**Forest Biometry**
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**Lecture – 19**
**Angular measurements**

[FL] today we shall have a look at how to measure angles. Now if you can remember you can measure the height of a tree using trigonometry, if you know the distance to the tree and the angular measurement, you can measure the height of a tree.

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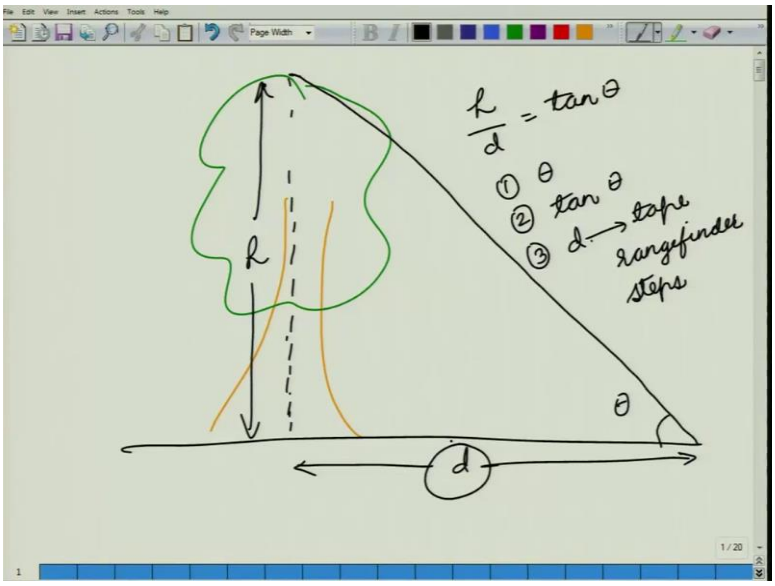


So, to recap if this is our tree this is the ground level. Suppose you know this angle theta and suppose you know this distance, if you know this distance d then we can figure out the height h of the tree by using the trigonometric relation h by d is equal to tan theta. So, in that case we will have to measure one the theta value 2 we will have to figure out the tan theta and 3 we will need to measure out D. Now d can be measured by using a tape by using a rangefinder or just by counting the steps as we have seen in the previous lecture.

So now that we know d how do we figure out theta is the next question. Now you must have used a device known as protractor in your earlier classes.

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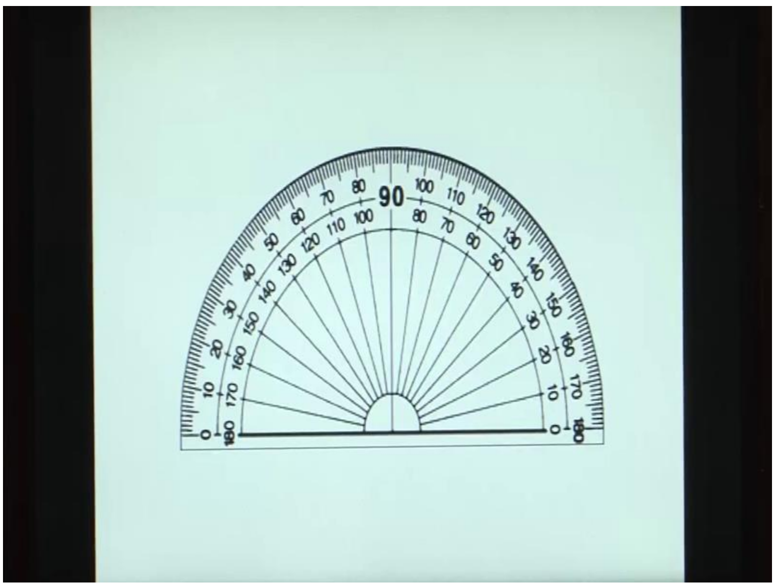

So, this is how a protractor looks like. So, this is an instrument it is generally transparent and all the degrees are shown here, from 0 degrees to 180 degrees on both sides.

Now, a protractor can very easily be used to measure an angle on a sheet of paper. So, for instance if you Wanted to measure this angle.

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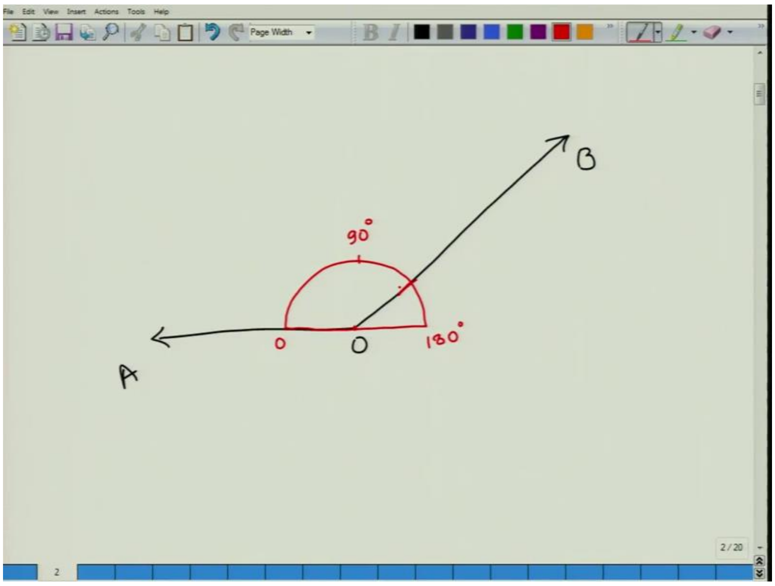


AOB what we would do is we would place the protractor here; such that this point corresponded to it is center. And then we would have all the degree measures from 0 degrees to 90 degrees to 180 degrees. So, at right at the point where this ray cuts the protractor we can read the reading from the protractor.

Now, it is easy to do it on a sheet of paper, but can that be used in the measurement of a tree height that is the next question. So, it turns out yes we can use it to measure a tree height, this is how it will look theoretically.

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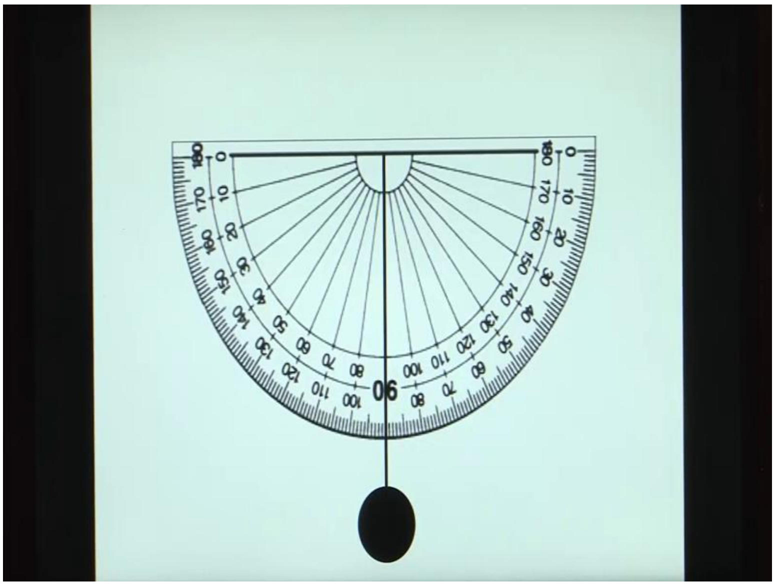

So, what we have done here is we have taken a protractor and right through it is 0 point we have drilled a hole and we have put a string through it. And on that string we can suspend a weight. Now if a weight is suspended then this string is always held vertical. So now, if this protractor is moved to measure any angles it can be used. So, let us now see how it would look in practice.

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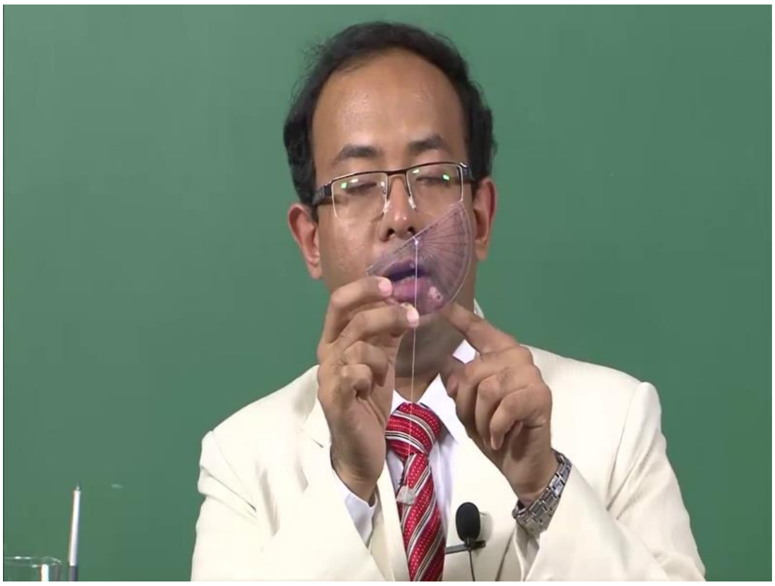

So, this is a protractor it is a very small instrument it is a very thin instrument. So, you can keep it in your pocket you can always drill a hole and keep a thread connected to it.

And at the end of the thread you can connect maybe a pebble or a small twig or any weight that you can easily get hold of in your forest. So now, if I hold it.

So now when I am holding it like this you can see that this angle is coming out to be 90 degrees. So now, if you want to measure the elevation of some point. So, when we turn it like this then the angle that we are interested in measuring is this angle. So, how can we reach to that angle? Well, we can directly measure that this point it now says 120 degrees. So, we can measure 120 degrees minus 90 degrees which would give us this angle.

So, how would we use it to measure say the angle of the top of a tree. We will use these 2 points. So, one point is this end of the protractor the other point is this end of the protractor, and if you have a tree somewhere there then we would try to adjust this protractor in such a way, that this point this point and the tip of the tree fall in the same line.

So, for instance if I considered this room as a tree and if I wanted to measure the angle of elevation of that point I would use it like this. So now, we need to keep in mind that this string with a weight attached to it would act like a pendulum. So, it would always be stringing. So, you can always dampen it by just turning this protractor to one side and this edge of the of this protractor would then dampen this string down.

So, if I need to measure the top of that roof it would give me this reading. So now, I can turn this protractor to it is side. So, that now this reading is constant. Now this reading is not moving. And now if I read it says 107 degrees.

So, what is the angle of elevation of the top of this from when measured from my eye level. So, how would I get to that? So, let us now again have a look at the tablet. So, what we have done is we have taken this protractor.

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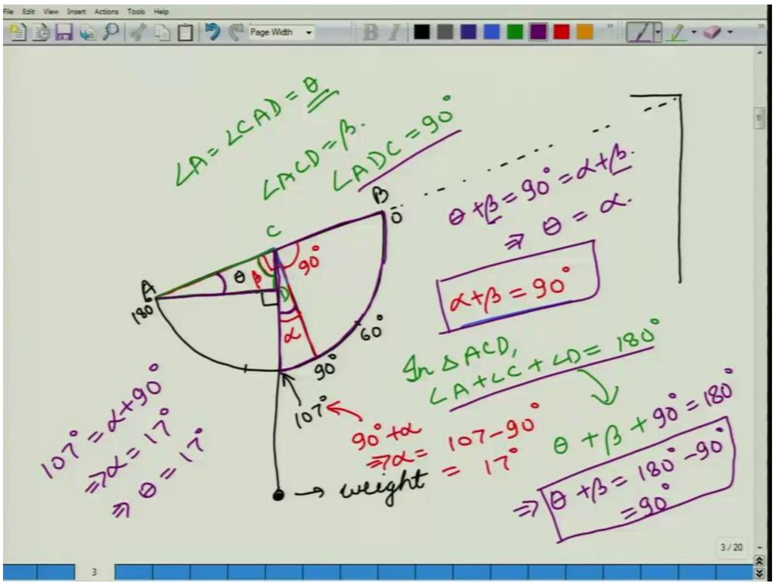


Inverted at and here we have a string to which a weight is attached. We have kept it such that this point A B they are in straight line as the top of the roof let us draw it like this. So, I am trying to measure this angle. The angle I am interested in is this angle theta. The reading that I got from this device. So, this is 90 degrees. So, it goes from 0 then 60 degrees 90 degrees. So, I am getting here and it goes up till 180 degrees now the angle that I have measured here is 107 degrees. I am interested in this angle. So, how would I get to that angle? So, if we extend this line then this is 90 degrees.

Now, we also know that this angle is 90 degrees. So now, how do we get to this angle theta? So, we have measured this angle let us call it alpha. Now if this angle is alpha and this angle suppose is beta. Now we know that alpha plus beta is equal to 90 degrees, because alpha plus beta corresponds to this angle. Or what exactly that we have measured is this again is 90 degrees. So, this angle 107 degrees it is 90 degrees plus alpha. So, we will get that alpha is equal to 107 minus 90 degrees is 17 degrees.

Now, in this when we are looking at this angle we have gotten the relation that alpha plus beta is equal to 90 degrees. Now consider this triangle. So, let us call it triangle ACD. So now, in triangle ACD angle A plus angle C plus angle D is equal to 180 degrees, because the sum of all 3 angles of a triangle is 180 degrees.

Now, angle A or angle CAD is equal to theta and this is the value that we are interested in knowing. So, from here we get theta plus angle C now angle C is angle ACD. Now angle ACD is beta this angle. So, theta plus beta plus angle D angle D is angle ADC which is 90 degrees. Because this is our horizontal lines and this is our vertical line. So, both are going to intersect at 90 degrees.

So, angle ADC is 90 degrees. So, this sum is equal to 180 degrees from here. So, this would imply that theta plus beta is equal to 180 degrees minus 90 degrees is 90 degrees.

Now, we already know we have already derived that alpha plus beta is equal to 90 degrees. So, from here we are getting that theta plus beta is equal to 90 degrees. So, theta plus beta is equal to 90 degrees is equal to alpha plus beta. So now, subtracting beta from both the sides we would get that theta is equal to alpha. So now, this angle that we are trying to measure is equal to this angle.

Now, what we have measured using our protractor is this complete angle which is 107 degrees. Now 107 degrees is equal to alpha plus 90 degrees. So, alpha is equal to 17 degrees. And so, theta also equal to 17 degrees.

Now, let us consider the situation in the case of a forest. Currently we showed how to measure the angle of elevation of a roof. Now similarly if we went into a forest and if we got That there was a tree whose elevation has measured from the from the point of the observers I if this were 17 degrees.

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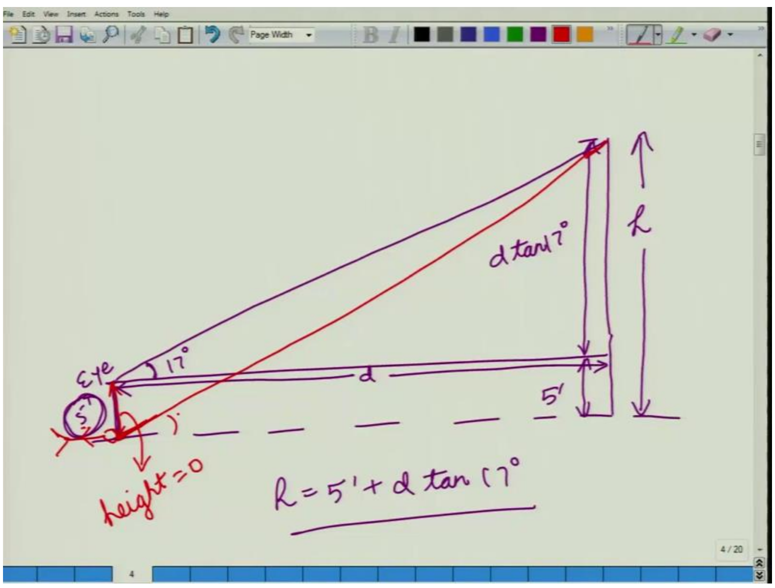

And the height of the eye from the ground level suppose it was say 5 feet and if you measured this distance that was d we would very easily be able to calculate the height of the tree. Now the height of the tree would be in 2 parts this portion is equal to d tan 17 degrees. And this position is equal to 5 feet. So, we will get h is equal to 5 feet plus d tan 17 degrees.

Now, this height 5 feet is a height that varies with every observer, but every observer can figure out what else is eye level is compared to the ground. So, this value is known to the observer and he can very easily calculate this angle 17 degrees. So, you can calculate the height of the tree just by using this small device.

Now, when you are carrying this device into the forest there is no need to carry this pebble along with it because you can always get up get a small pebble or a stone or maybe you can even carry a twig, we could in place of putting this pebble here we could have put a small twig. That would all have also given us the same gradient, because it is just acting as a weight to keep this string vertical. Now when you are carrying it into the forest you can always keep it in your pocket because it is a very small and handy equipment.

So now knowing this let us look at San example question.

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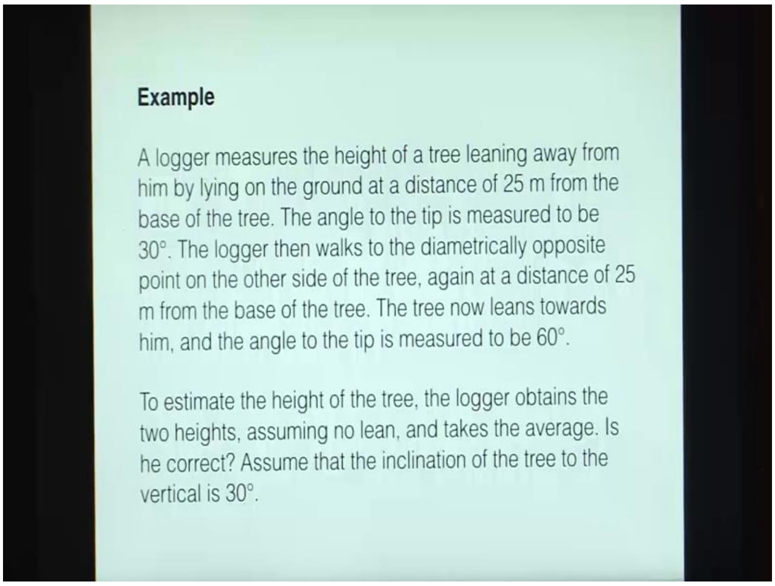

A logger measures the height of a tree leaning away from him by lying on the ground. So, when we say lying on the ground then in if you look back at the tablet this eye level this height this height is equal to 0. Because what the observer is doing is that the observer is lying on the ground and then taking the measurements. So, that it is as close to 0 height as possible. So, he is measuring this angle.

Now, coming back to the question, a logger measures the height of a tree leaning away from him by lying on the ground at a distance of 25 meters from the base of the tree. And the angle to the tip is measured to be 30 degrees. So, if you represented this graphically how would it look like.

So, if this is the ground level.

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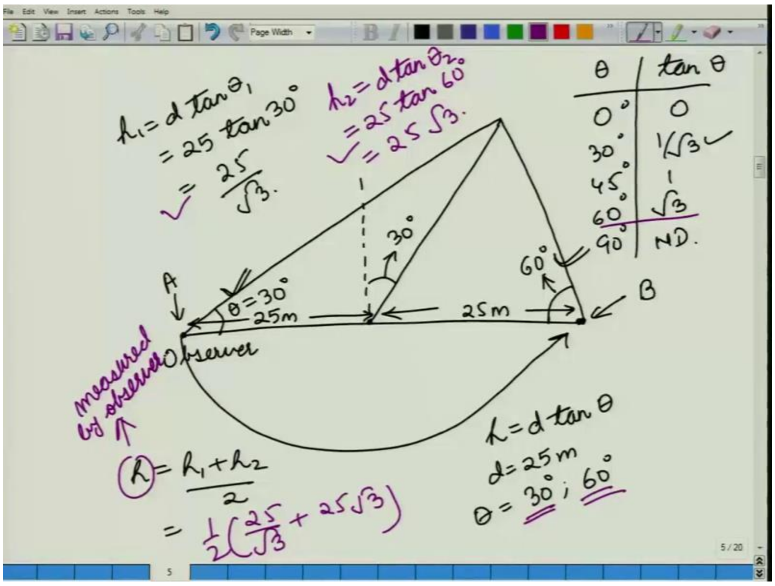


And there is a tree that is leaning away from the observer. So, this is the point of the observer. So, he lies on the ground at a distance of 25 meters from the base of the tree, and he measures the angle to the top. So, he has measured this angle.

Now, coming back to the question, the angle to the tip is measured to be 30 degrees. So, this angle theta is equal to 30 degrees. Now in the question it says the logger then walks to the diametrically opposite point on the other side of the tree, again at a distance of 25 meters from the base of the tree.

So now what the logger has done as he has moved from this position the first position to another position that is diametrically opposite, this position again is at a distance of 25 meters from the base of the tree. And is again measuring the angle of elevation of the tree. So now, looking back at the question.

So, the logger then walks to the diametrically opposite point on the other side of the tree again at a distance of 25 meters from the base of the tree. The tree now leans towards him and the angle to the tip is measured to be 60 degrees. So, he has measured this angle and this angle comes to 60 degrees.

Now, in the question it further states to estimate the height of the tree the logger obtains the 2 heights assuming no lean and takes the average, is he correct? Assume that the inclination of the tree to the vertical is 30 degrees. So now, coming back to the tablet. What it states is the angle of inclination of the tree to the vertical. So, this angle is 30 degrees. Now what the logger has done is he has taken 2 readings. One at point A one at point B. Now he has taken 2 readings of the tree he has assumed this tree to be having no leaned.

So, essentially what is measuring is h is equal to d tan theta. Now he is measuring this. So, once he is at a distance of 25 meters and in the other case also he is at a distance of 25 meters. So, d is equal to 25 meters in both the cases.

The first angle that he has measured is 30 degrees. And the second angle is 60 degrees. So, he has calculated both these heights and he has calculated h is equal to h 1 plus h 2 upon 2. Because he has taken the average of both these readings. So, how much would that height be?

So, h 1 h 1 is equal to d tan theta 1 is equal to 25 times tan 30 degrees. Now if we lose the relation between theta and tan theta. So, that would come to 0 30 45 60 and 90 degrees. And it comes to 0 1 by root 3 one root 3 and not defined. So, these are the relations of between theta and tan theta.

So, 25 tan 30 degrees will be equal to 25 upon. So, it is 25 upon root 3. So, that is h 1. Then he measures the second height from the second position, h 2 is equal to d tan theta 2. Now theta 1 was 30 degrees theta 2 is 60 degrees. So, it is 25 tan 60 degrees. Now tan 60 degrees is root 3. So, it is 25 root 3.

So now this is h 1 this is h 2 he calculates the height h to be the average of both of these. So, it is 25 upon root 3 plus 25 root 3 whole multiplied by half. So, this is the angle this is the height of the tree as measured by observer. Now how much is the actual height of the tree? How do we figure that out?

So now let us draw the diagram again.

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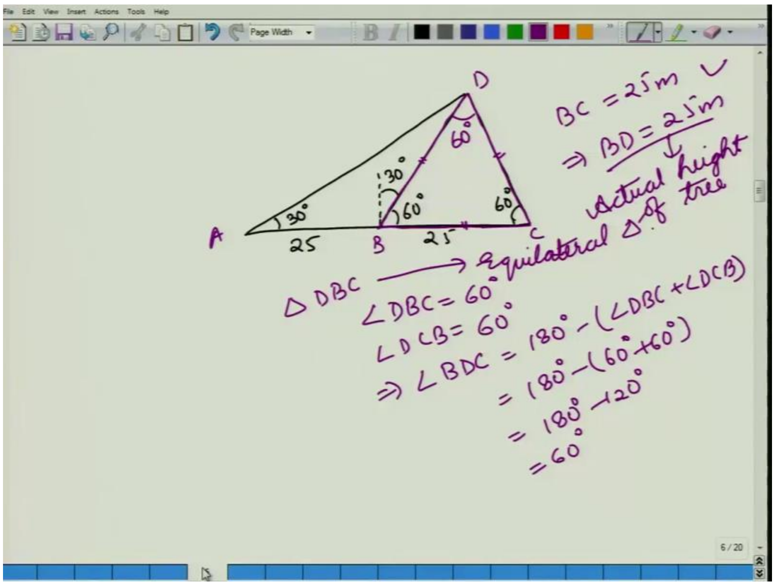


So, in the diagram we had this centered point. So, this was 25 meters this was 25 meters this angle is 30 degrees, this angle is 60 degrees. And the angle of inclination is 30 degrees, which would give us that this angle is 60 degrees.

Now, how much is the actual height of the tree? So, there are 2 ways of solving this problem. One the easier method you can always see that in this triangle. So, let us call these A B C D. So now, consider this triangle. So, here we have angle DBC is the. So, we are considering triangle DBC. So, angle DBC is 60 degrees angle D C B is also equal to 60 degrees. So, what is angle BDC? Bangle BDC will be 180 degrees minus the sum of the other 2 angles angle DBC plus angle D C B is 180 degrees minus 60 degrees plus 60 degrees is equal to 180 degrees minus 120 degrees is equal to 60 degrees. So, we have figured out that this angle is also 60 degrees.

Now, in this triangle all the 3 angles are equal to 60 degrees. Which means that this is an equilateral triangle. So, this is an equilateral triangle. Now in an equilateral triangle all the 3 sides are equal. So, this side is equal to this side is equal to this side. Now because we are given that B C is 25 meters. So, we can get that BD is also equal to 25 meters. So, this is the height of the tree actual height. So, this is the actual height of tree.

So, what the logger had measured was this, what we have received here is 25 meters. Now this happened to be an easy question because we had both these angles to be 60 degrees. But what if these angles were not 60 degrees. So, how would you figure out the height of the tree. So now, let us draw another diagram.

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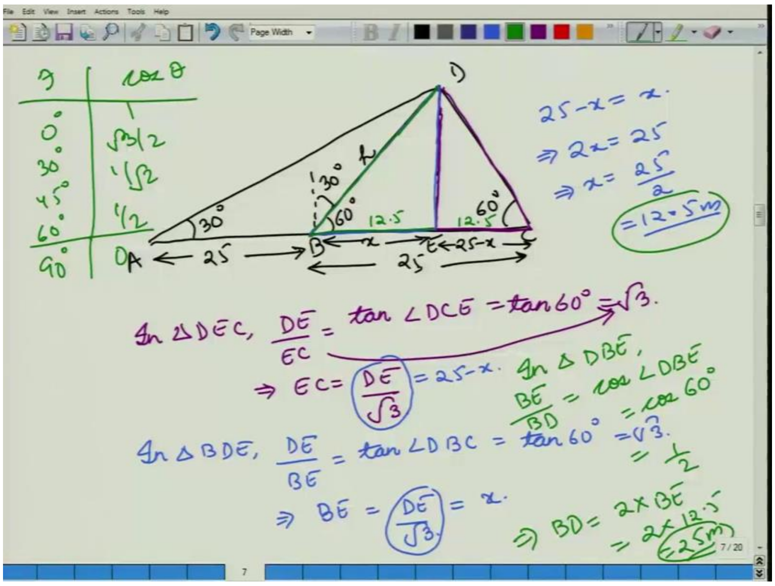


A B C d this angle is 30 degrees this angle is 60 degrees this angle is 30 degrees. So, this angle is 60 degrees.

Now, we can always draw a perpendicular from D to the side AC let us call it E. Now we are interested in knowing h. Now this is given to be 25 meters, this is also equal to 25 meters. So now, let us consider BE to be equal to x. So, in that EC will be 25 minus x.

Now, in this triangle DEC in triangle DEC we have DE upon EC is. So, this is DE that is the perpendicular upon the base, is equal to tan angle DCE is tan 60 degrees that we know to the root 3. So now, we can figure out that d that EC is DE. So, putting this on the other side. So, we have EC is equal to DE upon root 3.

Now, let us calculate for another triangle let us consider this triangle BDE. So, here we have in triangle BDE we have DE upon BE is again the perpendicular upon the base is equal to tan angle DBC is tan 60 degrees is root 3. So, we get that BE is equal to DE upon root 3. Now we know that BE is equal to x and EC is equal to 25 minus x.

Now, as we can see 25 minus x is DE upon root 3 here also we have d upon root 3. So, we can get that 25 minus x is equal to x or 2 x is 25 or x is 25 upon 2 is 12.5 meters. So, we have calculated x. So, this portion is divided into 2 halves which are 12.5 and 12.5.

Now, we are interested in knowing this side that is DB. Now in triangle DBE. We have BE upon DB. So, BE upon DB which is equal to cos angle DBE is cos 60 degrees. Now how much is cos 60 degrees. So, if you looked at the relations again. So, if we have theta verses cos theta 0, 30, 45, 60, 90 the relation goes like 0 half 1 by root 2 root 3 by 2 and 1. So, cos 60 degrees is half.

So, here we have BE upon BD is half. So, we can calculate BD is equal to 2 into BE is equal to 2 into, now BE is x if figured out to be 12.5 is 25 meters. So, whether we use this equilateral triangle concept or whether we calculated it through trigonometry we got the same answer that the height of the tree is 25 meters, but what did the logger measure the height of the tree to be he calculated it to be half of 25 by root 3 plus 25 root 3.

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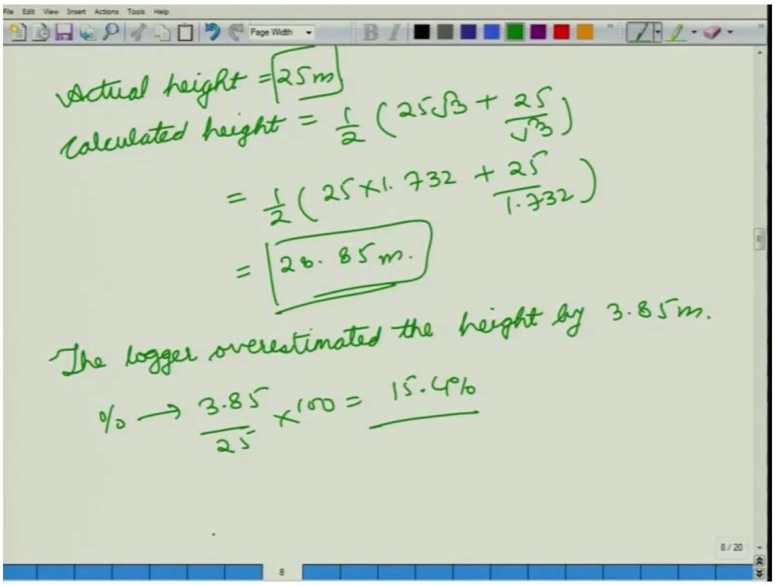


So, we have Actual height is 25 meters and the calculated height is equal to half of 25 root 3 plus 25 by root 3. Now root 3 is equal to 1.7352 meters. So, from here we would get this value to be equal to half of 25 into 1.7352 plus 25 upon 1.7352 and this we get to be 28.85 meters.

So, this is the actual height this is the height that was calculated. So, the logger over estimated the height by 3.85 meters or if you wanted it in percentage we can calculate it by 3.85 divided by the actual height into 100 is equal to 15.4 percent.

So, essentially if you wanted to calculate the height of a tree, you do not have to take the average of these 2 heights, but you should go either by this method of geometry or the method of trigonometry to calculate the correct height.

Now, in our demonstration we have seen this protractor. That can be used to measure the angle. Now what if there were now if I am measuring this angle this portion of the string it has to be kept vertical, but then it will act as a pendulum. So, I can dampen it down, but then measuring this angle becomes a bit tricky, because while taking the measurement itself the string could move a little. So, could we have another instrument that will give us these angular values easily.

So, if you look at your screen now.

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So, this is that instrument it goes by the name of hagas altimeter hagas altimeter.

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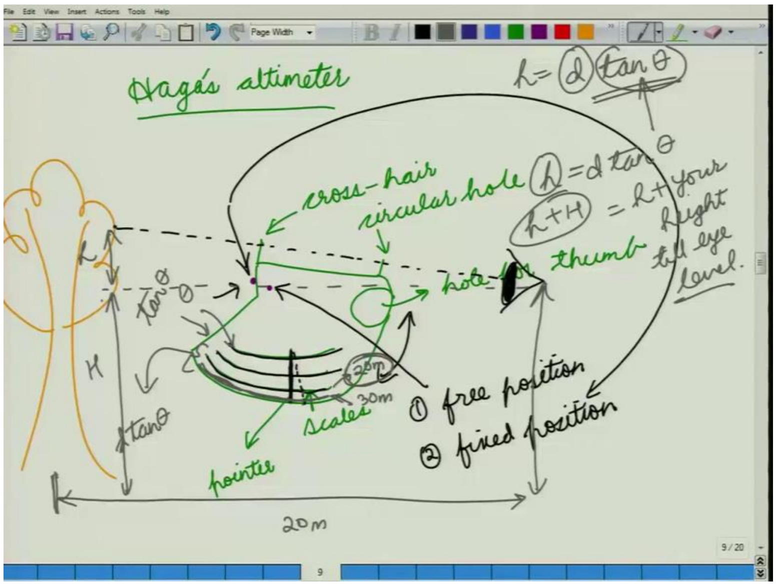


So, how does his equipment function? Now coming back to the on to the slide. So, as you can see this instrument on it is right side it has a circular hole in which you can put your thumb, then on the top you can see 2 ends on the left end and the right end. Now both these ends have to be matched together and the line of site should go to the point where you want to take the measurements. And then downside you can see a number of scales that can be used for measurement.

You can also see that on that scale we have a pointer that is currently put to the right side now when we deploy this instrument we use it like this. So, we have 2 measures. So, this is these two. So, here we have a cross hair and here we have a circular hole, then here we have a hole for thumb and here we have number of scales, and we also have a pointer.

Now, if we went back to the slide again. So, as you can see on the left side you can see a small button that is right beneath the cross hairs. Now there would be 2 buttons actually. So, you will have one button here and the other button here. So, if there are 2 buttons we will see it in another picture.

Now, how do you take the measurements? So, suppose there were a tree somewhere, you need to observe the top of the tree from this side. So, suppose this is your eye. So, you need to ensure that any point that you need to measure. So, suppose this is the point we are trying to measure. So, this point the crosshairs the circular hole and your eye, they all have to be on the same line.

Now, this pointer can be put into 2 positions. One position at the free position. So, a free position is reached when you press this button. So, when it is in the free position then this pointer is always vertical no matter how much you tilt your equipment this will always be vertical. So, it is very similar to this case.

So, when the pointer is free to move it is very much like this. So, whether you are tilting your equipment to any side this line is always vertical. Now when you press the other button. So, coming back to the tablet when you press this button, it puts your pointer into the fixed position. So, what that does is to take the example of the protractor. So, this is currently in the in the free position when you press the other button. So, suppose it was at this position when you press the other button this pointer will become fixed.

So now you can always take it out you can move it in a in any side, but this line will be fixed the line of the pointer. And so, you can very easily measure the angles using your scales. So, coming back to the slide now to the tablet. So, here this pointer were into the free positions. So, it would become vertical, once it has become vertical you press this button. And then it will become fixed and then you can take them the measurements using these scales.

Now, why are they multiple scales here? So, let us first look at some slides. So, here we had the instrument to measure the height. You would first measure the distance from the base of the tree that you can do with the tape or with the range finder or maybe with your steps. Then you would stand to one side and then you will arrange your instrument such that your eyes the circular hole that is for your optical hole, the cross hairs in the front and the top of the tree came in the same line. Then you would press this button.

So, when you press this button the pointer has become fixed and then you can take the readings from this scale. But this instrument also does something very smart. So, coming back to the tablet we know that the height is equal to d tan theta. So, if you used a protractor if you used this instrument, you would just be able to measure theta. So, once you have measured theta you will have to calculate tan theta or look it up look at it is values in a table or maybe on a calculator then multiply it with your distance d, but what this device does is it directly calculates tan theta and shows it on one of the scales.

So, one scale would show you theta another scale would show you tan theta. So, you have directly come to the measurement of this. There would be other scales as well on this device that would give u d tan theta for 2 different distances. So, consider. So, let us suppose that these 2 scales were giving you d tan theta. So, there would be one scale for say 20 meters and the other scale for say 30 meters.

So, to use this instrument you would take your position from the base of the tree such that this distance was 20 meters. Once you have done that when you take the readings from the from this device you can locate this is scale of 20 meters and you will directly get the height.

Now, this height would be the height of the portion of the tree that is above your eye level and you can add your height to it to get the height of the tree. So, basically what you are getting is h d tan theta, but this h is the is this height you need to add your height to the figure to get h plus h. So, this was say small h this one is your capital h the height of the tree is h plus h. So, you just need to take readings from your device for the same scale 20 meters. So, you got h plus you added your height, or the height of your height till eye level. So, in this way you would directly get the height of the tree.

So, today we looked how to measure the angles using a protractor using an altimeter and how to get the height of a tree from these angular measurements.

Thank you for your attention, [FL].