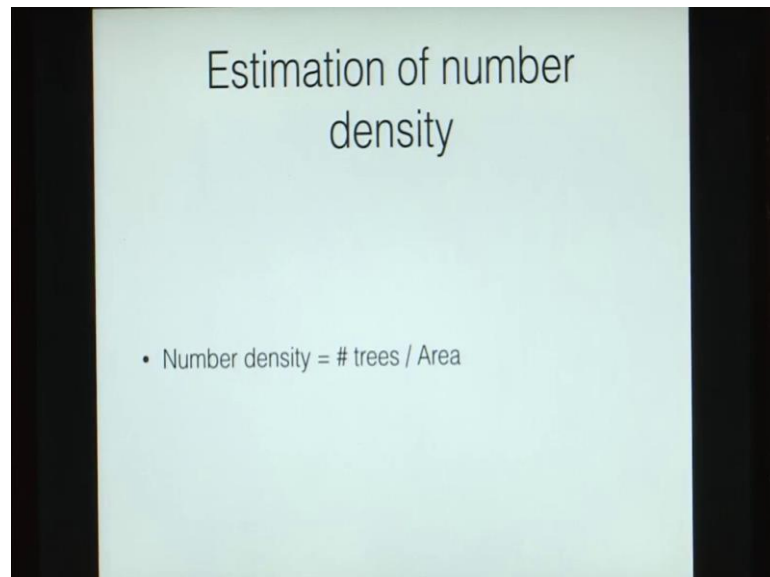


Forest Biometry
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Lecture - 30
Number density and sample calculations

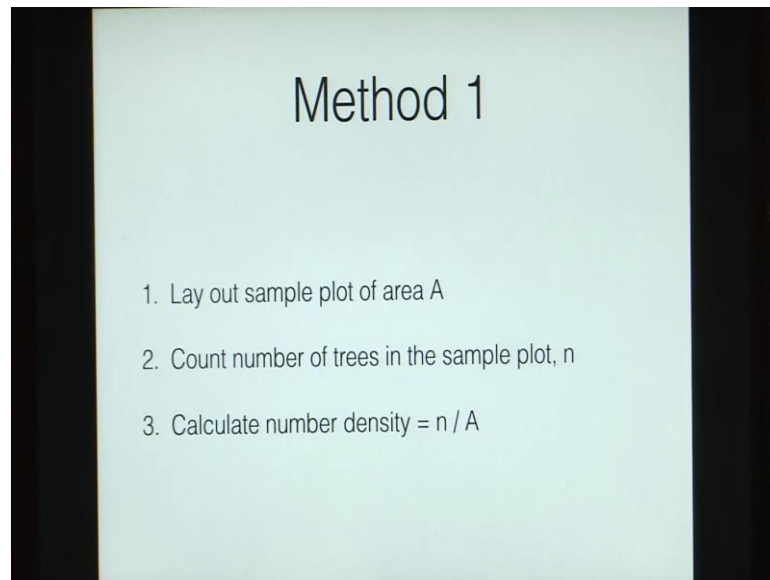
[FL]. Today we will further continue with our discussion on number density and look at some sample calculations.

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So, to recapitulate we looked at a number of methods of estimating number density or the stand density which is given as the numbers of trees per unit hectare of the forest.

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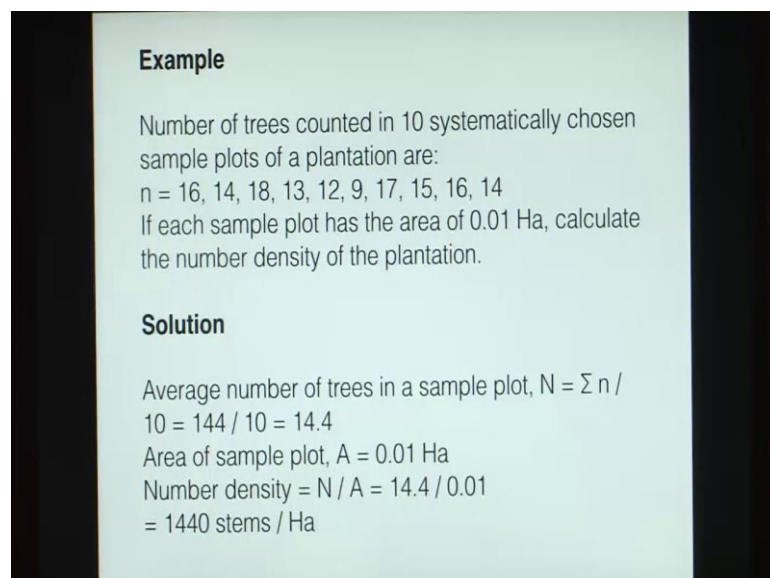
Method 1

1. Lay out sample plot of area A
2. Count number of trees in the sample plot, n
3. Calculate number density = n / A

So, the first method which is the simplest method is to lay down a sample plot of area, count the number of trees and then, calculate the number density as n by A.

Now, in this case the area of the sample plot should be in hectares to get a number density in number per unit hectare.

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Example

Number of trees counted in 10 systematically chosen sample plots of a plantation are:
 $n = 16, 14, 18, 13, 12, 9, 17, 15, 16, 14$
If each sample plot has the area of 0.01 Ha, calculate the number density of the plantation.

Solution

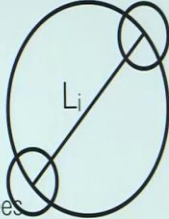
Average number of trees in a sample plot, $N = \Sigma n / 10 = 144 / 10 = 14.4$
Area of sample plot, $A = 0.01$ Ha
Number density = $N / A = 14.4 / 0.01$
= 1440 stems / Ha

We also looked at an example problem and how to solve it.

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Method 2

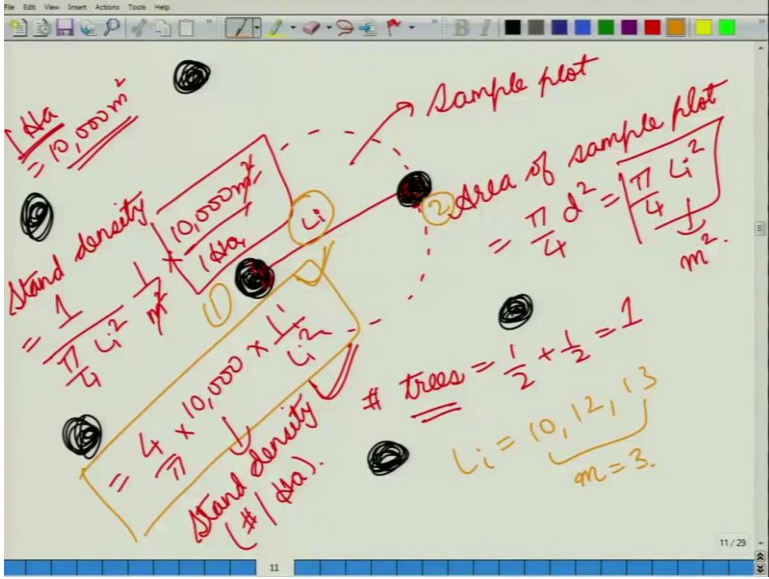
1. Find distance between nearest trees, L_i (in m)



2. In area $\pi / 4 \times L_i^2$,
trees = 1
 \Rightarrow In area of 10,000 sq m, # trees
 $= 10000 / (\pi / 4 \times L_i^2)$
 $= 4 / \pi \times 10000 \times 1 / L_i^2 \Rightarrow$ Number density

We also derived another equation for the second method.

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Area = 10,000 m²

Stand density = $\frac{1}{\frac{\pi}{4} L_i^2}$

Area of sample plot = $\frac{\pi}{4} d^2 = \frac{\pi}{4} L_i^2$

trees = $\frac{1}{2} + \frac{1}{2} = 1$

$L_i = 10, 12, 13$
 $n = 3$

Stand density (#/Area) = $\frac{4}{\pi} \times 10,000 \times \frac{1}{L_i^2}$

So, now coming to the tablet, we derived that in the case of having two trees, this is our first tree and the second tree and the nearest distance between both of these is given by L_i . So, we have the stand density given by this equation 4 by π into 10000 into 1 by L_i square. Now, this is a very simple method of getting to the stand density just by looking at two nearest trees and getting one measure.

Now, as is the case we normally do not take just a single measure. We take n number of measures. So, suppose this is the first value L_i . So, suppose it was L_i is equal to suppose 10 12 13. So, these are three values that we calculated. So, how do we get to one value of the number density? So, we will take average of all these three. So, when we take the average, suppose if we take m number of values. So, in this case we have m is equal to 3. So, then what will be the average?

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The image shows a whiteboard with handwritten mathematical equations. The first equation is:

$$\# \text{ density} = \frac{4}{\pi} \times 10000 \times \frac{1}{L_i^2}$$

The second equation shows the average over m observations:

$$= \frac{4}{\pi} \times 10000 \times \sum \frac{1}{L_i^2} \times \frac{1}{m}$$

A red checkmark is visible to the right of the equations. The whiteboard interface includes a toolbar at the top and a status bar at the bottom showing '12 / 29'.

So, the number density in the first case is given by 4 by pi into 10000 into 1 by L_i square. Now, suppose we have m number of readings. So, in that case we will take an average. So, by taking an average, we will sum up all these values n divided by m. So, this becomes 4 by pi into 10000 is a constant. So, we can take it out of the equation. So, 4 by pi into 10000 into sum of 1 over L_i square into 1 upon m. So, that is our number density or the stand density when we have taken m observations. So, to consolidate this equation, let us now look at an example.

So, on your screen you can see the example. Now, coming to the slides.

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Example

The distance between nearest two trees measured in 10 systematically chosen sample plots of a plantation are:

$L = 2.25, 3.75, 1.95, 3.65, 2.75, 2.9, 3.1, 3.45, 3.6, 2.85$

Calculate the number density of the plantation.

So, the distance between nearest two trees measured in 10 systematically chosen sample plots of a plantation are these. So, these are the values of L_i and we are required to calculate the number density of the plantation. So, we have taken 10 readings of the nearest two trees. The values are given and we need to calculate the number density. So, which equation are we going to use? We are going to use this equation. This is the equation that we will be using.

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$L = 2.25, 3.75, 1.95, 3.65, 2.75, 2.9, 3.1, 3.45, 3.6, 2.85$

$$\# \text{ density} = \frac{4}{\pi} \times \frac{10,000}{10 \text{ ha}} \times \sum \frac{1}{L_i^2}$$
$$= \frac{4}{\pi} \times \frac{10,000}{10} \left[\frac{1}{2.25^2} + \frac{1}{3.75^2} + \frac{1}{1.95^2} + \dots + \frac{1}{3.6^2} + \frac{1}{2.85^2} \right]$$

1586 stems / ha.

So, now coming to the problem, we have L is given as 2.25, 3.75, 1.95, 3.65, 2.75, 2.9, 3.1, 3.45, 3.6 and 2.85.

Now, number density, so the number density is given by $4 \pi \times 10000$ divided by m into sum over $1/L_i^2$ and this case because we have 10 readings, so m is equal to 10. So, this equation will become $4 \pi \times 10000$ divided by 10. Now, the distance between two trees is given in meters. So, in this case, your sum $1/L_i^2$ will become $1/2.25^2$ plus $1/3.75^2$ plus $1/1.95^2$ plus $1/3.6^2$ plus $1/2.85^2$. So, now when we do the calculation, we get a value of 1586 stems per hectare.

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L_i	$1/L_i^2$
2.25	$\frac{1}{2.25^2} = x$
3.75	$\frac{1}{3.75^2} = y$
1.95	$\frac{1}{1.95^2} = z$
...	...
$\sum \frac{1}{L_i^2} = x + y + z + \dots$	

Now, how do you do this large calculation on a sheet? So, what we can do is, we can make a table of L_i and $1/L_i^2$. So, we will have the values of 2.25, 3.75, 1.95 and so on. So, now $1/L_i^2$ is $1/2.25^2$ will get a value. See it is x here. We have $1/3.75^2$. Let us call it y . Here we have $1/1.95^2$, let us call it z and then, once we have calculated all these values x y z and so on. Then, we will calculate $1/L_i^2$, its sum which is given by the sum of these values. So, it is $x + y + z$ plus so on.

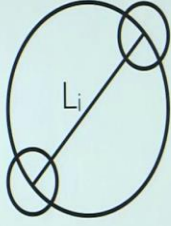
So, once we have this $1/L_i^2$, we will put it into this equation. So, we will replace this portion by the value that we have calculated here and then, multiplied by 4

by 5 into 10000 to get the number of stems per hectare. So, that is done when you have the nearest two trees.

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Method 3

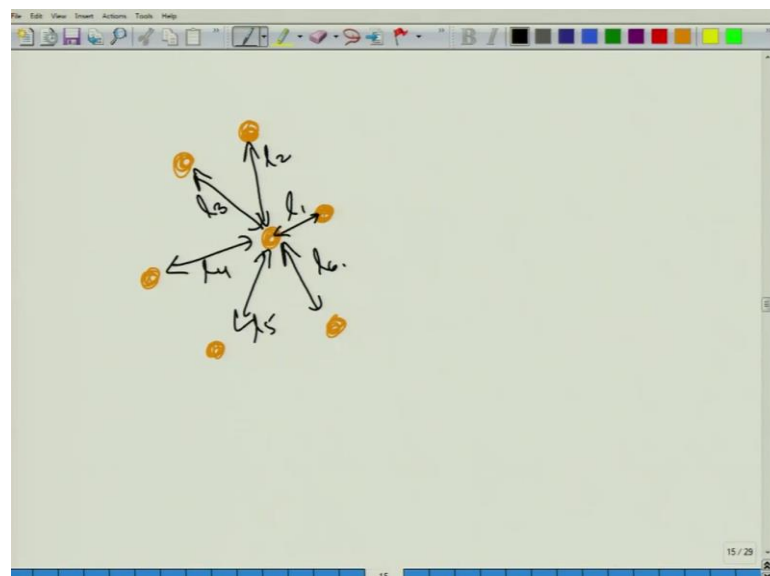
For multiple samples (m) of nth nearest neighbours,
Number density
 $= (n - 0.5) \times 10000 / (m\pi) \times \sum (1 / k_i^2)$



The diagram shows a large circle labeled L_i with a central point. Two smaller circles are shown overlapping the boundary of L_i . A line segment connects the central point to the center of the larger of these two overlapping circles, representing the distance to the nth nearest neighbour.

Now, suppose we have taken multiple samples that is m. So, as in the previous case like m was 10 in the equation in that case because we had taken 10 samples; here also we are taking m samples, but here suppose in place of the nearest neighbours, we take the nth nearest neighbour. Now, what do we mean by that? So, what is the nth nearest neighbour?

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So, suppose let us consider this to be the first tree. So, this is the first nearest neighbour. So, let us consider these to be the nearest neighbours. Let us remove this. So, now the nearest neighbour is this one. So, this is the first nearest neighbour and then, suppose these distances are 1 1, 1 2, 1 3, 1 4, 1 5, 1 6.

Now, we are trying to generalize these equations. So, in piece of the first nearest neighbour, we are putting all these numbers into ascending order and we are calculating it for the nth nearest neighbour. So, what would the equation be? So, coming back to the slide in this case, the equation becomes n minus 0.5 into 10000 divided by m pi into sum over 1 upon k i square. So, in place of L i is for the nearest neighbours, we use the term k i. So, to consolidate this fact let us look at an example problem.

So, here we have the distance between the fourth nearest trees. So, here n is equal to 4. The distance between the fourth nearest trees measured in 10 systematically chosen sample plots of a plantation and so on. So, here we have m is equal to 10, that is 10 number of samples and we are calculating the distance between the fourth nearest trees. So, n is equal to 4 and we are given the values of k. So, here also we need to calculate the number density of the plantation. So, how do we do that?

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The whiteboard shows the following handwritten work:

$$k = \begin{matrix} 4.8, & 6.2, & 5.4, & 6.1, & 5.7, & 6, & 5.8, & 5.6, & 6.2, & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{matrix}$$

$$n = 4$$

$$m = 10$$

$$\# \text{ density} = (n - \frac{1}{2}) \times \frac{10000}{m\pi} \times \sum \frac{1}{k_i^2}$$

$$= \frac{3.5 \times 10000}{10\pi} \left[\frac{1}{4.8^2} + \frac{1}{6.2^2} + \frac{1}{5.4^2} + \dots + \frac{1}{6.2^2} + \frac{1}{6^2} \right]$$

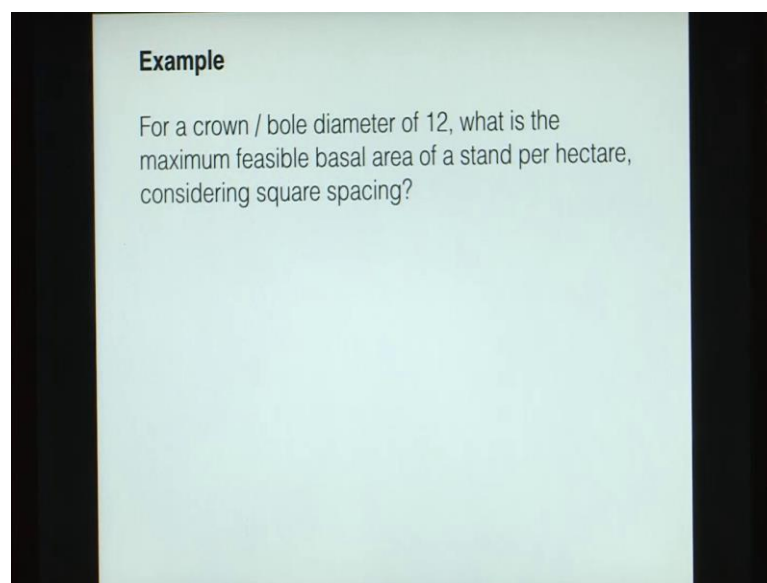
$$= \underline{\underline{339 \text{ stems/ha}}}$$

So, here we are given k is equal to 4.8, 6.2, 5.4, 6.1, 5.7, 6.6, 5.8, 5.6, 6.2 and 6 1 2 3 4 5 6 7 8 9 10. So, we have 10 values n is given to be 4. So, it is the fourth nearest tree. M is given to be 10 because we have 10 samples. So, the number density is given by n minus

half into 10000 upon π into sum over 1 upon k^2 . So, this becomes 4 minus half is three and a half, 3.5 into 10000 upon π . So, 10π into sum over 1 over k^2 . So, that is 1 over 14.8 square plus 1 over 6.2 square plus 1 over 5.4 square and so on till we reach the last value 1 over 6 square. So, when we do this calculation, we get a value of 339 stems per hectare.

So, we can calculate the number density for the nearest two trees or for the n th nearest neighbours. So, now let us look at another concept.

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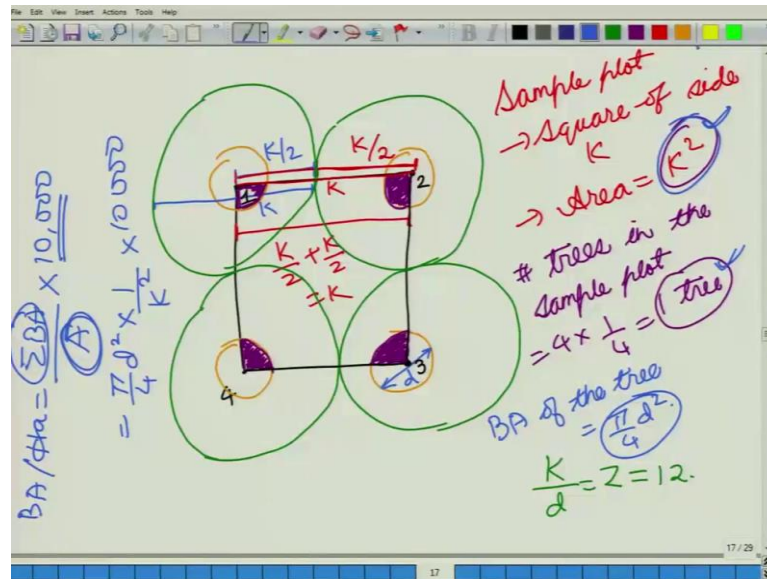
So, coming back to the slides now, here we have for a crown over bole diameter of 12. So, bole is the main trunk or the stem of the tree, crown is its canopy. So, for a crown over bole diameter of 12, what is the maximum feasible basal area of a stand per hectare considering square spacing. So, what are we given here, we are told that if the stem has a diameter of 1 meters, then the crown has a diameter of 12 meters, but we do not know. Then, the actual diameters of the bole or the crown we are just given a ratio and we are considering that this ratio is a fixed ratio for every tree which is generally the case in a stand.

So, for a fixed crown to bole diameter of 12, what is the maximum feasible basal area of a stand per hectare? Now, basal area of a stand per hectare is when you take any sample plot in your forest stand and you measure the basal areas of all the trees that I need your sample plot. When you sum them up and then, divide it by the area of the sample plot,

this gives you the basal area of the stand per hectare. So, for the given crown over bole diameter of 12, what is the maximum feasible basal area of a stand per hectare considering square spacing?

So, how do we solve this problem? So, let us consider a square spacing.

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So, when we have a square spacing, when this is the position of the first tree, this is the position of the second tree, this is the position of the third tree and this is the position of the fourth tree and there are no other trees in between. So, here we have four trees. Now, what is the maximum feasible basal area?

So, when the basal area is the maximum, at that time we are also given that in the case of a tree, the crown diameter divided by the stem diameter. So, here the stem diameter is measured at the breast height. So, let this diameter be k and let this diameter be d and then, we are given that k by d a factor z is equal to 12. In this case, when your basal area is the maximum, then all the trees are touching each other. So, when they are touching each other, the crowns will be touching each other and not the stems. So, this will be the situation. So, let us remove this and here we have the crowns that are touching each other.

So, let us consider these two be circular crowns. So, now these crowns are touching each other and below each crown will be a stem. So, what is the side of the square? Now, each

side of the square if you consider the half of the side, then this is equal to the crown radius. So, it is $k/2$, this half. The other half is also $k/2$. So, because this is a crown, this portion will be $k/2$. So, half of it is $k/2$. Here also we have $k/2$. So, what is the side of the square? So, the side of the square will be $k/2 + k/2$, it is equal to k . So, this side is of a length k . So, now, consider in this area of k by k .

So, let us consider this square to be our sample plot. So, we have a sample plot that is square of side k . So, the area is equal to k square. Now, in this case square area what is the number of trees that are there. So, if you look here, here we have a quarter of a tree that is inside the square. So, this is one-fourth of a tree, here also we have one-fourth of a tree, here also we have one-fourth of a tree and here as well we have a quarter of a tree.

So, the number of trees in the sample plot is 4 into a quarter tree which is equal to 1 tree. So, now in this area of k square, we have 1 tree. Now, how much is the basal area of that 1 tree; so basal area of the tree now because this tree has a diameter of d . So, the basal area of tree is given by $\pi/4 d^2$. Now, that is the basal area of 1 tree, but in our sample plot of k square area, we only have 1 tree. So, what is the basal area per of the stand per hectare? So, that is given by the sum of the basal areas of all the trees divided by the area of the stand into 10000.

So, if the area of the stand is given in square meters, then we are multiplied by 10000 to get a value in hectares. So, here the sum of the basal areas is equal to the basal area of 1 tree and the basal area of 1 tree is given by $\pi/4 d^2$. So, we divide it by the area a which is given by k square and we multiply by 10000. Now, in this equation we are given, so let us write it here.

(Refer Slide Time: 16:41)

$$\text{BA/ha} = \frac{\pi}{4} d^2 \times \frac{1}{k^2} \times 10000$$

we're given that $\frac{k}{d} = z = 12$.

$$\frac{d^2}{k^2} = \frac{1}{z^2}$$
$$\text{BA/ha} = \frac{\pi}{4} \times \frac{1}{z^2} \times 10,000$$
$$= \frac{10000 \pi}{4 \times 12^2} = \frac{10000 \times 3.14}{4 \times 12 \times 12} = 54.5 \text{ m}^2/\text{ha}$$

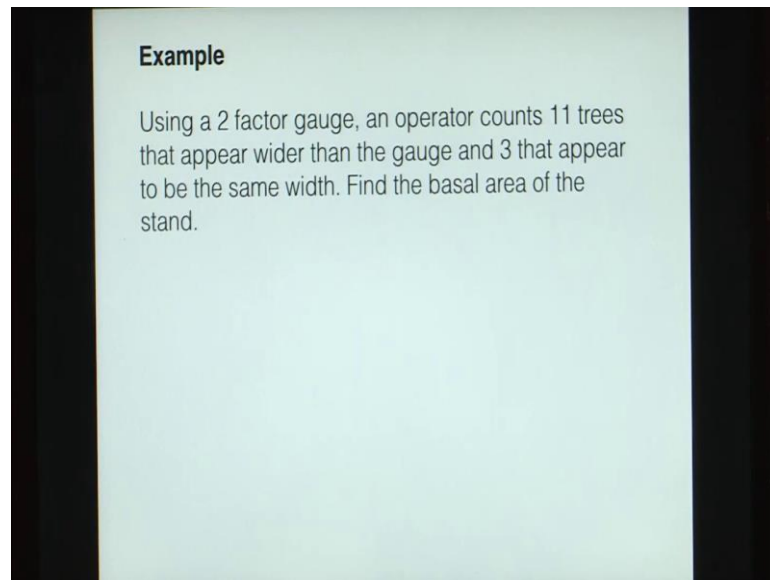
So, the basal area per hectare is given by pi by 4 d square into 1 by k square into 10000.

Now, we are given that the ratio of k by d which we can also refer to by this number is z. So, we are given that this ratio k by d or z is equal to 12. Now, in this equation let us replace k and d by z. So, we have d square by k square. So, d square by k square is equal to 1 by z square. So, we have the basal area per hectare is equal to pi by 4 into 1 by z square into 10000 or we can also write it as 10000 pi upon 4 z square. Now, here we are given that the value of z is 12. So, this value becomes 10000 into 3.14 divided by 4 into 12 into 12. So, we get this value to be 54.5 square meters per hectare.

So, essentially what this tells us just by using a single value of the crown to bole diameter which we call it z. So, just by using this single value of crown to bole diameter that is k by d, we can calculate the maximum basal area, the maximum feasible or the maximum possible basal area per hectare considering any kind of spacing. So, here in the case of square spacing, we got our equation as 10000 pi upon 4 z square, but we can do the same thing for any other kind of spacing as well. So, what does this tell us? If we have this ratio of z which is equal to k by d for any spacing, we can calculate the maximum basal area that is possible.

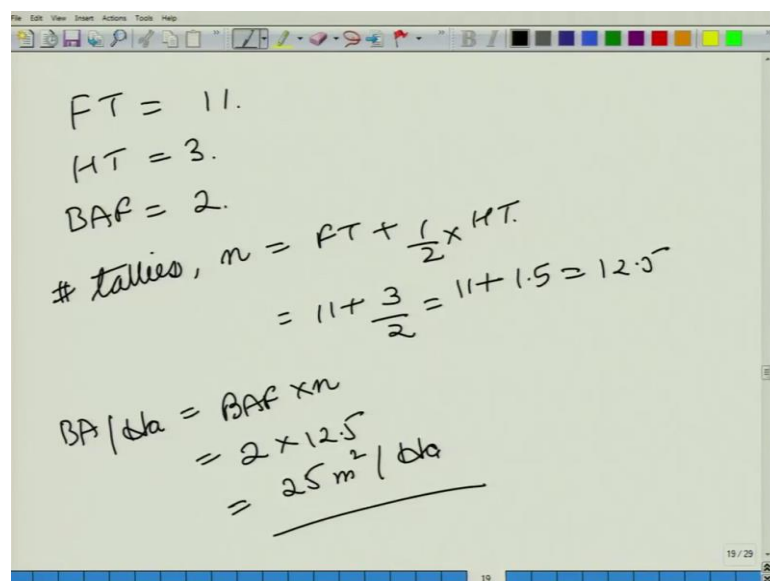
So, let us now look at another example.

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So, on your screens now. So, it states that using 2 factor gauge an operator counts 11 trees that appear wider than the gauge. So, when we say 2 factor gauge, it means that the basal area factor is or BAF is 2. Now, the operator counts 11 trees that are wider than the gauge. Now, if we consider 11 trees that appear wider than the gauge, so these are the full tallies. So, the number of full tally is 11 and 3 that appear to be the same width. So, trees that are the same width as that of the gauge are half tallies and we are required to find the basal area of the stand. So, how do we do that?

(Refer Slide Time: 19:39)



So, we are given that the number of full tally is 11, the number of half tally is 3, the basal area factor is equal to 2. So, the number of tallies which we call n is equal to the number of full tallies plus half. The number of half tallies is 11 plus 3 by 2. Now, 3 by 2 is 1.5. So, this is 12.5.

Now, the basal area per hectare is given by BAF times n. So, here BAF is 2 times 12.5 is 25 square meter per hectare. So, this is how we calculate the basal area density of a stand if we have BAF or the basal area factor and the number of trees in full tally and in half tally.

So, to continue let us look at one more example. So, it states that the distance between the nearest two trees measured in 12 systematically chosen sample plots of a plantation are, so here we have 12 samples. So, we have m is equal to 12 and here we have the nearest 2 trees. So, we are given these distances L as 3.25, 2.75 and so on and we are required to calculate the number density of the plantation.

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The image shows a handwritten derivation on a whiteboard. At the top, it lists $m = 12$ and a list of 12 distances L_i from 1 to 12. Below this, it calculates the number density using the formula $\frac{4}{\pi} \times 10000 \times \sum \frac{1}{L_i^2} \times \frac{1}{m}$. The sum is calculated as $\frac{1}{3.25^2} + \frac{1}{2.75^2} + \frac{1}{2.05^2} + \dots + \frac{1}{3.05^2} + \frac{1}{3^2}$. The final result is 1504 stems/ha.

$$m = 12$$

$$L_i = \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3.25, & 2.75, & 2.05, & 3.55, & 2.85, & 2.9, & 3.1, & 3.35, \\ 9 & 10 & 11 & 12 \\ 3.6, & 2.65, & 3.05, & 3 \end{matrix}$$

Number density

$$= \frac{4}{\pi} \times 10000 \times \sum \frac{1}{L_i^2} \times \frac{1}{m}$$

$$= \frac{4}{3.14} \times 10000 \times \left[\frac{1}{3.25^2} + \frac{1}{2.75^2} + \frac{1}{2.05^2} + \dots + \frac{1}{3.05^2} + \frac{1}{3^2} \right] \times \frac{1}{12}$$

$$= 1504 \text{ stems/ha}$$

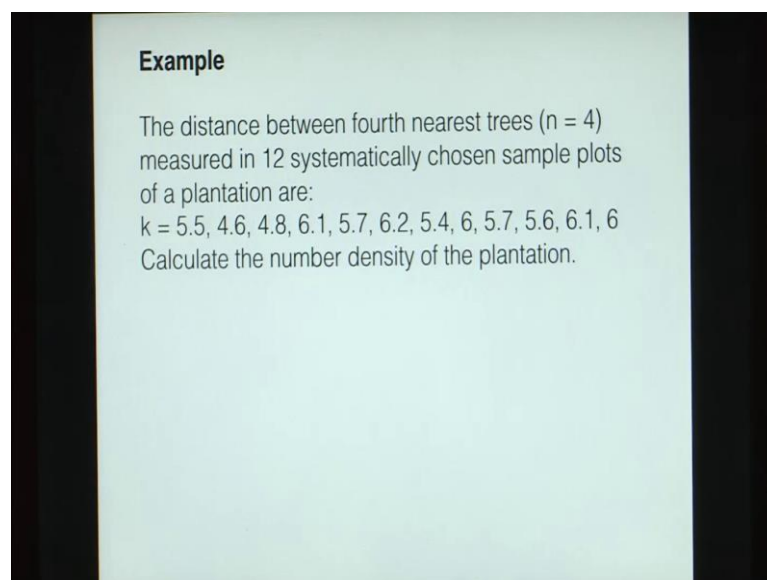
So, what are we given here? We are given that m is equal to 12, L_i is given as 3.25, 2.75, 2.05, 3.55, 2.85, 2.9, 3.1, 3.35, 3.6, 2.65, 3.05 and 3. So, 1 2 3 4 5 6 7 8 9 10 11 12. So, we have 12 samples and all the L_i are given and we are required to calculate the number density of the plantation.

So, we will use the formula number density is given by $4 \pi \sum_{i=1}^m \frac{1}{L_i^2}$ that we have already calculated previously.

So, what is this? We have $4 \pi \sum_{i=1}^{12} \frac{1}{L_i^2}$ where L_i values are 3.25, 2.76, 2.05, 1.3, 0.5, 3, 1.2, 1.3, 0.5, 3, 1.2, 1.3. So, when we do this calculation, we get the value of 1504 stems per hectare. So, this is just to recapitulate. So, if we have m number of samples that in which the stems that are closest together are separated by the distances given by L_i , then we can calculate the number density by using the formula $4 \pi \sum_{i=1}^m \frac{1}{L_i^2}$.

So, let us take one other example there on your screen.

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Example

The distance between fourth nearest trees ($n = 4$) measured in 12 systematically chosen sample plots of a plantation are:

$k = 5.5, 4.6, 4.8, 6.1, 5.7, 6.2, 5.4, 6, 5.7, 5.6, 6.1, 6$

Calculate the number density of the plantation.

Now, the distance between the fourth nearest trees n is equal to 4 measured in 12 systematically chosen sample plots. So, here we have m is equal to 12 of a plantation are given. So, the k values are given and we need to calculate the number density of the plantation. So, in this case, we do not have the nearest neighbour. We have fourth nearest neighbours. So, how do you do that?

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So, critical angle is given by the angle subtended by your object divided by this length. So, the angle that is obtained from your eye is the critical angle. Anything that subtends an angle that is greater than the critical angle is taken as a full tally. Anything that subtends a lesser angle, then the critical angle is taken as a no tally. Anything that subtends an angle that is equal to that of the critical angle is taken as half tally. The total tally is given by the full tally plus half of the half tallies. So, that gives you the full tally.

Once you have the full tally and you know basal area factor of your device from the critical angle, so that can be computed. So, if you multiply n that is the total tally comprising of the full tally and the half tally, if you multiply this n with the basal area factor, you will get the basal area of the stand per hectare. Also, we can use this point sampling to calculate the stand density or the number of trees per hectare. We can also use it to calculate the number of trees that are in each diameter class.

Then, we also looked at a method of calculating the stand density apart from the point sampling. So, one is that you find out, you would mark a sample plot on your forest stand, you take its area, you calculate or you count the number of trees that are there in the sample plot divided by the area of the sample plot in hectares and you will get the number density of the stand density as the number of trees per hectare of the forest.

You can also use this method of sample plots to compute the basal area density of the stand which is sum of the basal areas of all the trees in your sample plot divided by the

area of the sample plot in hectares. So, that will give you the basal area density of the stand which is also given in the case of point sampling by n into BAF.

Now, we also looked at a method of computing the stand density just by measuring the distances between the two nearest neighbours. So, if you have the values of L_i , we can compute the stand density. If you have multiple L , then we take the average given by sum of 1 over L_i square into 1 by m , where m is the number of samples that you have taken in your measurements.

We can also compute the stand density using the n th nearest neighbours using the formula n minus half into 10000 by m pi into 1 over k_i square its sum. So, that is another way of computing the stand density.

Thank you for your attention. [FL].