Forest Biometry
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Lecture – 03
Recap of trigonometry

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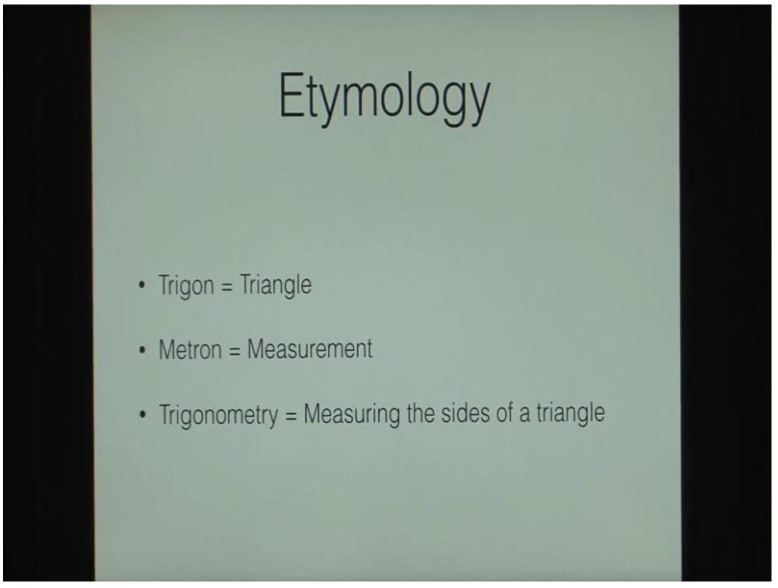

[FL]. Today, we will have a recap of trigonometry. Let us begin with the word roots. Our trigon is a triangle; metron refers to measurement as in biometry. This trigonometry is the science of measuring the sides of a triangle.

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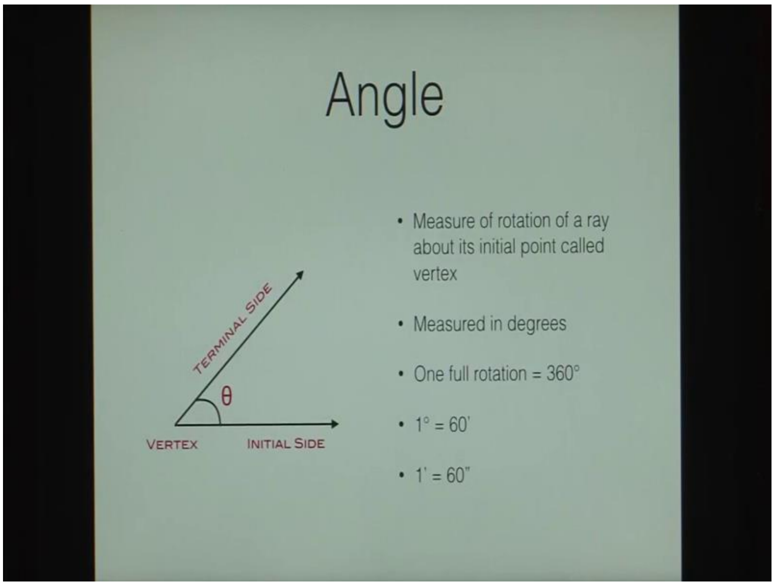

An angle is a measure of the rotation of ray about its initial point called vertex. It can be measured in degrees or radians one full rotation is arbitrarily assigned 360 degrees; 1 degree is further divided into 60 minutes and 1 minute into 60 seconds as we have in a clock.

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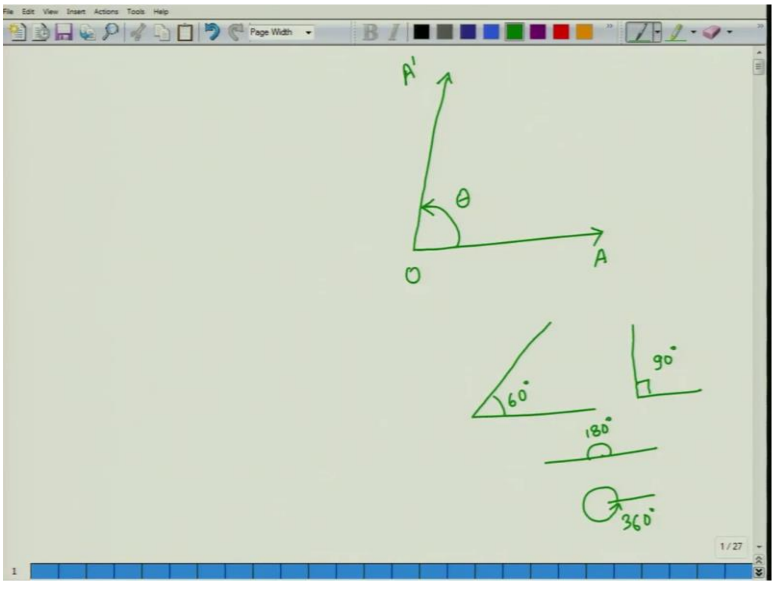

So, essentially if we have a ray which has an origin O. And suppose we make it to rotate about its origin. So, when it goes like this, this was the original position this is the final position this is this measure of the rotation is called the angle it is represented by theta.

Now some very common angles are 60 degrees, 90 degrees, 180 degrees and 360 degrees. This value of 360 degrees has been taken as an arbitrary value. An angle can also be measured in radians; the radian measure is the ratio of the length of an arc to the radius of the arc.

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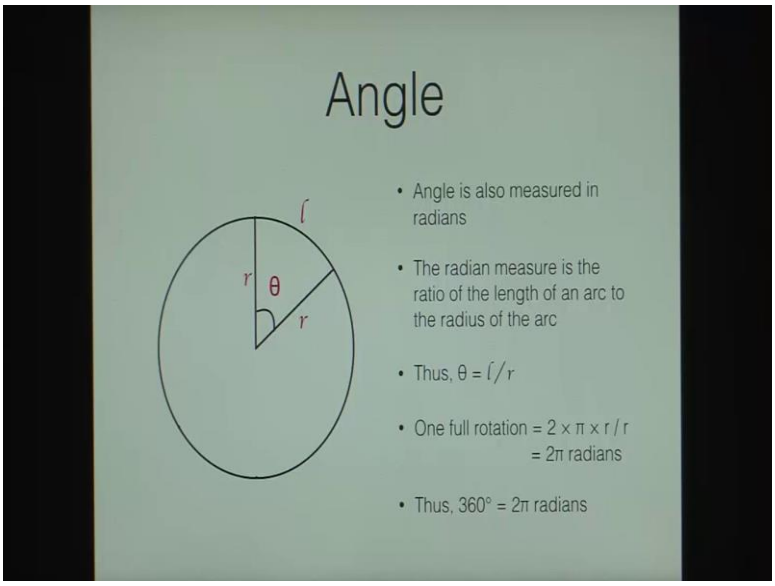

So, suppose we have a circle with its center O, and if we consider this arc.

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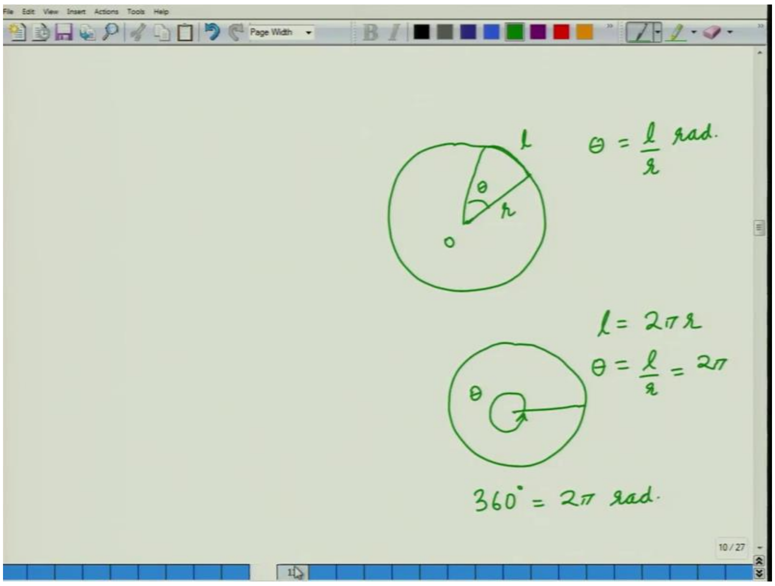

So, the length of the arc l divided by the radius it also gives you that the theta value. So, theta equals l upon r radians also referred to by r a d – rad. When we consider one full rotation in a circle if this angle under wind a full rotation from here to here the length of the arc would be equal to the perimeter of the circle that is 2 pi r. So, the theta value will be equal to l by r is equal to 2 pi radians. So, now this full rotation is equal to 360 degrees. So, 360 degrees equals 2 pi radians.

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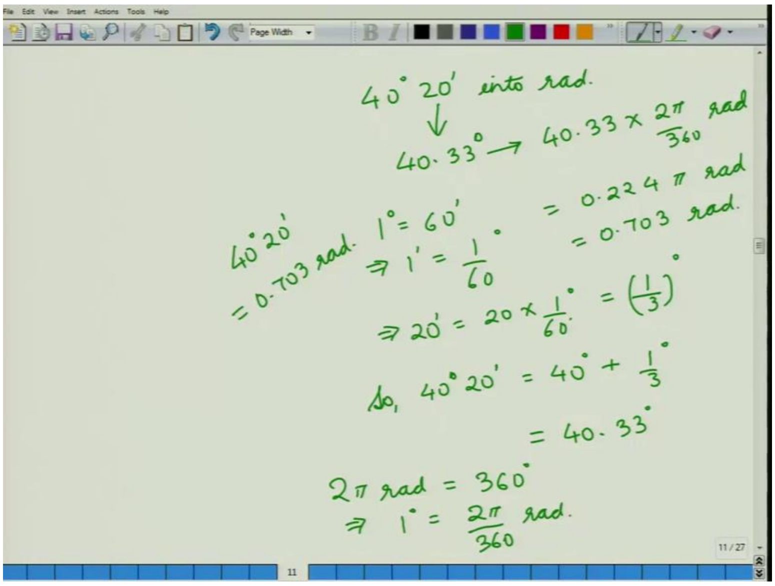

So, let us now consider an example. Convert 40 degrees, 20 minutes into radians. So, I will give you some time to think about it, how do we convert 40 degrees and 20 minutes into radians. So, now let us look at how do we do this conversion. First of all, we convert 20 minutes into degrees because we want everything to be in degrees. Now, 1 degree is equal to 60 minutes. So, 1 minute is equal to 1 by 20 degrees. So, 20 minutes is equal to 20 into 1 by 60 degrees equal to 1 by 3 degrees. So, 40 degrees 20 minutes is equal to 40 degrees plus 1 by 3 degrees or 40.33 degrees. So, this 40 degrees 20 minutes is 40.33 degrees

Now, we know that 2 pi radians is a is equal to 360 degrees. So, 1 degree is equal to 2 pi upon 360 radian. So, 40.33 degrees will be equal to 40.33 into 2 pi upon 360 radians which is equal to 0.224 pi radians or 0.703 radian. So, essentially 40 degrees 20 minutes is equal to 0.703 radians. So, now, we can convert a degree measure into a radian measure, this is how the calculation goes.

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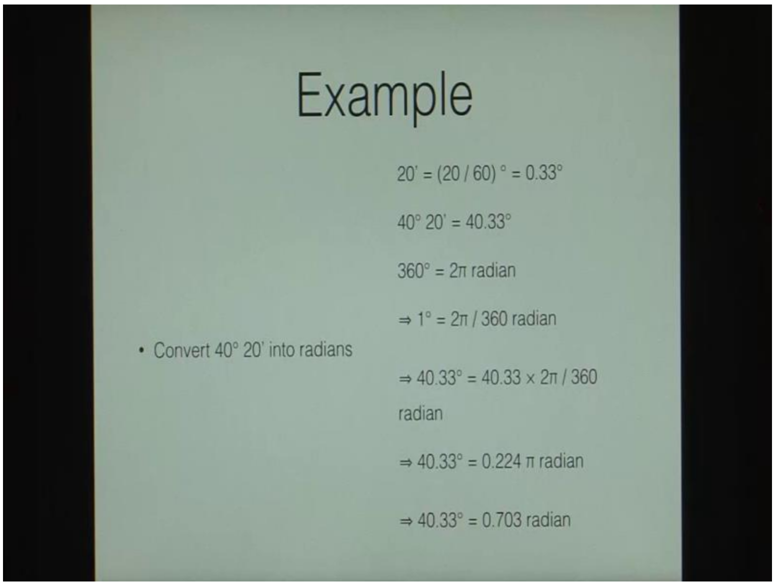

Now, can we convert radiance into degrees, it follows a very similar procedure. So, let us look at this next example. Convert 6 radians into degrees.

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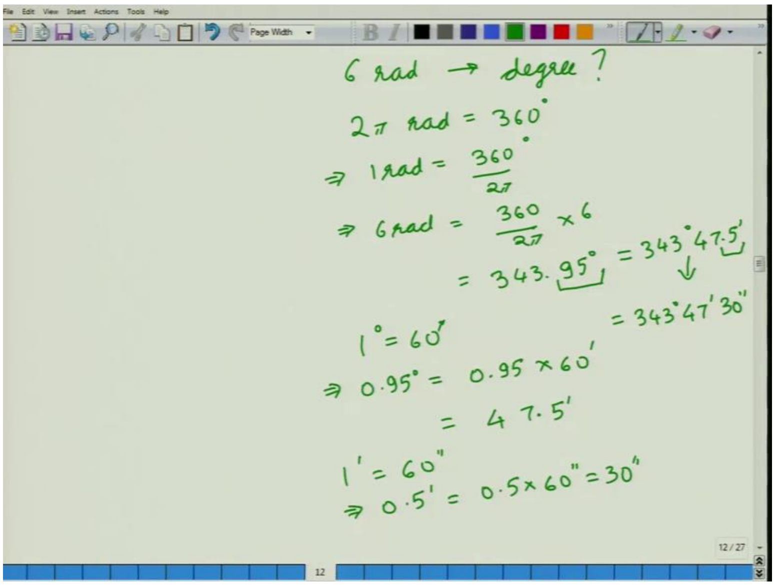

So, we have a measure of 6 radian, and we want to convert into degrees. So, now again 2 pi radian is equal to 360 degrees. So, 1 radian is equal to 360 upon 2 pi degrees. So, 6 radians is equal to 360 upon 2 pi into 6, which is equal to 343.95 degrees. Now, we could stop here at 343.95 degrees, but we could all also convert this 0.95 degrees into minutes and seconds. So, now 1 degree is equal to 60 minutes. So, 0.95 degrees is equal to 0.95

into 60 minutes is equal to 47.5 minutes. So, essentially this is equal to 343 degrees and 47.5 minutes. Now, we could stop here or we could convert these 0.5 minutes into seconds as well. Now, 1 minute is equal to 60 seconds. So, 0.5 minute is equal to 0.5 into 60 seconds or 30 seconds. So, this value is also equal to 343 degrees 47 minutes and 30 seconds. So, in this way, we can convert from degrees to radians and vice versa.

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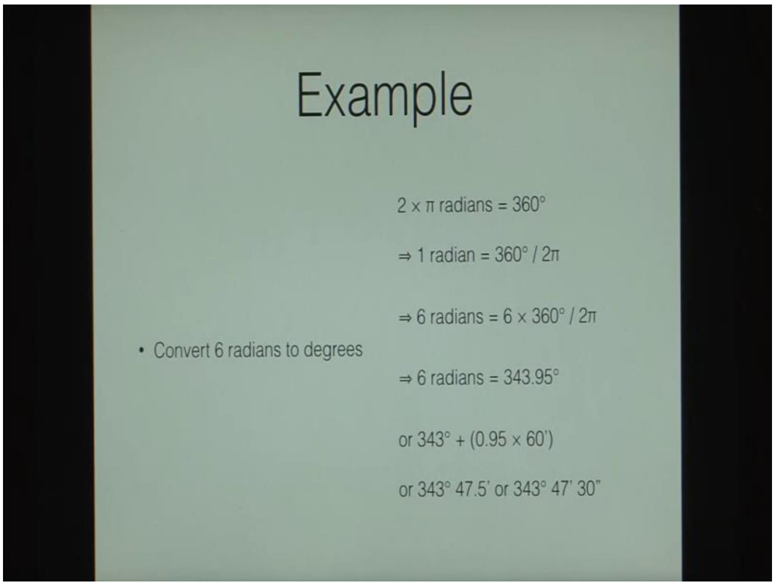

So, here is how the calculation goes.

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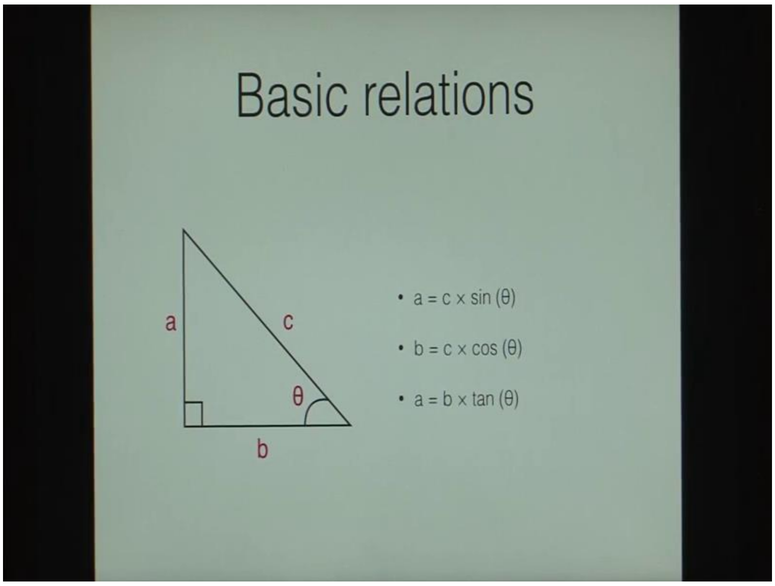

So, now, let us look at some basic relations of trigonometry. So, in the case of trigonometry, we start with a right angle triangle. So, this is the right angle triangle, this is 90 degrees, this angle is theta.

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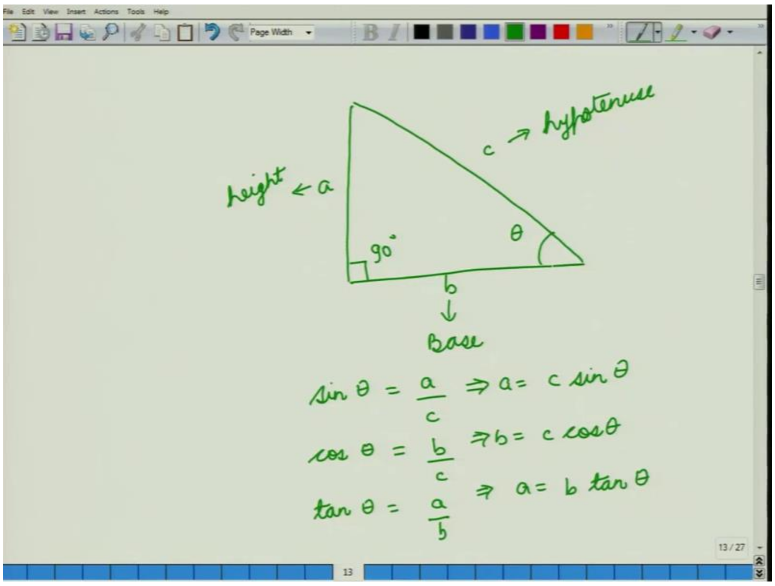

Now, this triangle has got three sides, the sides are represented by a, b and c. Now, b is also called the base of this triangle, a is the height of the triangle and c is the hypotenuse of the triangle. Now, the basic relations of trigonometry are like this, sin theta is equal to a upon c; cos theta or the cosine of theta is equal to b upon c; and tan theta is equal to a upon b. So, essentially a equals c sin theta, b equals c cos theta, and a is also equal to b tan theta.

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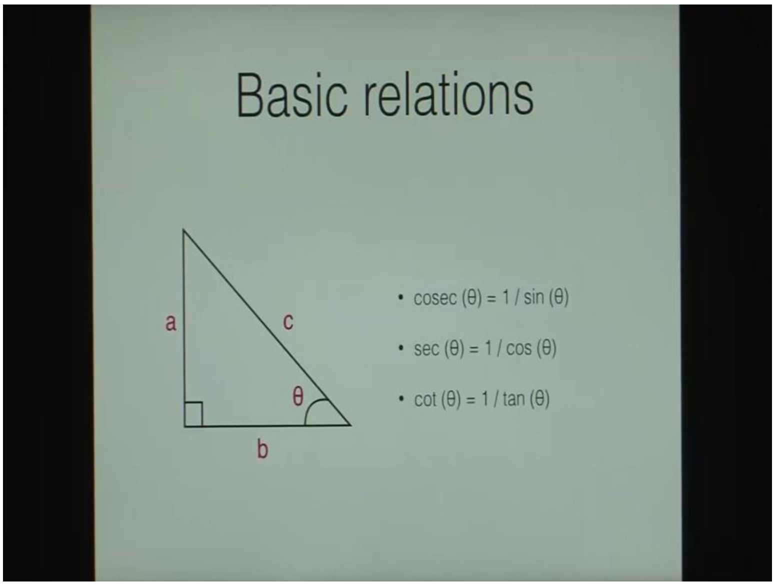

We also define inverses. So, we have cosec theta is equal to 1 upon sin theta; sec theta is equal to 1 upon cos theta.

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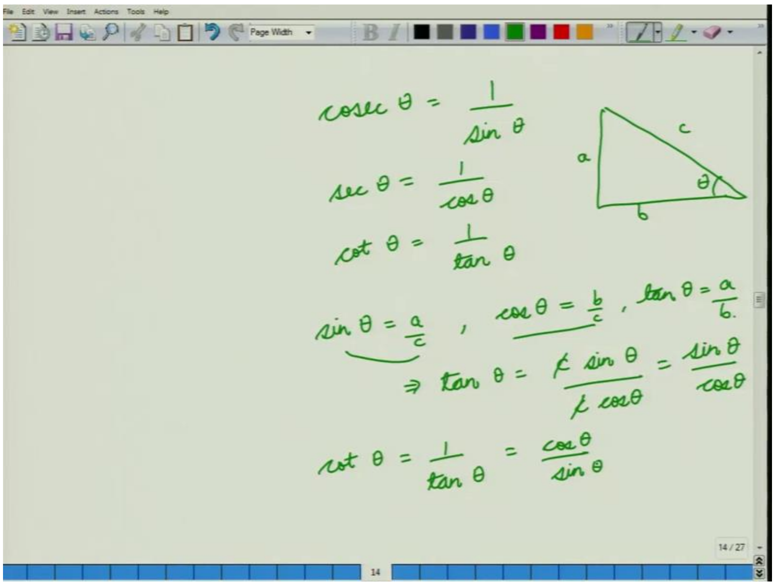

And cot theta is equal to 1 upon tan theta. Now, with this we can also define some other relations. Now, in this triangle, we have sin theta equals a upon c, cos theta equals b upon c, and tan theta equals a upon b which would imply that tan theta equals now a we can take from here. So, a is c sin theta; and b we can take from here b is equal to c cos theta, c and c get canceled out. So, tan theta equals sin theta upon cos theta. Similarly, cot theta equals 1 upon tan theta equals cos theta upon sin theta.

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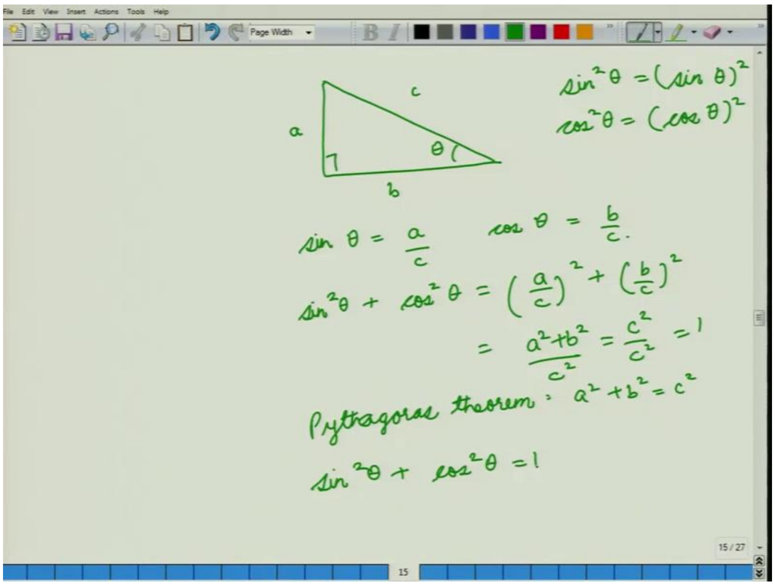

Now, let us look at one other relation. We have sin theta equals a upon c, cos theta equals b upon c. Now, let us look at sin squared theta plus cos square theta. So, when we write sin square theta, it means sin theta whole square and cos square theta equals cos theta whole square, so, sin square theta plus cos square theta will be a by c square plus b by c square equals a square plus b square upon c square. Now by Pythagoras theorem, we have a square plus b square equals c square. So, this value becomes c square by c square this is equal to 1; so we have sin square theta plus cos square theta equals 1.

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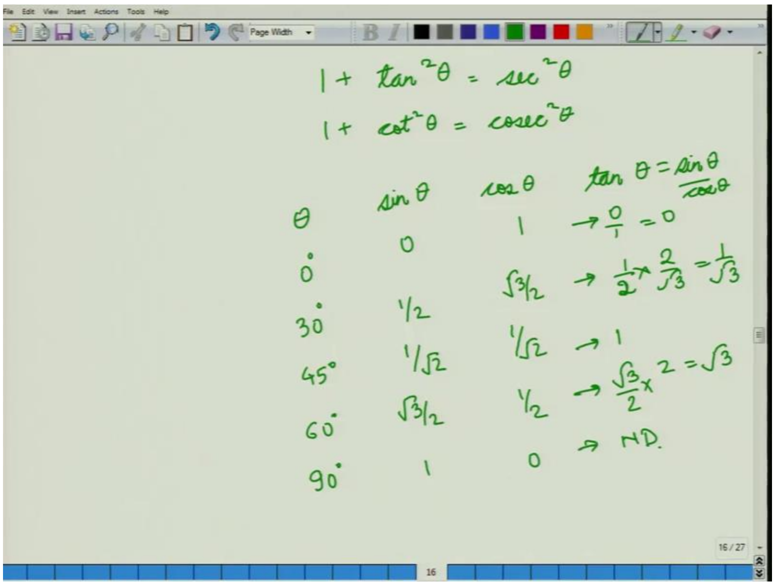

Similarly, we can also prove some other relations like 1 plus tan square theta equals sec square theta and 1 plus cot square theta equals cosec square theta. Now, as your assignment, you are going to prove these.

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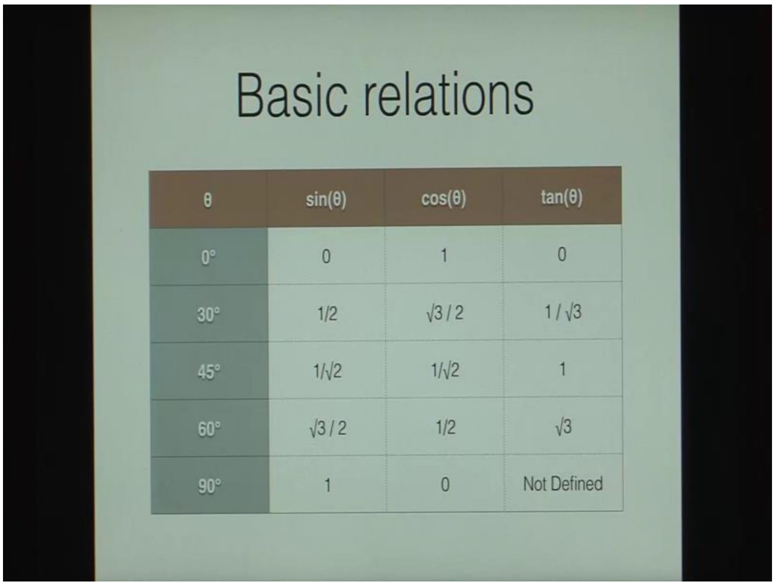

Now, the values of the sin co sin and tangent of several angles have been calculated. Now, some of these are given in this table. Now, we can very easily remember these. So, for this theta values 0, 0 degrees, 30 degrees, 45 degrees, 60 degrees and 90 degrees. The sin theta goes like 0, half, 1 by root 2, root 3 by 2, and 1. Cos theta goes in the opposite direction. So, it becomes 0, half, 1 by root 2, root 3 by 2, and 1. And tan theta it is the ratio of sin theta upon cos theta. So, basically this would become 0 by 1 equal to 0, this would become half into 2 by root 3 equals 1 by root 3, this would become 1. This becomes root 3 by 2 into 2 equal's root 3, and this becomes not defined because division by zero is not defined. Now, we can use these values and there are trigonometric relations to measure a triangle.

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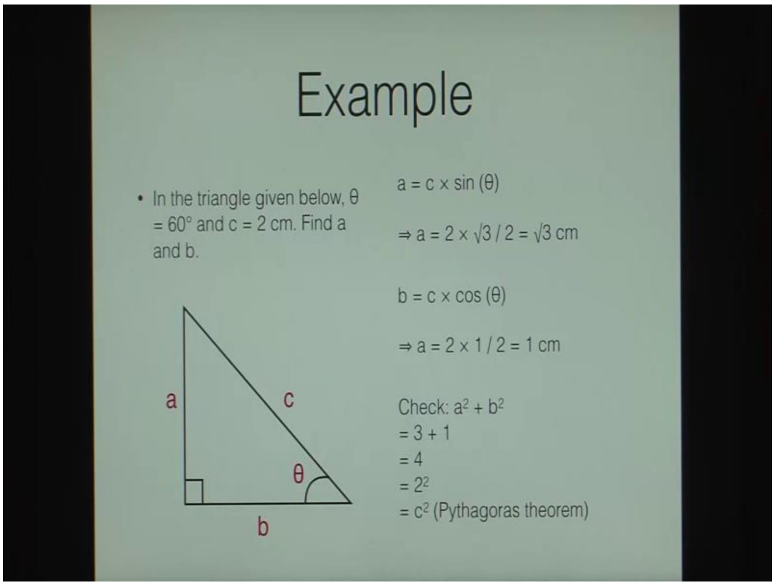

So, let us look at an example in the triangle given below theta is 60 degrees and c is 2 centimeters. Find a and b.

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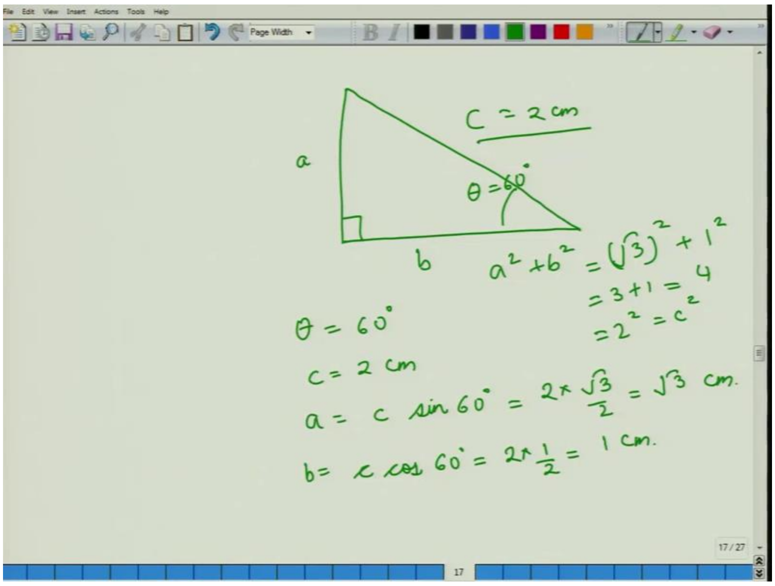

So, let us solve this question. So, we have a triangle where this theta is given to be 60 degrees, this is 90 degrees. Now, we are given that theta equals 60 degrees; and c equals 2 centimeters, this is 2 centimeters. Now, what is a, a given by c into sin 60 degrees or c sin theta. Now, sin theta from the last page we know that sin theta is root 3 by 2. So, this becomes 2 into root 3 by 2 equals root 3 centimeters; b equals c cos 60 degrees or c cos theta. Now, cos of 60 degrees is half. So, this becomes 2 into half equals 1 centimeter. Now, how do we know that these values are correct will be again I am deploys the Pythagoras theorem. So, let us look at a square plus b square here it will become root 3 square plus minus square equals 3 plus 1 equals 4 equals 2 squares, now c is given as 2 centimeters. So, this is c square. So, now, we can check our calculations by this method.

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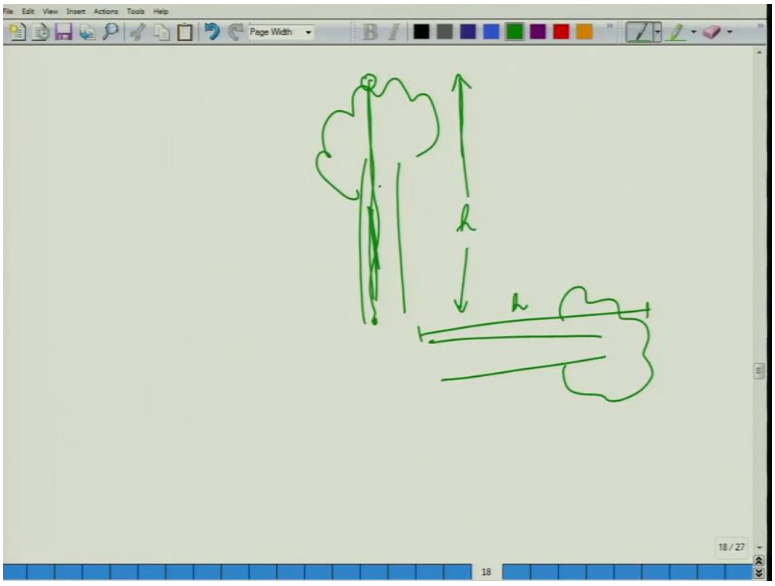

So, now, that we know how to use trigonometry to measure the size of the triangle how are we going to use it in the field. So, suppose you have a tree and you want to figure out the height of the tree; obviously, we have got some base one you can always chop this tree down and then measure the length. So, when this tree falls on the ground you can measure the length which will be equal to the height, but of course, we do not want to chop these trees. So, do we have any other option? Yes, the second option which is called as direct method involves that you go on the top of this tree. So, you set here and then you take a wire or a string with a weight attached to it, you throw it down. When the string is tight enough, you measure this length, and this length will be equal to height, but of course, out there in the field we do not have such situations we cannot climb on each and every tree that we want to measure. So, trigonometry gives us one solution.

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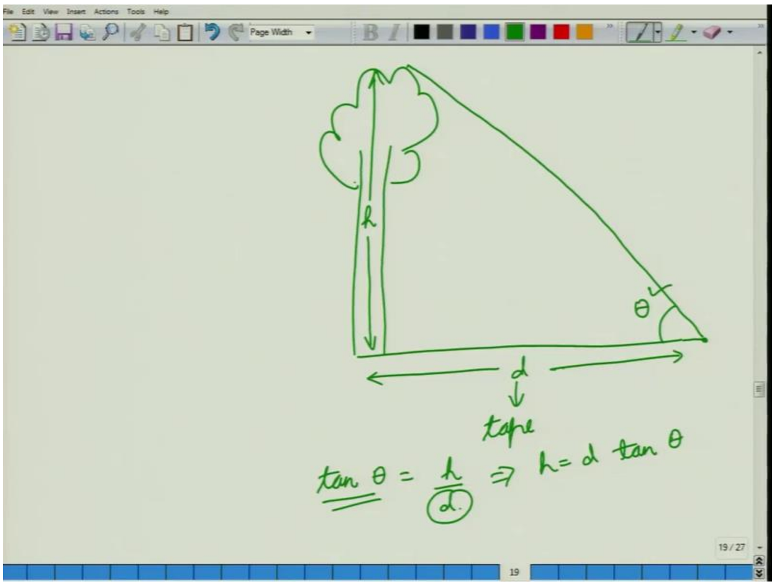

So, suppose you have this tree whose height you wish to measure. Suppose, you go at a distance of d from the base of the tree and you take this angle that is subtended by the top of the tree to the ground this angle would become theta. So, d can very easily be measured with a tape. Height is something that we wish to measure, theta is something that we can very easily measure with a protractor or by using some specific equipments that we have. So, how do we make a get the height of the tree? Well, we can use the relation tan theta equals h by d. So, essentially if you get to if you know theta you can find out tan theta you know d, so h equals d tan theta. So, h can be easily measured, of course, but this example requires you to be on the ground. What about if you are holding your equipment at your eye level?

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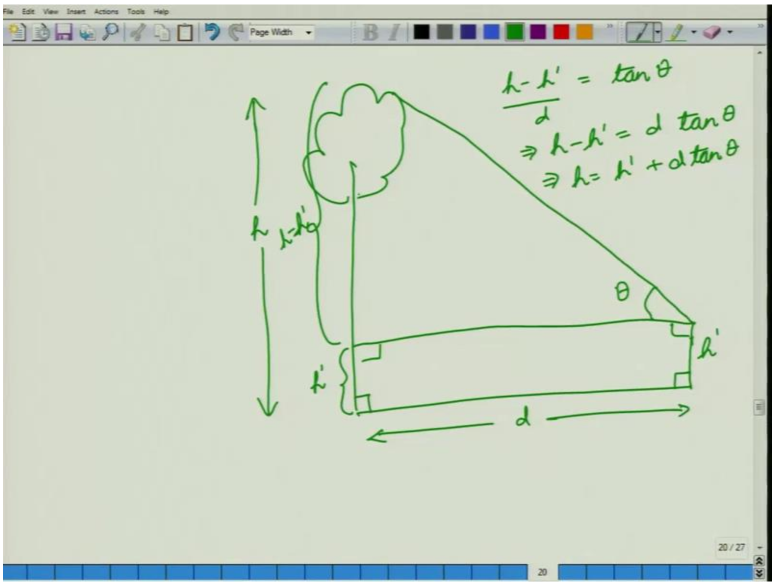

So, suppose this is your tree, you are standing here at your eye level, and you are measuring the angle that is subtended by this tree to your height. So, essentially you want to measure this height h you know this distance d we can figure out this angle theta and we also know r height, which is h prime. So, now can we measure the height of the tree? Well, yes, we can if you draw a straight line from ourselves to the tree, this portion becomes a rectangle, so this side is also equal to our height this portion will then become h minus h prime. So, now, we can use the relation h minus h prime a upon d equals tan theta in which case will have h minus h prime equals d tan theta or we will have height equals r height plus d tan theta. So, in this way also we can measure the height of the tree. Now, we will have similar questions in your assignments that will be uploaded shortly.

Thank you for your attention, [FL].