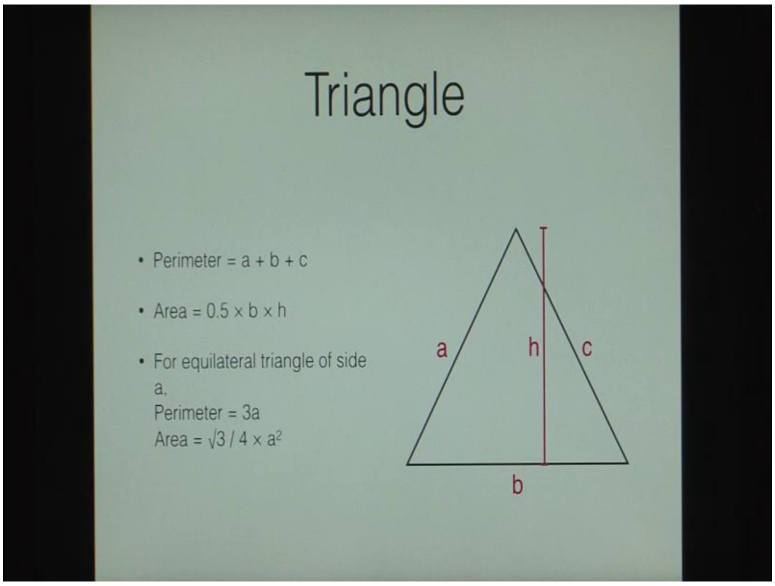
Forest Biometry
Prof. Mainak Das
Dr. Ankur Awadhiya
Department of Biological Sciences & Bioengineering & Design Programme
Indian Institute of Technology, Kanpur

Lecture – 02
Recap of formulae: Area and Volume

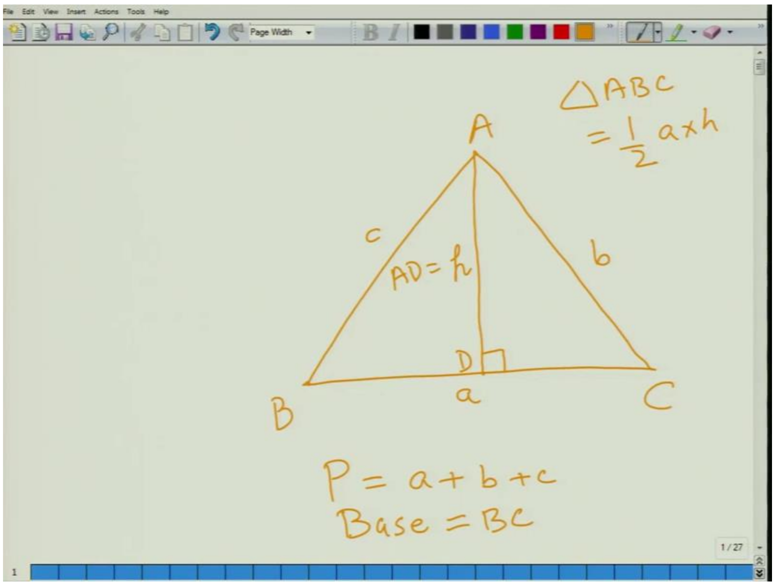
[FL]. This is the week of for introductions. And we shall recap many of the formulae that you already know. This everybody is on the same platform for better understanding of the topics.

(Refer Slide Time: 00:25)



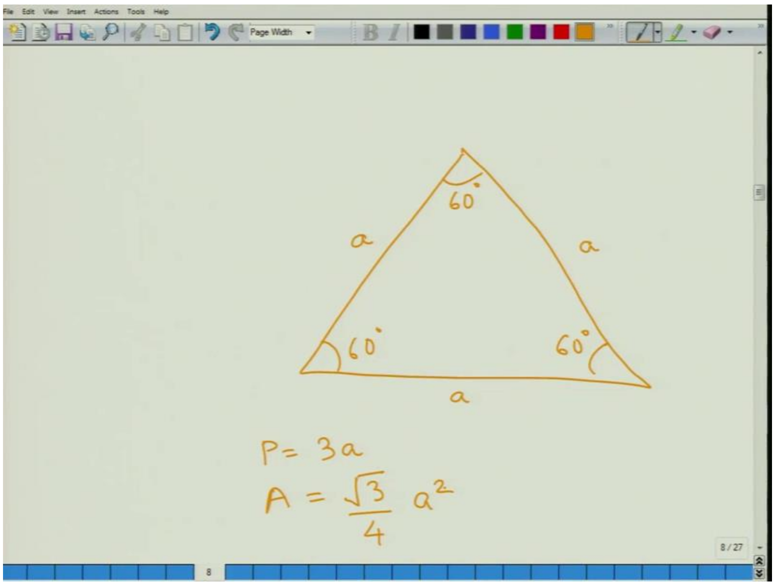
A triangle is the plane figure with three sides and three angles; its perimeter if the sum of the lengths of the three sides; the area is half of base length times the height. Now, how do we figure out the height? A triangle is a plane figure with three sides and three angles.

(Refer Slide Time: 00:52)



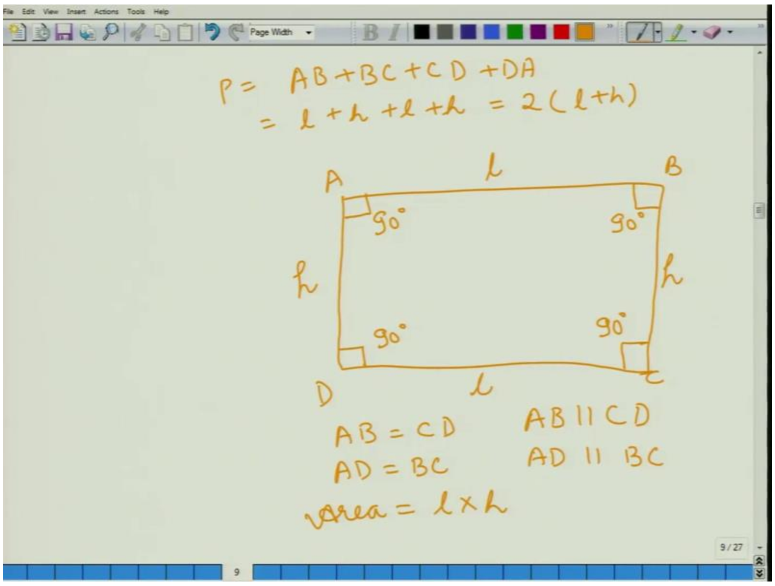
Now let us draw a triangle. Suppose, this is a triangle that we refer to as ABC, now these sides have lengths of small a, small b and small c. The perimeter of a triangle is the sum of the lengths of the three sides. So, P or the perimeter equals a plus b plus c. The area of the triangle is half of the base length times the height. Now, suppose we take the base to be BC, so your base equals BC. So, how do we find the height we take 0.8 and we draw a perpendicular to BC, suppose it intersects at point D. Now, the length AD is equal to h. So, the area of the triangle, triangle ABC would be equal to half of a into h. And you can do this for any of the other sides as well.

(Refer Slide Time: 02:15)



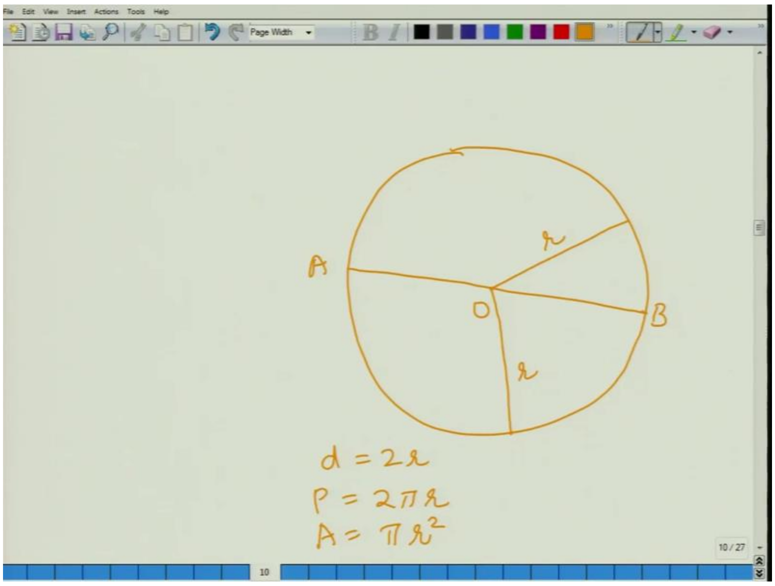
An equilateral triangle is a triangle in which all the three sides are equal and all the three angles are 60 degrees. So, if you draw an equilateral triangle will have all these three angles as 60 degrees and all the sides are of the same length, suppose we call it a small a. So, its perimeter would be a plus a plus a or thrice of a that is three times the length of a side. The area of an equilateral triangle can be found by using this formula A equals root 3 by 4 times a square.

(Refer Slide Time: 02:56)



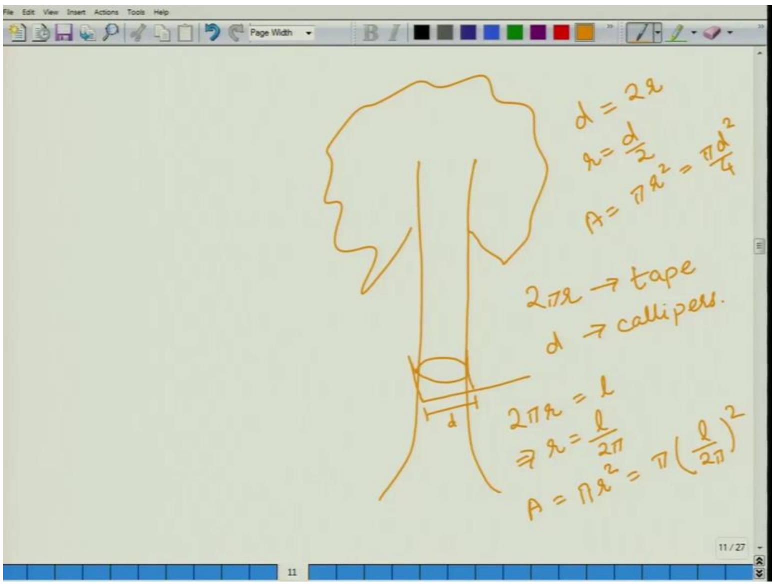
Rectangle on the other hand is a plane figure with four sides and four right angles. Suppose, we call this rectangle ABCD, now, it has got four sides AB is equal to CD and both of these are parallel that is AB is parallel to CD. Similarly AD is equal to BC and AD is parallel to BC. So, the opposite sides are equal and opposite sides are parallel. All the angles are 90 degrees. Now, suppose we call CD as having the length l. So, AB will also be having the length l. If we say that BC has a height of h then AD also has a height of h. The area is equal to base into height that is l into h. The perimeter will be equal to AB, so perimeter equals AB plus BC plus CD plus DA, AB is l plus h plus l plus h equals twice of l plus h. So, the perimeter is twice the sum of the lengths of sides and the area is of the product of the lengths of sides.

(Refer Slide Time: 04:38)



Let us consider the next figure it is a circle. Let us draw a circle. So, a circle is a plane figure it has a center that we represent as O, and all the points on the on the circle are at the same distance r from this the circle or from the center. So, this is r, this is also r. The diameter is called that so it is a line that passes through the center and it cuts the circles at two points A and B. Now, diameter is twice the radius the perimeter is given by 2 pi r. And the area of the circle is given by pi r square. Now, why do we need to know about a circle? Many trees especially coniferous trees have a circular cross section.

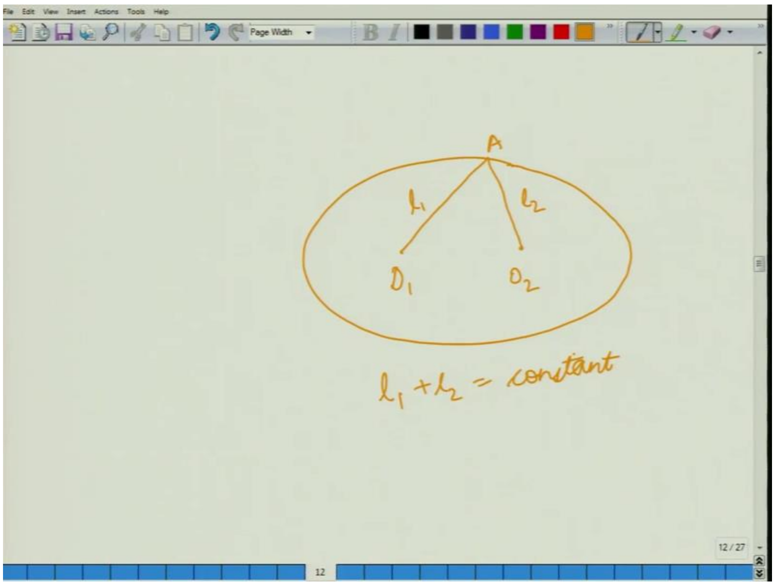
(Refer Slide Time: 05:41)



So, suppose we have a tree and suppose this cross section is a circle. Now, if we know the circumference of this circle we can find out 2 pi r. Now, how do we find out the circumference of the circle will begin use a tape. So, you take a tape; move it all around the circle and the length of that tape will give you the perimeter of the circle. Alternatively you can use callipers. So, when you use callipers, you can put one end of the calliper at this point the other end of the calliper at this point and you can figure out this length and this length will be equal to d. So, either we can find out 2 pi r using a tape or we can find out d using callipers.

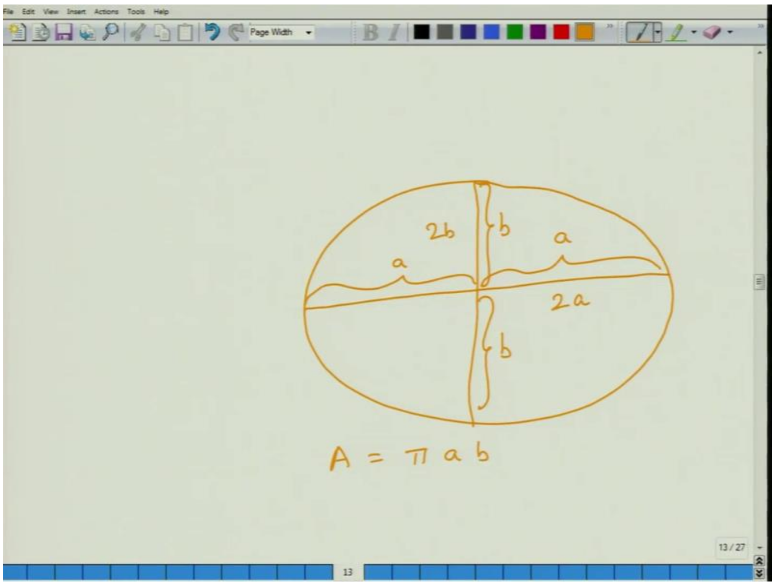
Now in both these situations if you wanted to figure out the basal area that is the area of this circle you can do it. So, suppose you know this 2 pi r equals length l so that would imply that r equals l upon 2 pi and the area which is pi r square will be given by pi into l by 2 pi square. Alternatively, if we knew d we know that d equals twice of radius, so r equals d by 2, and the area or the basal area would be pi r square equals pi d square by 4. So, in the case of coniferous trees, we can very easily find out the basal area either by using the perimeter measures with a tape or by using the diameter as measured with callipers.

(Refer Slide Time: 07:35)



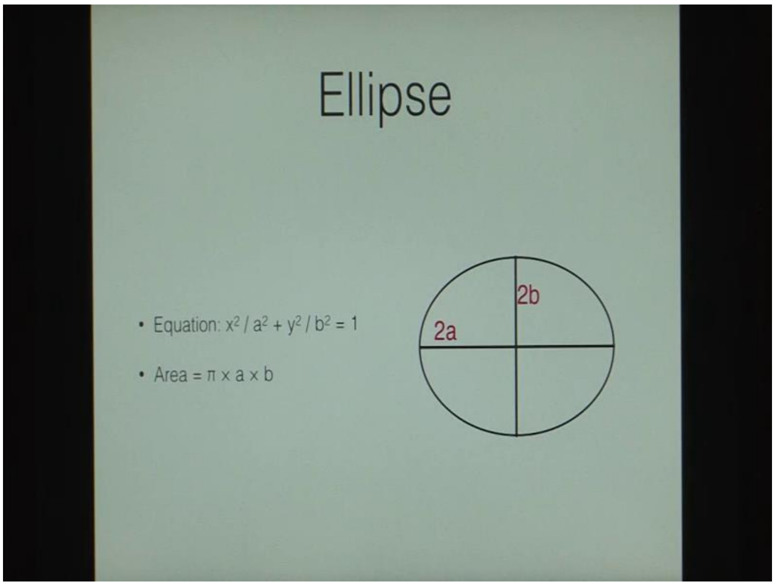
The next figure is called an ellipse. Now, an ellipse is a plane figure in the form of a regular oval. So, it has got two origins; let us call it O 1 and O 2. And all the points on this ellipse are such that the length l 1 plus l 2; every point a on this ellipse to these origins O 1 and O 2, so in the case of an ellipse l 1 plus l 2 is a constant.

(Refer Slide Time: 08:19)



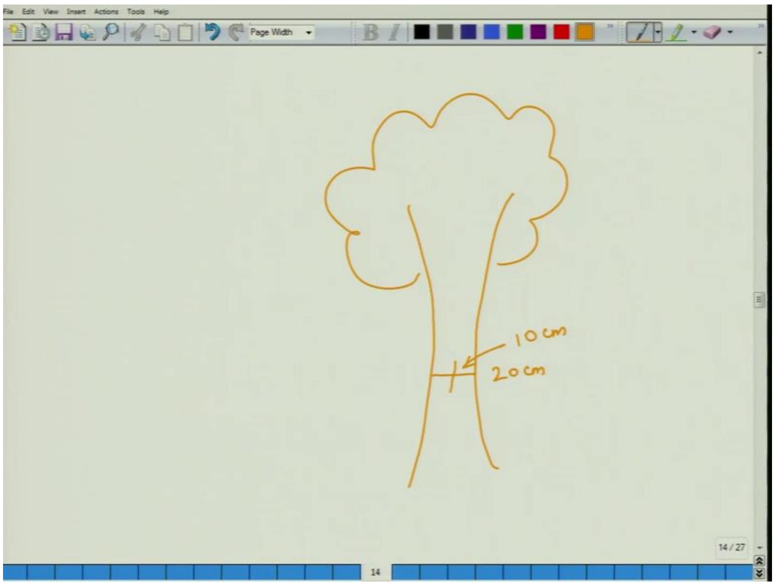
At the same time, an ellipse is said to have two axes. So, this is called the major axis and this thing is called the minor axis. Now, major axis is generally represented by twice of a; and minor axis is represented by twice of b. So, half of the major axis is a and half of the minor axis is b. The area of an ellipse is given by pi times a times b.

(Refer Slide Time: 09:07)



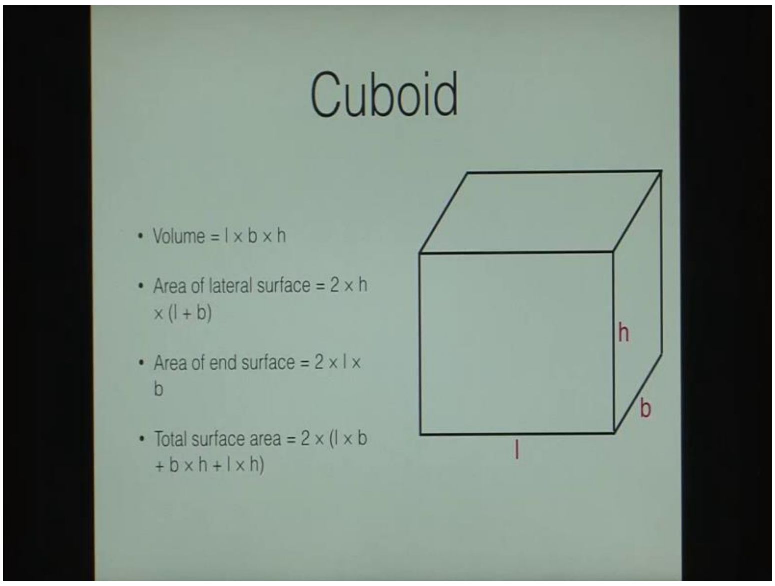
Now, why do we need to know about an ellipse? Well because again many trees have elliptical cross-sections. In the case of a tree that has an elliptical cross-section, you can find out, which side has the greatest length.

(Refer Slide Time: 09:16)



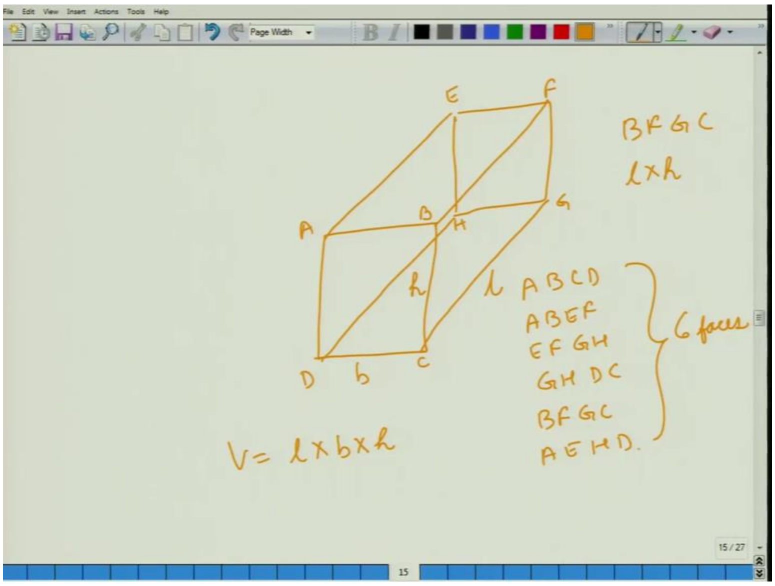
Suppose, this has a length of say 20 centimeters; and on the opposite end if you measure across the minor axis, suppose that length comes to be 10 centimeters. So, now you can figure out the basal area of this tree.

(Refer Slide Time: 09:33)



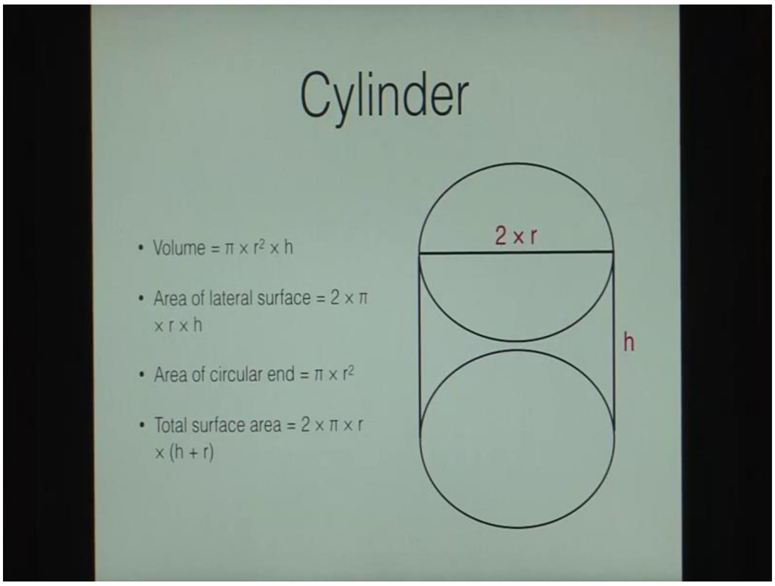
A cuboid is a solid figure; it has six regular faces at right angles to each other.

(Refer Slide Time: 09:40)



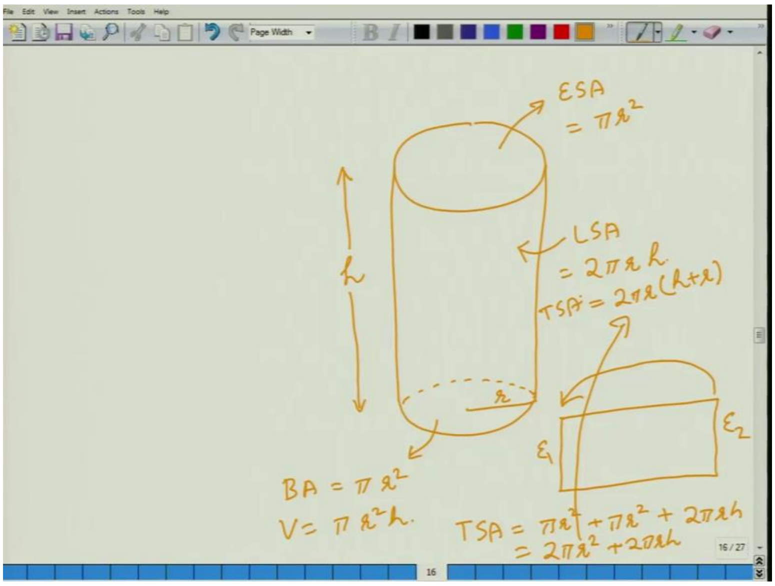
So, let us draw a cuboid. So, now, this cuboid has got six faces ABCD, ABEF, EFGH, GHDC, BFGC, A E H D. So, it has got six faces. The opposite faces are parallel to each other; and the faces that are adjacent to each other are at right angles. A cuboid is said to have a length a breadth and a height. The volume of a cuboid is given by length times breadth times height. The area of a lateral surface, so suppose we consider a lateral surface as BFGC. Now, because a lateral surface is a rectangle, so its area would be given by its length times the height. So, in this case, it is l multiplied by h. The area of the lateral surface of the cuboid is given by the sum of the areas of the lateral rectangles. The area of the end surface is the sum of the areas of the end rectangles. The total surface area is the sum of the lateral surface areas and the end surface areas.

(Refer Slide Time: 11:35)



The next solid figure is a cylinder.

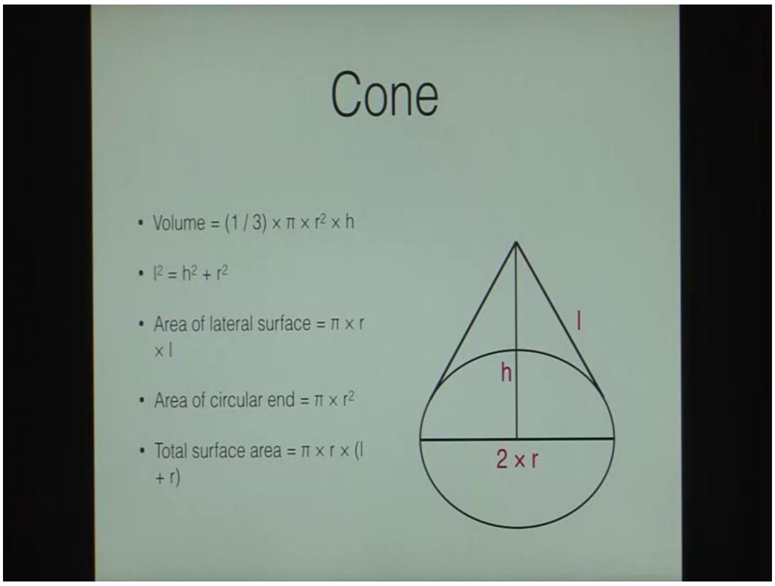
(Refer Slide Time: 11:41)



Now, you can construct a cylinder by taking a circle and moving it perpendicular to the circle. Now, if we move this circle on this direction, and suppose it reaches at this point. So, this is a cylinder. Alternatively, you can construct a cylinder by taking a rectangle and rolling it, so that both these edges e 1 and e 2 get jointed together. So, a cylinder is a solid geometrical figure with straight parallel sides and a circular or in oval cross section. The volume of a cylinder is given by the basal area, so because this is a circle, suppose it has radius of r. So, the basal area is given by pi r square. The volume is given by multiplication of the basal area with the height of the cylinder. So, the volume is equal to pi r square h.

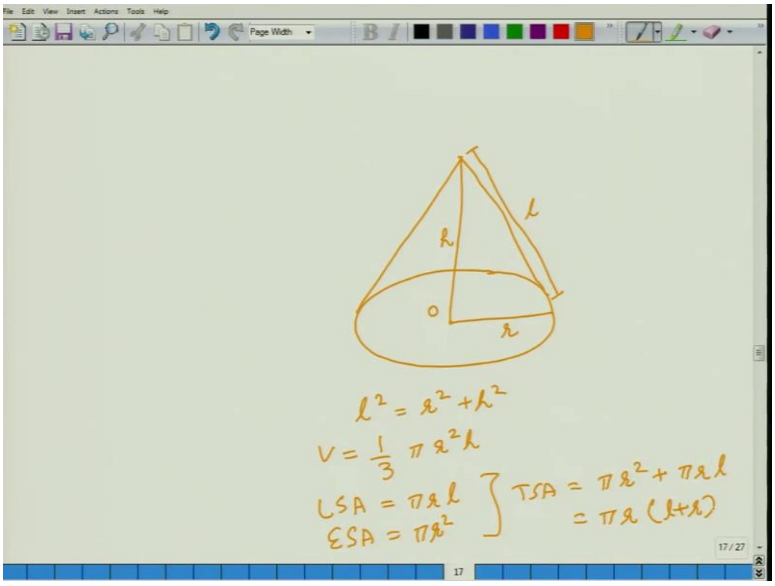
Now, suppose we wish to figure out the lateral surface area. Now, the lateral surface area is this surface area; the surface area that is on the sides of a cylinder. Now, the lateral surface area is given by the circumference of this circle multiplied by the height. So, the lateral surface area is 2 pi r into h. The end surface area because this is a circle, it is given by pi r square. So, the total surface area of a cylinder is given by pi r square plus pi r square for the two ends plus the lateral surface area is 2 pi r h. So, it becomes 2 pi r square plus 2 pi r h or we can also write it as 2 pi r bracket h plus r. So, this is the total surface area of a cylinder.

(Refer Slide Time: 14:12)



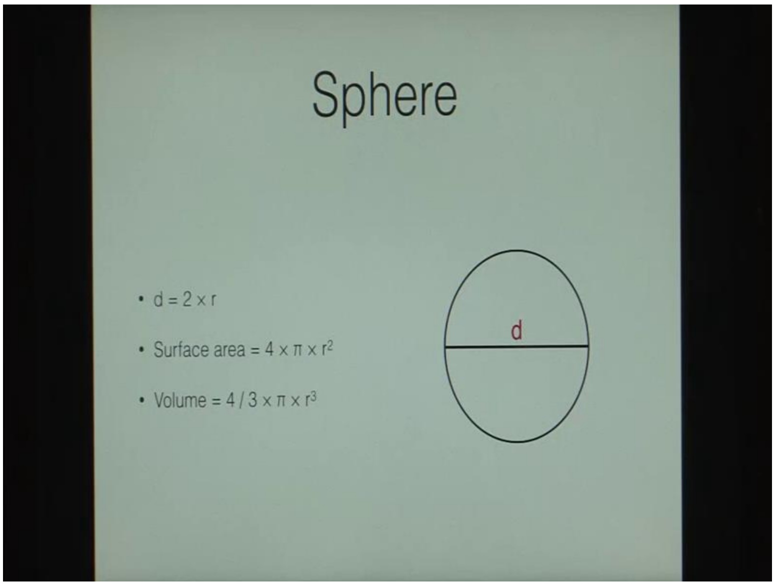
Solid figure is a cone.

(Refer Slide Time: 14:18)



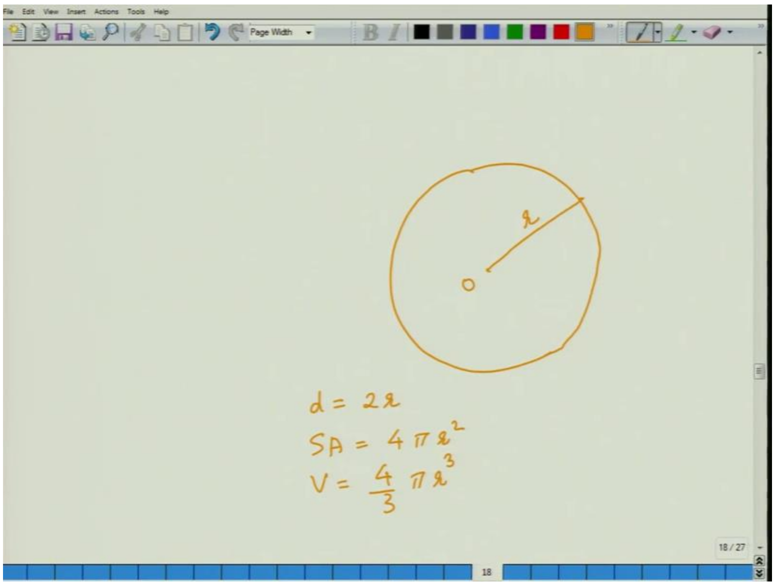
So, a cone consists of a circle and all the points are made to taper to a single point. So, it has a circular base which has a center o and a radius r; and it has a height that is given by h. So, if the height of the figure is h and the base has a radius of r, then we can find out the slant height. Now, slant height is this length; it is given by l. Now, l can very be very easily figured out by using the Pythagoras theorem l square equals r square plus h square. The volume of a cone is given by one-third of the volume of a cylinder of similar radius and similar height. So, the volume would be given by 1 by 3 pi r square h the area of the lateral surface. So, the lateral surface area is given by pi r l; and the area of the end because it is a circle it is given by the end surface area equals pi r square. The total surface area then is given by pi r square plus pi r l equals pi r l plus r.

(Refer Slide Time: 15:40)



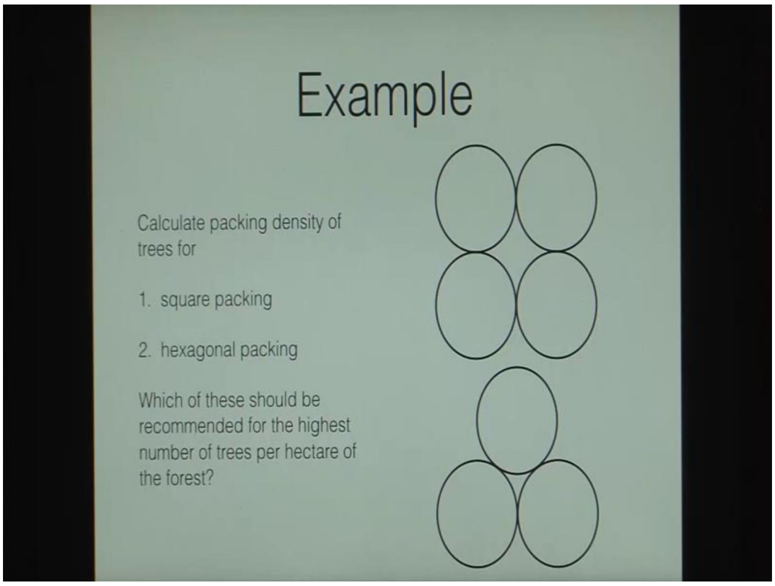
The next solid figure is a sphere.

(Refer Slide Time: 15:45)



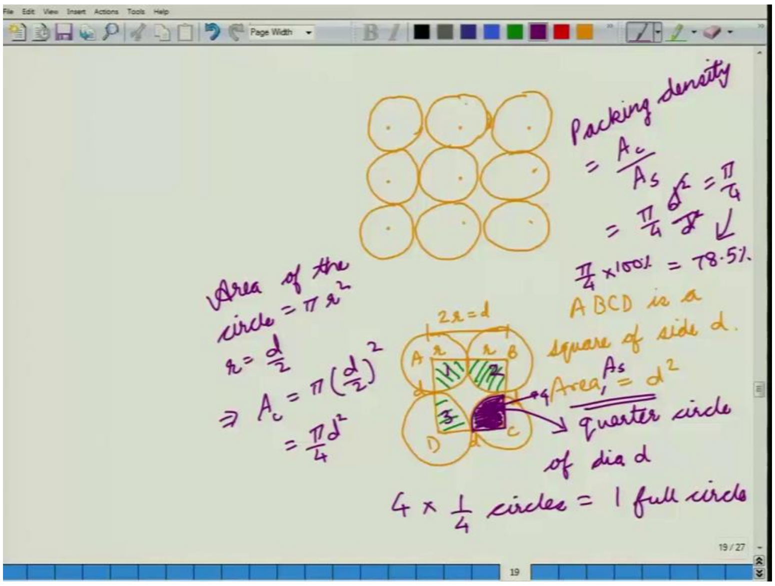
So, similar to a circle or sphere consists of a center O, and it consists of all the points that are at the same r from the center in the three dimensions. As in the case of circle here also we can define diameter as twice of the radius the surface area of a sphere the surface area is given by 4 pi r square; and the volume of a cylinder is given by 4 by 3 pi r cube. Now that we know about these plane and solid figures, let us have a look at some practical problems from the field.

(Refer Slide Time: 16:34)



Calculate packing density of trees for square packing and hexagonal packing as depicted in these figures. It is important if we wish to have the highest number of trees per hectare of the forest. So, how do we calculate a packing density suppose you planted trees in a regular grid.

(Refer Slide Time: 16:58)

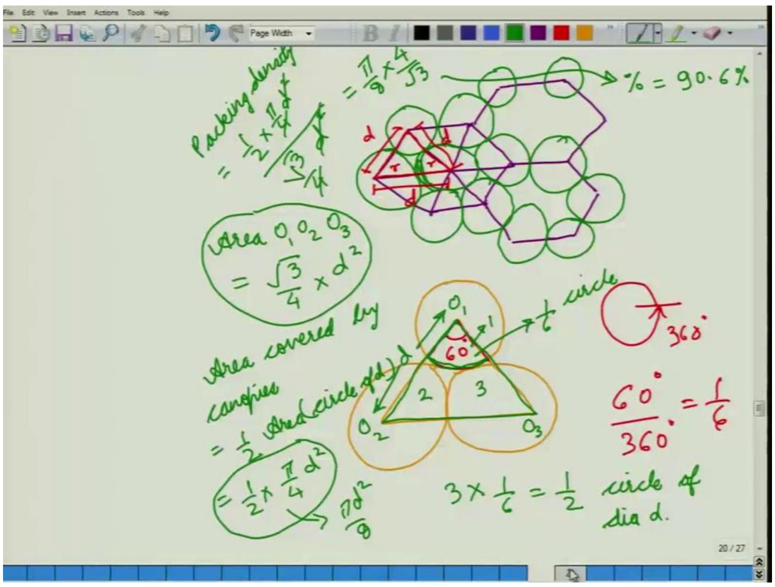


And this is a rectangular grid. So, later on this tree will have a canopy like this the next tree will also have a canopy, now because all these trees are of the same species and at the same site. So, this packing would become a square packing. Now, we wish to figure out how much of the area of the land is covered by these canopies. So, how do we do it. Consider just four points and we have circles there, and these circles are touching each other these are their centers. Now, consider this length this is r, this is r. So, this total is equal to 2 r or d. Similarly, this side is d, this is d, and this is d. So, in this case, if we consider this square that is ABCD, so ABCD is a square of side d. So, what is the area of ABCD, it will be given by d square.

Now, we consider these areas. Now, this is the area covered by the canopies. Now, this area is one-fourth of a circle. So, this is a quarter circle. And what is the diameter of this circle of dia d. So, because we have four quarter circles 1, 2, 3 and 4 that is covered in this square. So, we have four into a quarter circles that becomes one full circle. Now, the area of the circle is given by pi r square, r is equal to d by 2. So, the area of the circle is equal to pi d by 2 square or pi by 4 d square.

Here we saw that the area of the square is given by d square. So, how much of the area of the square is covered by the canopies. So, the packing density is given by the area covered by the circles divided by the area of the square which is pi by 4 d square by d square is get cancelled out is equal to pi by 4. Now, pi by 4, if you convert it into percentage terms, it will be pi by 4 into 100 percent which comes to 78.5 percent. So, the packing density in the case of a square packing comes to 78.5 percent.

(Refer Slide Time: 21:00)



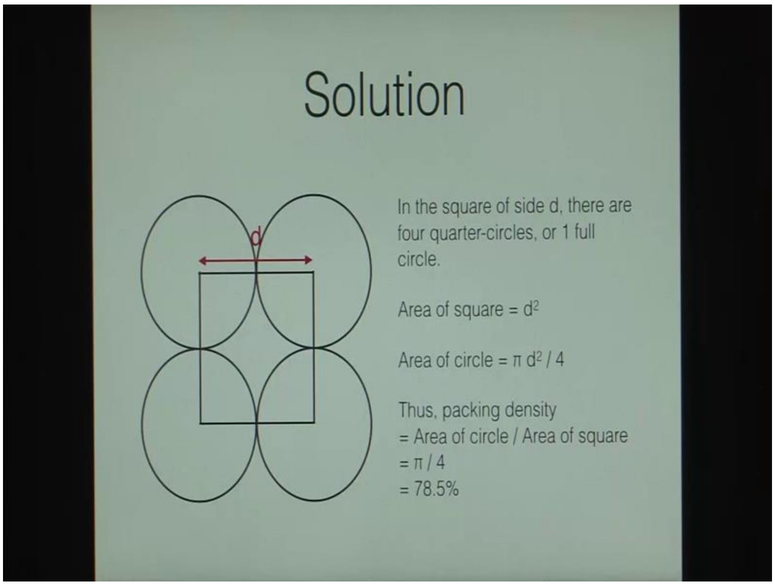
Now, consider a hexagonal packing in the case of a hexagonal packing you have trees that are arranged at the vertices of a regular hexagon. So, when this tree is grow their canopies will touch each other in a hexagon. Now, this hexagon will further go out. And we will also have fries here. Now, the point here is to figure out the area that is covered by these hexagons.

Now, if we look at these circles, and we also have a point at the center that is touching all the other canopies, in this case, if we look at this portion, we have three circles that are touching each other. Now, what is this length is again d because it is r plus r, this length again is d, this length again is d. So, essentially what we have here is we have an equilateral triangle that has three canopies that are touching each other. Now, if you look at this portion the angle subtended here is 60 degrees because it is the angle of an equilateral triangle if you take a ray. And if you make it go all around it, it becomes 360 degrees. So, what about this sector that is covering only 60 degrees.

So, it becomes 60 upon three 60 degrees or 1 by 6 of a circle. So, this region is one-sixth of a circle. Now, in this triangle we have three such one-sixth circles. So, let us call this triangle o 1, o 2, o 3. Now this triangle has a diameter of as a side of d, now because this is an equilateral triangle, the area of the triangle is given by root 3 by 4 into the side d square which in this cases d square. If we consider these three sectors of the circles, so these three you have sector one, sector two, and sector three and all these three sectors are one-sixth of a circle. So, we have three into 1 by 6 circles or we have half circle here of dia d.

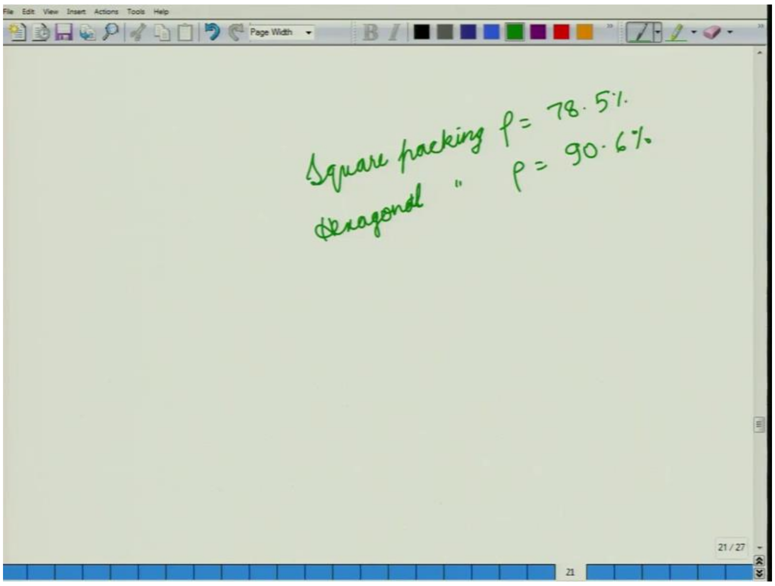
Now, the area covered by the canopies is equal to half the area of a circle of dia d. So, it becomes half into pi by 4 into d square. Now, if you want to calculate the packing density what will that be. So, packing density this area divided by the area of the triangle. So, it becomes half into pi by 4 d square whole divided by root by 4 d square d square and d square get cancelled out 4 and 4 gets cancelled out and so it becomes pi by 8 into this pi d square by 8 and this thing is root 3 by 4 into d square. So, d square and d square get cancelled out and this area becomes pi by 8 into 4 by root 3 or when expressed in percentage terms it becomes 90.6 percent.

(Refer Slide Time: 26:28)



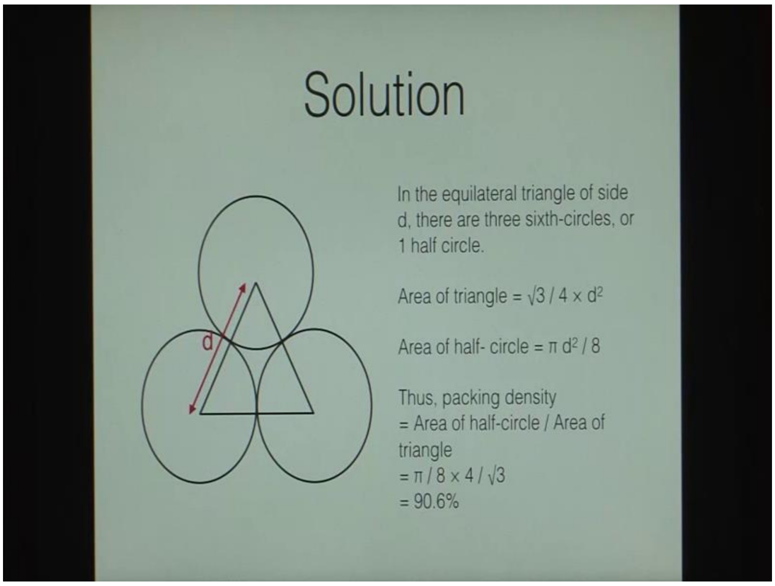
So, in the in the last slide we saw that the packing density in the case of a square packing is 78.5 percent. So, if you consider a square packing density is 78.5 percent.

(Refer Slide Time: 26:30)



Whereas in the case of a hexagonal packing the density is equal to 90.6 percent thus a hexagonal packing will give us a higher number of trees per hectare of the forest as compared to the square packing.

(Refer Slide Time: 27:05)



And which is why a hexagonal packing is preferred when we want to extract as much of the word as possible from the stands.

Thank you for your attention, [FL].