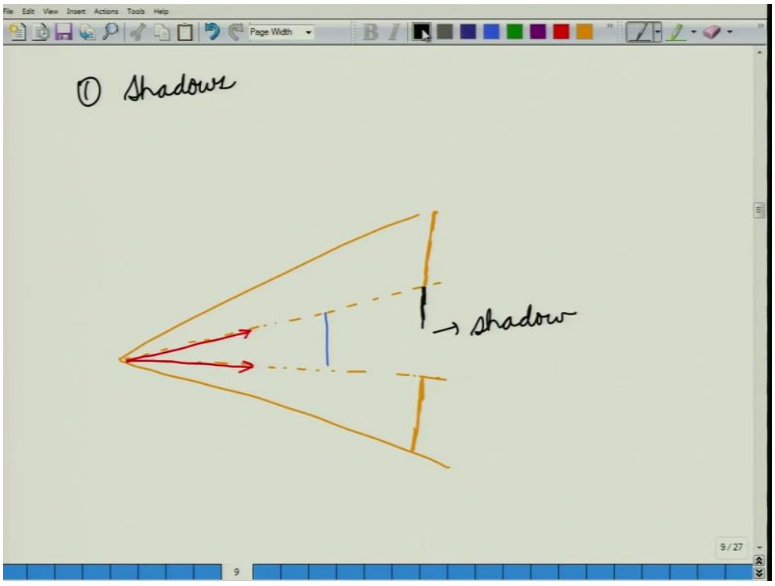


So, the first way of using this method would be using, now as we know if we have a source of light and if we keep some object in front of it then the rays of light get blocked by this object and on the screen would be having a shadow this region would be black in colour because it is not receiving any light from the source whereas, the top and the bottom portions would be illuminated.

So, here you have a shadow, so the first concept here is how shadows are formed. So, a shadow is formed because these light trees are moving in straight lines, because if they were able to curve around this object then maybe they would have reached this portion or maybe they would have reached this portion as well, but this does not happen because light travels in straight lines. The second concept would be to know about how these rays of light are at a distance so for instance.

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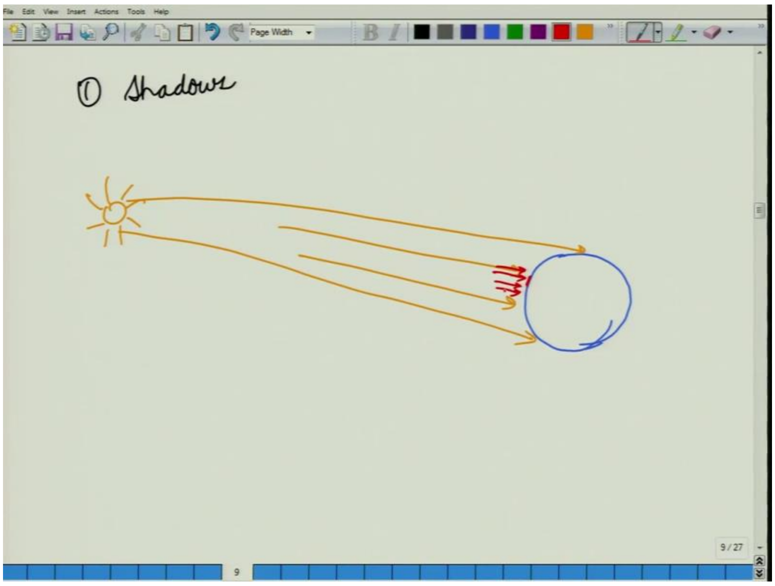


If we consider the sun and we have the earth, that is at a very great distance from the sun. So, here the rays of light would appear to be nearly parallel. So, these are parallel rays of light and especially if we consider a small portion on the surface of the earth then we can take this approximation very easily that all the rays of light are parallel, when they are reaching this point this is because the sun is at a very great distance from the earth.

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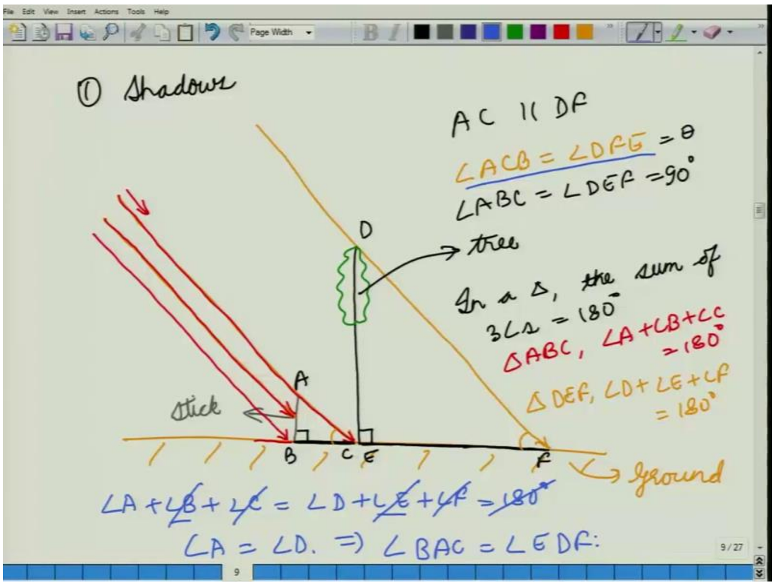


So, now that we know that the rays of light are parallel. So, suppose we have this ground and suppose let us draw 2 parallel rays, now consider two objects when object is your tree. So, here you have a tree and the second object is say a small stick. So, we have a stick and we have a tree.

Now, the rays of light let us draw another ray that is parallel to this, now this ray is intercepted by the object and this not able to reach the ground. So, this thing is your ground, so this ray of light this ray of light it is intercepted by the object and. So, is unable to reach the ground whereas, this ray of light it has just touched the surface the top of the object and is able to reach the ground, any day to the right of it would be able to reach the ground because it is not intercepted by the object.

So, what would that result in it would result in a shadow, so here we have a shadow, here if you had another the ray of light it would be able to reach the surface and. So, this portion is illuminated whereas, the portion to the right of the object is not illuminated similarly in the case of this tree will it will because of its greater height it will form a larger shadow.

So, this is the shadow of the tree, now let us consider these 2 triangles, we have triangle A B C and the triangle D E and F, now because the rays of light A C is parallel to the ray of light D f. So, we can say that these two angles are equal because these are two parallel rays of light that have been intercepted by the ground. So, the angle subtended by these two days to the ground would be the same, so we can say that angle A C B is equal to angle D F E.

Now, because both of these objects are said to be vertically above the ground, so both these angles are equal and equal to 90 degrees. So, angle A B C is equal to angle D E F is equal to 90 degrees and suppose this angle is theta.

Now, we know that in a triangle the sum of 3 angles is equal to 180 degrees. So, in the first triangle in triangle A B C we have angle A plus angle B plus angle C is equal to 180 degrees similarly in triangle D E F, we have angle D plus angle, E plus angle, F is equal to 180 degrees., from both of these we can say that angle A, plus angle B, plus C is equal to angle D, plus angle E, plus angle F, is equal to 180 degrees.

So, now we have seen here that angle C and angle F, angle C is the same as angle A C B and angle F is the same as angle D F E. So, angle C and angle F are equal angle D and angle E D and E are equal to 90 degree. So, they also cancelled out, so we have angle A is equal to angle D, angle A can also be written as angle D A C is equal to angle D or E D f.

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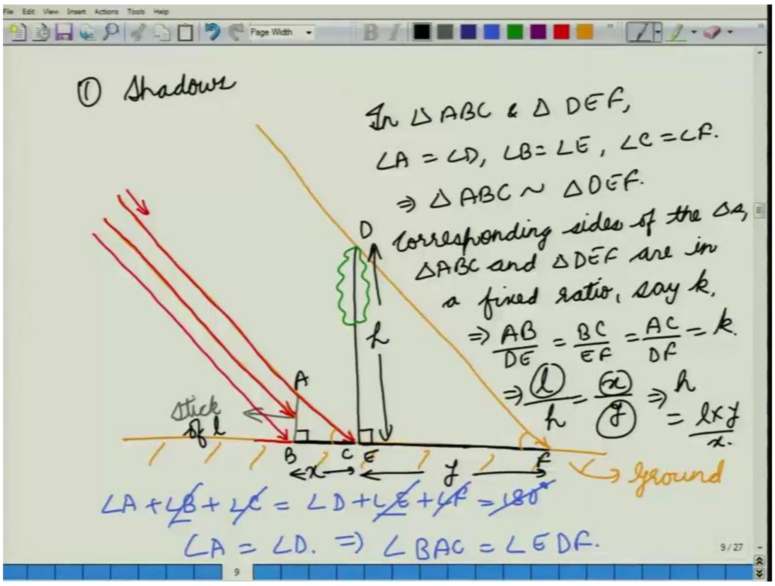

So, in these 2 triangles, so let us now again consider these triangles, so we have seen that in triangle A B C and triangle D E F, we have angle A is equal to angle D, angle D is equal to angle E, and angle C is equal to angle F, which implies that triangle A B C is similar to triangle D E f. So, what does that tell us it tells us that the corresponding sides of the triangles, triangle A B C and triangle D E F are in a fixed ratio say k, which means that A B upon D E, is equal to B C upon E F, is equal to A C upon D F, is equal to k.

Now, we are interested in knowing the height h of this tree, now because you have taken this stick. So, we can measure its length and suppose it is l. So, this equation would tell us that A B which is equal to l upon D E which is equal to h is equal to B C, now B C is the length of the shadow that is casted upon by the stick at this time. So, B C is equal to x E F is the length of the shadow casted upon by the tree so x upon y.

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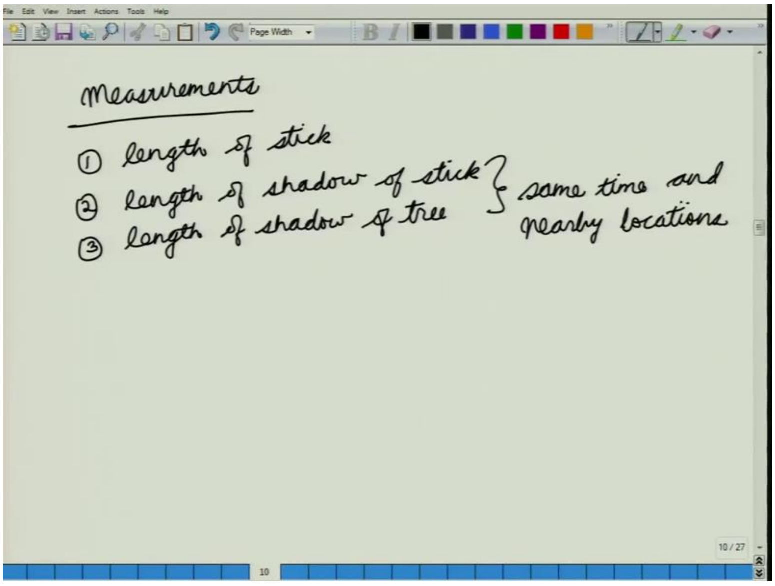


So, in this equation we know l we know x and we know y, so we can get h is equal to l into y upon x, so by using the method of shadows we can find out the height of a tree, what are the measurements that we need we would need the length of stick, length of shadow of stick and the length of shadow of tree, now both these measurements have to be at the same time and nearby locations. So, by using this we can figure out the length of the tree.

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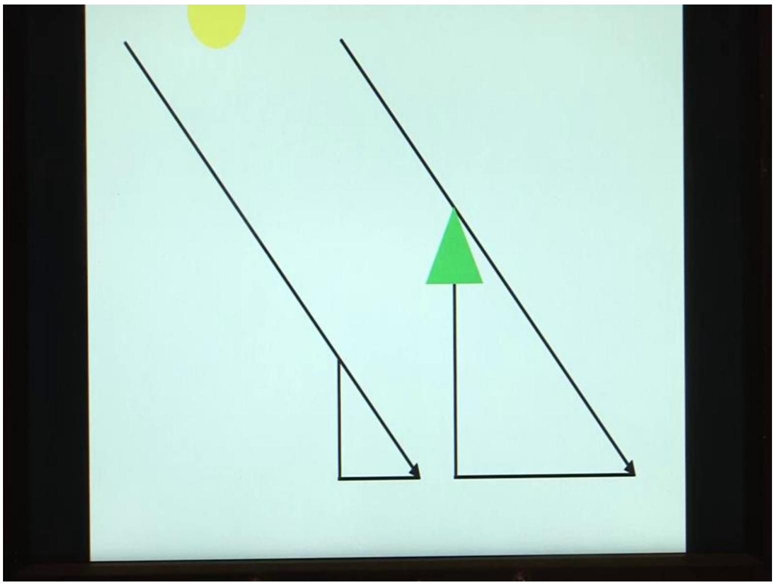

Now, another way in which you we use the method of similar triangles is called the single pole method. So, we are considering the methods of similar triangles, now what is the single pole method, in the single pole method we have an observer who extends his arm and then holds a stick to his arm in such a way that the length of the stick. So, suppose this is our observer.

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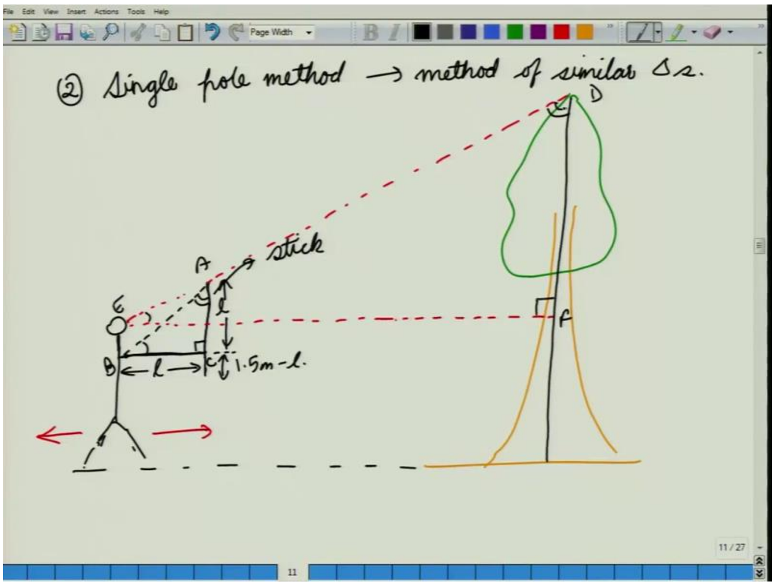


So, this is his arm and he is holding a stick, so this is the stick and he holds it in such a way that the length of his arm and the length of the portion of the stick, that is above his arm. So, this stick the total length is say 1.5 meters so this length would be 1.5 minus l.

So, he is holding this stick in such a fashion that the length of his arm and the length of the portion of the stick above his arm is the same, now he takes the stick and then he goes into the field and there he sees a tree, so what does he do now? So suppose, so he is standing at the same level.

Now, our observer will either go towards the tree or will go away from the tree to ensure that the from his eye this, the top of the stick and the top of the tree come in the same line. So, when that is done let us form 2 triangles, now draw a line from his eye to the tree suppose this is the axis of the tree, so in these, so now, let us take a vertical from here.

Now, consider these 2 triangles the first triangle is this A B C and the second triangle is this D E and F and because this length is a small length. So, we can consider that in these 2 triangles will have this angle is equal to this angle both these angles are 90 degrees and this angle is equal to this angle. So, which would make are both these triangles similar, so now, let looking at the slide now.

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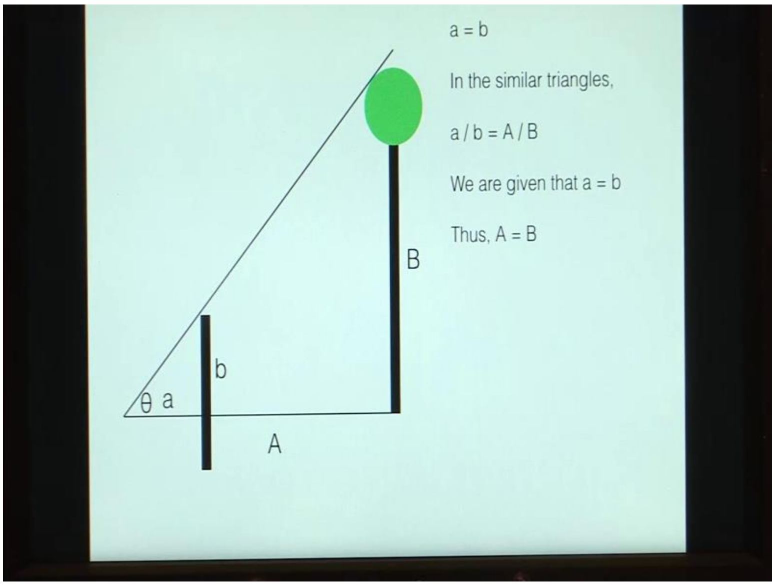


So, now we have a tree, and we have a tree of height B at A distance capital A from the observer, the observer has held the stick in such a form that A is equal to B, where A is the length of his arm and B is the length of the stick on the top of the arm, and now the observer has moved forward and backward such that the top of the stick and the top of the tree are in the same line.

So, then we will have 2 similar triangles and in these 2 similar triangles, will have small A by small B is equal to capital A by capital B, because in these 2 triangles.

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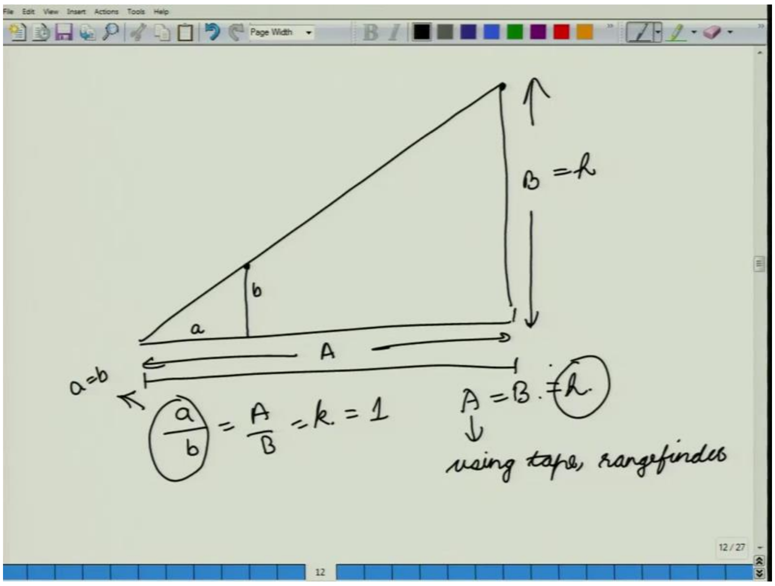


We have small A and small B and we have capital A and capital b. So, in these 2 triangles we have small A by small B is equal to capital A by capital B is equal to some constant k.

Now, because small A and small B have been kept in such a way that, A is equal to b. So, this ratio is equal to one so once the observer has achieved this location such that the top of the stick and the top of the tree are in the same line. So, at that point we can see that the length A will be equal to the length B, now the length B is equal to the height of the tree and the length A is this distance from the observer to the tree, which can very easily be measured using the tape or say a range finder or by some other ways. So, when this is achieved then we can very easily get the h,.

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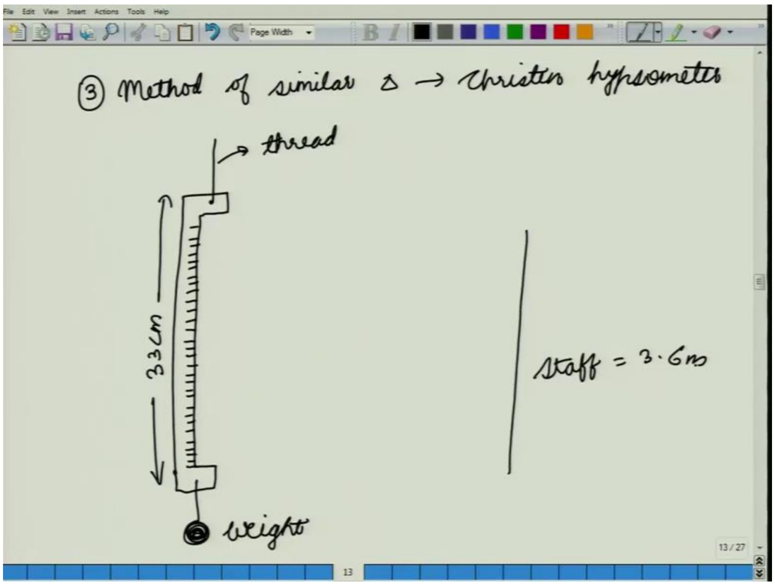


A third way of using the method of similar triangles is the use of a device known as a christen hypsometer. So, looking at the slides now a hypsometer is a very simple device.

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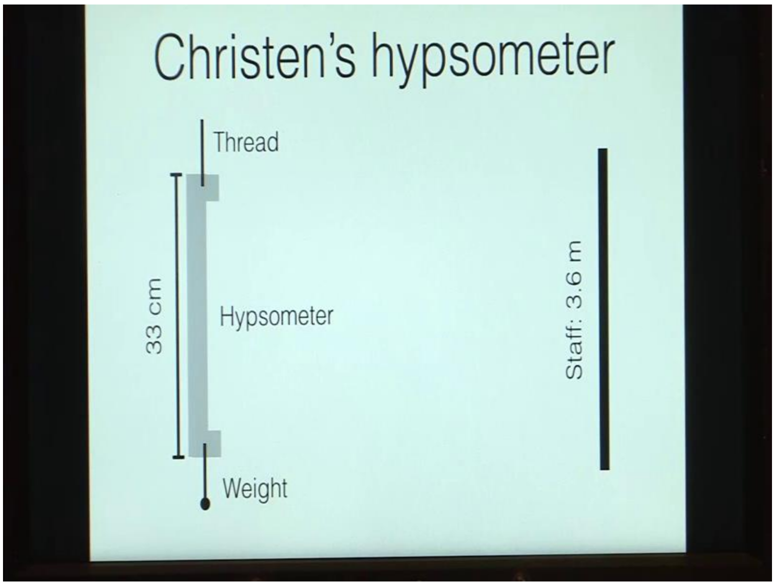


So, it is either made of wood or it is made out of metal or made out of plastic or say cardboard. So, it can be made out of any simple material the instrument is in the form of a strip with two flanges.

So, suppose this is your instrument made out of cardboard on this side we will mark the scales. So, you can take measurements from this side, one end of the hypsometer would be connected to a thread to suspended and the other end would be connected to a weight. So, that the hypsometer is kept vertical at all times the length of a hypsometer is generally taken to be 33 centimetres or roughly (Refer Time: 19:46).

The hypsometer is always used with a staff, now a staff is a pole, that is generally taken to be of the length of 3.6 meters. So, how is this hypsometer used if we have a tree and this is your ground level?

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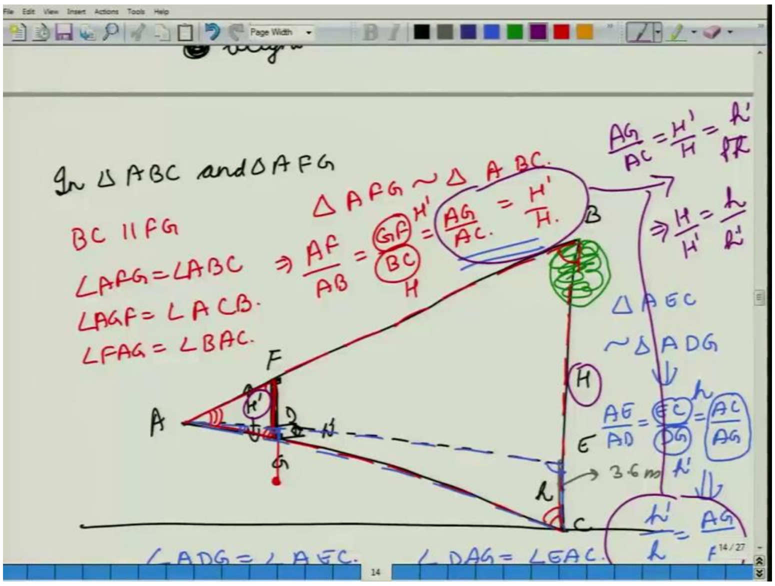

So, you have a tree here, so you keep the staff right next to the tree. So, you know that this length is equal to 3.6 metres. Now, the observer stands at a distance from the tree and he ensures, that when he is looking at the top and the bottom of the tree when he keeps the hypsometer in such a way, that the top and bottom of the tree subtend the same ray as the top and bottom of the hypsometer, and then he takes he looks at the top of the staff and then he takes a reading h.

So, suppose so this length is h prime and the total length of the hypsometer is capital h prime, suppose the height of the tree is capital h and the height of the staff is small h. So, in this situation we get that in this triangle. So, let us label the triangles, so A B C D and E, so now in triangle D F G. So, in triangle A B C and A F G, so we are considering these 2 triangles, 1 triangle is A F G and the second triangle is the extension.

So, in both these triangles we have that B C is parallel to F G, because both of these are vertical to the ground because your hypsometer is attached to a weight and this tree is growing vertically on top of the ground. So, both these sides B C and F G are parallel to each other.

Now, this angle A F G is equal to this angle A B C, because we have two parallel lines that are cut by an intercepting line, we have angle A G F or this angle is equal to this angle A C B and this angle is a common angle. So, angle F A G is equal to angle B A C.

So, now we have the triangle A F G, is similar to triangle A B C. So, in this situation we will have that the corresponding sides are in a constant ratio. So, we have A F by A B is equal to G F by B C is equal to A G by A C, now G F this length is the length of the hypsometer or h prime B C is the height of the tree or H. So, this is equal to h prime upon H.

Now, similarly in we can say that in this triangle. So, now, we considered these 2 triangles, triangle A E C and triangle A D G in both these triangles we have a common angle, this angle and this angle. So, angle A D G is equal to angle A E C, because again we have two parallel lines that are intersected by A line also angle A G D is equal to angle A C E and the common angle, angle D A G is equal to angle E A C. So, both these triangles are also similar which would give us that the corresponding sides are in a constant ratio. So, we have A E by A D, is equal to E C by D G, is equal to A C by A G.

Now, here we have that D G is equal to small h prime or the of this part of the hypsometer that is being subtended by the staff, E C is the height of the staff which is normally taken to be 3.6 meters is equal to A C upon A G. So, this would give us that h prime upon h is equal to taking the reciprocal of this A G upon A C.

Now, previously we have seen that A G upon A C is equal to h prime upon h. So, this observation and this observation would together give us that A G upon A C is equal to H prime upon H, which is equal to small h prime upon small h. So, we have that H upon H prime. So, H is the height of the tree divided by the length of the hypsometer is equal to small h upon small h prime, small h is the length of the staff and small h prime is the portion on the hypsometer that is subtended by the staff. So, in this way we can figure out the lengths of the triangle.

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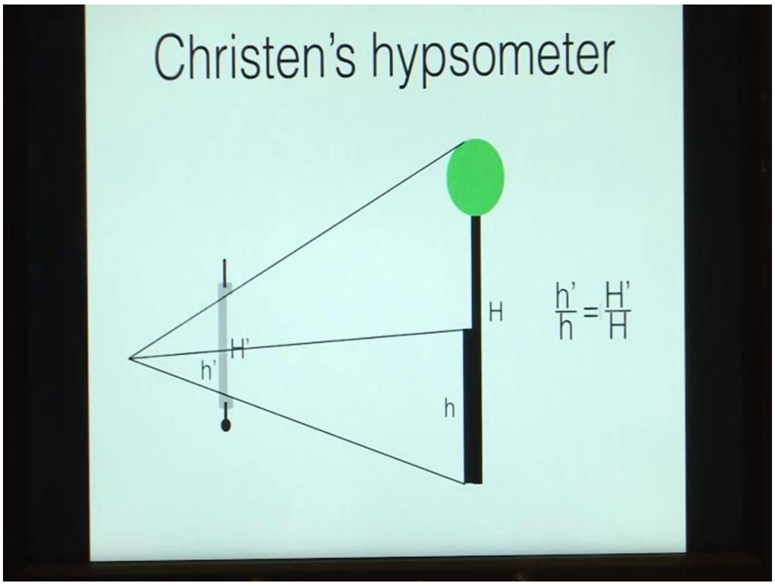


So, now let us look at an example, for it a 24 meters height tree the staff length is 4 metres.

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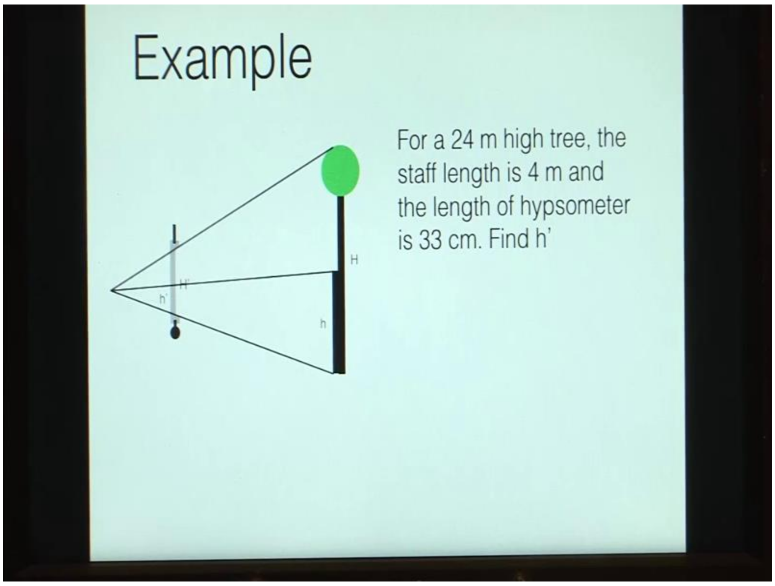


And the length of the hypsometer is 33 centimetres. So, we need to find h prime.

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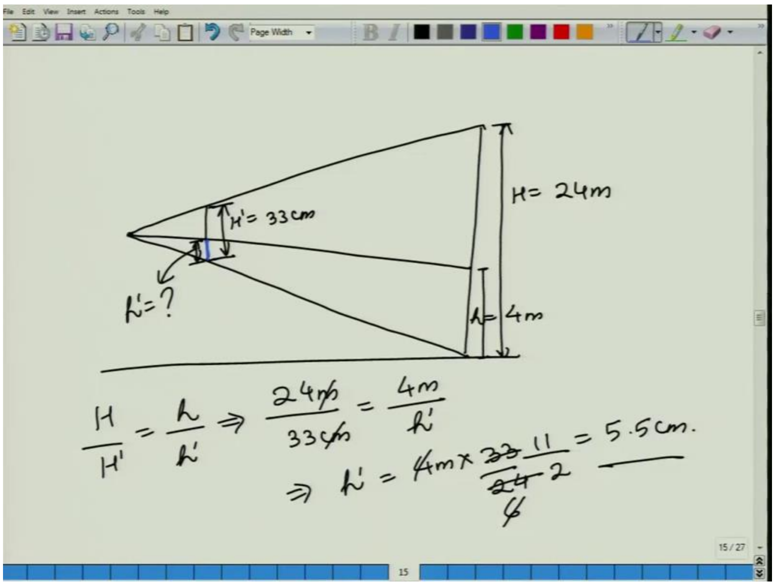


So, we have a tree this is our hypsometer and we are given that the height of the tree h is equal to 24 metres the length of the hypsometer or h prime is equal to 33 centimetres the length of the staff or small h is equal to 4 meters say it is not 3.6 in this case it is 4 meters.

We need to figure out, this length or h prime is equal to question mark. So, using the relation h upon h prime is equal to small h upon small h prime, we get capital H is 24 meters divided by h prime as 33 centimetres is equal to small h that is 4 meters divided by h prime. So, centimetres and centimetre it cancelled away we have h prime as 4 meters into 33 by 24, now 4 6 are 24 3 11 3 2 are 6 is 5.5 centimetres. So, we can figure out what is the length that is subtended by the staff on the hypsometer. So, this is how we are going to solve it

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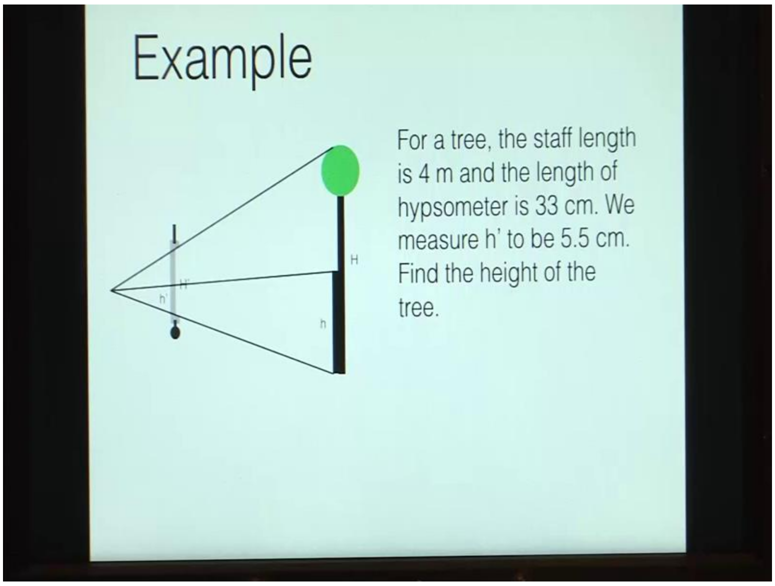


Now, let us look at another problem so as you can see on your slides. So, this is an inverse problem for a tree the staff length is 4 meters and the length of the hypsometer is 33 centimetres, we measure h prime to be 5.5 centimetres as we did in the previous case find the height of the tree. So, in this case we are asked to do the opposite thing.

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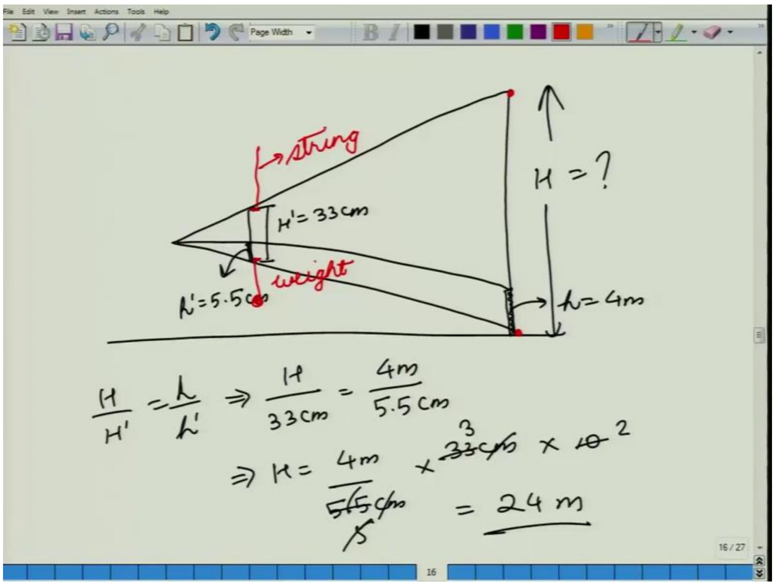


So, we have a tree given an observer, I was looking at the top and the bottom of the tree , through a hypsometer we have a staff that is added right next to the tree and the observer is looking at the top of the staff. So, here we are given that for a tree the staff length is 4 metres. So, we have small h is equal to 4 meters we are given that the length of the hypsometer is 33 centimetres. So, h prime is equal to 33 centimetres and we measured small h prime. So, this distance small h prime is 5.5 centimetres, we need to calculate capital H that is the height of the tree is question mark.

So, again using the relation H upon H prime is equal to small h upon small h prime, we get capital h upon h prime is 33 centimetres is equal to small h that is 4 metres upon 5.5 centimetres. So, we get h is equal to 4 metres by 5.5 centimetres into 33 centimetres. So, 11 3 are 33 11 5 are 55 5 2 are 10 yes 4 3 are 24 meters. So, we can calculate that the height of the tree is 24 metres.

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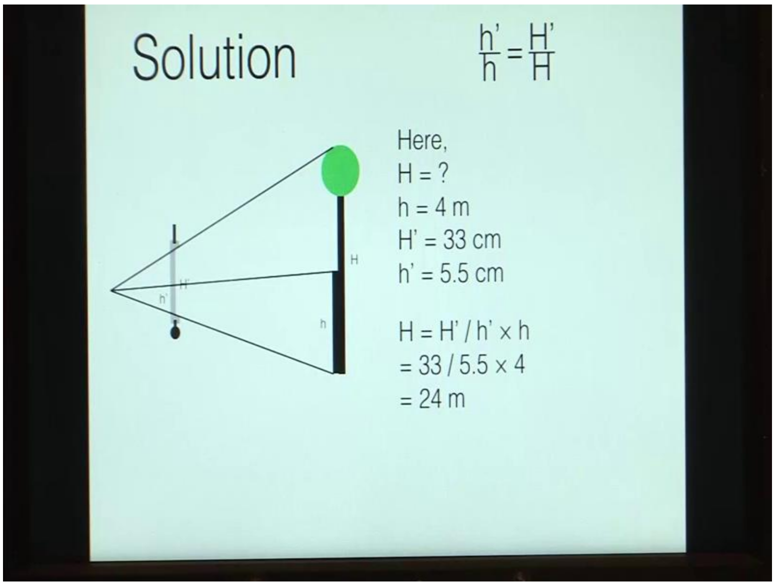


So, let us now look at some advantages of using this method or of using the hypsometer one the hypsometer is light.

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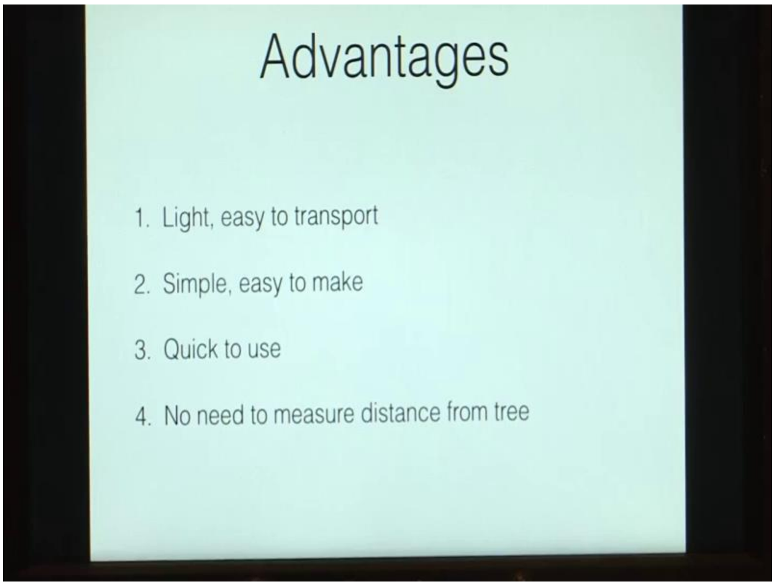


And is easy to transport it can be made out of wood, it can be made out of metal, it can even be made out of a piece of cardboard. So, it is very light and it is easy to transport it is very simple and easy to make you can make it at your home as well using a piece of cardboard it is quick to use out in the field and there is no need to measure the distance from the tree. So, you only need to measure the length of the staff and the length that the staffs subtends on the hypsometer these are the only two readings.

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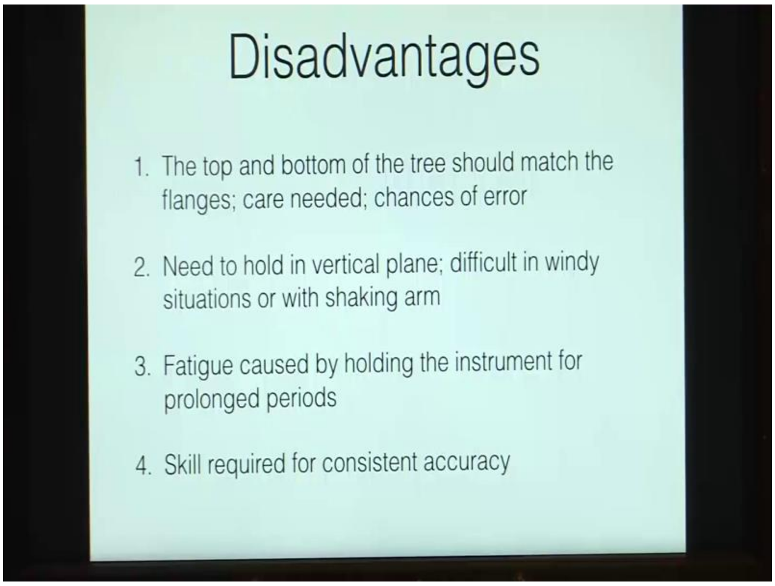

Now, the length of the staff will always be known to you because you will be using a fixed staff for your measurement. So, you need only one measurement and you can calculate the height of the tree.

Now, the disadvantages of using a hypsometer are these, the top and bottom of the tree should match the flanges and. So, as we saw in the diagram so if you move back to the tablet yeah. So, the top and the bottom of your hypsometer so this is the top this is the bottom it should match the tops and bottoms of the tree. So, you need to have some care while using this device and both of these need to match exactly otherwise you will have some errors.

You need to hold the hypsometer on a vertical plane. So, to hold it on a vertical plane you add this string on top and you add a weight at the bottom. So, this is just to keep it vertical, but even after doing this it might become difficult in situations when you have a breeze of air flowing through. So, if you have a windy situation your hypsometer might sway and when it sways then you have chances of error quipping in.

Then you will also have some amount of fatigue that is caused by holding your hypsometer like this in a vertical plane. So, when you are keeping it like this for prolong durations of time you will your hands will start aching and also you need to have considerable amount of skill to have a consistent accuracy. So, these are the disadvantages of using a hypsometer.

So, in this class we looked at the methods of similar triangle we looked at the methods of finding out the height of the tree using shadows using a pole and using a hypsometer. So, these are the shadow and stick methods of finding out the heights of a tree.

Thank you for your attention [FL].