Forest Biometry
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Lecture – 31
Volume: Direct calculations through sections

[FL]. In this week we shall start looking at volume computations. As you would remember computation of the volume is one of the most essential parts of doing forestry. Because whether we are doing forestry whether we are managing our forest stands for say timber or for carbon sequestration or for any other means. In most cases we require how much amount of biomasses there in the forest how much of it can be harvested, how much of it requires to be left in the forest and so on.

So, volume computations are paramount in managing any forest stand. Today we shall start looking at volume computations through sections. So, these are known as direct calculations. So, we have done one example before.

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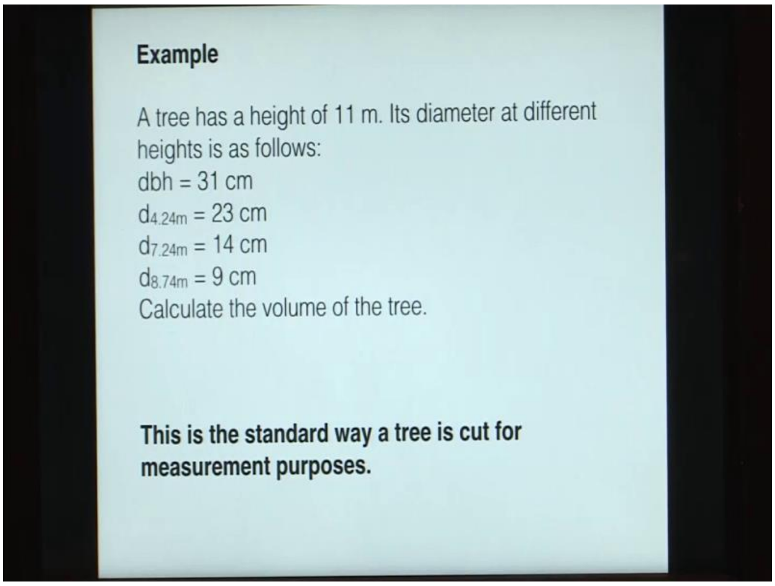

So, let us use this example to see how what we mean by direct computations. So, in this problem that we had done in the previous lecture, a tree has a height of 11 metres it is diameter at different heights is given and we were required to calculate the volume of the tree.

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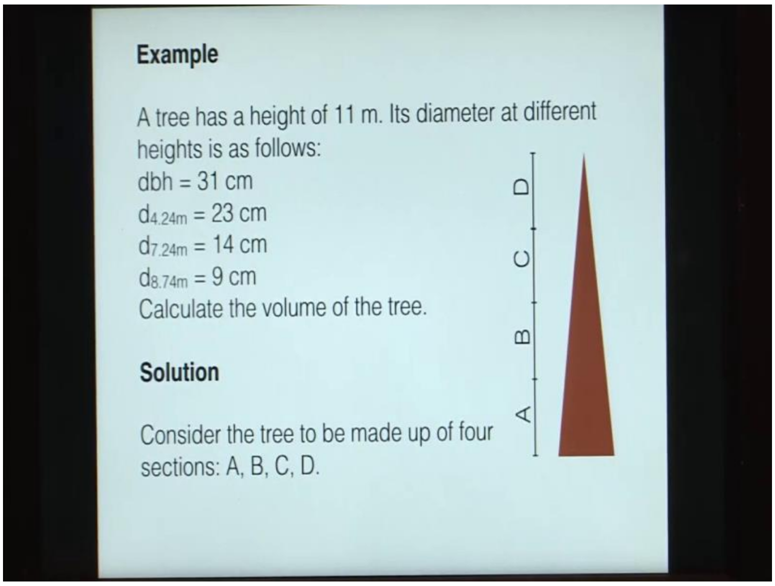

Now, this is the standard way in which a tree is cut for measurement purposes, and as we had discussed in that lecture we divided our tree into a number of sections, in which the first section is considered to be of a length of twice of breast height.

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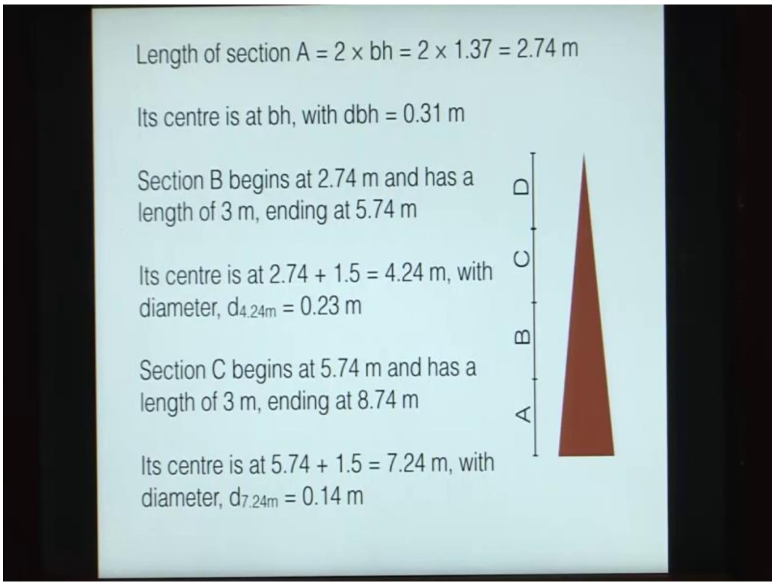

So, that we could use the middle diameter or the diameter at breast height to compute it is volume. The other sections were taken to be 3 meter sections. So, that not only can we use this tree for the computation of volume, but at the same time it can also be used, in a profitable manner by selling it.

So, 3 meter is the standard section length. And the top most section was considered to be a conical section.

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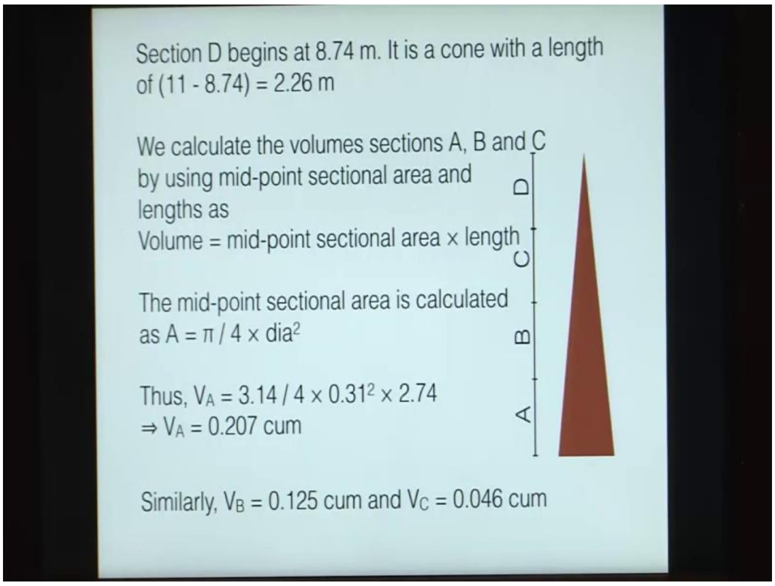

So, we computed the volumes of all the sections and then we added up those volumes to get the volume of the tree. We also dealt with the volumes of a number of different shapes.

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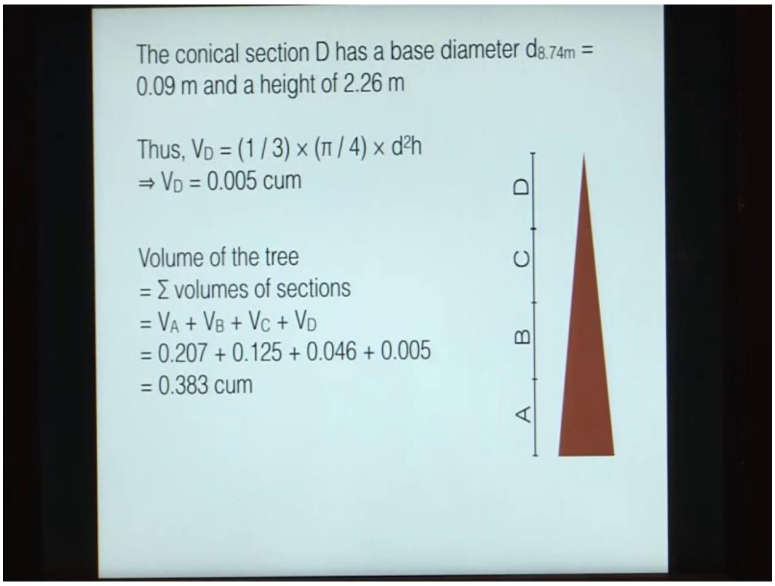

So now, today we will look at an extension of those equations.

So, as you would recall the volume of a cylinder is given by s into l. So, we had earlier referred to this volume by this equation.

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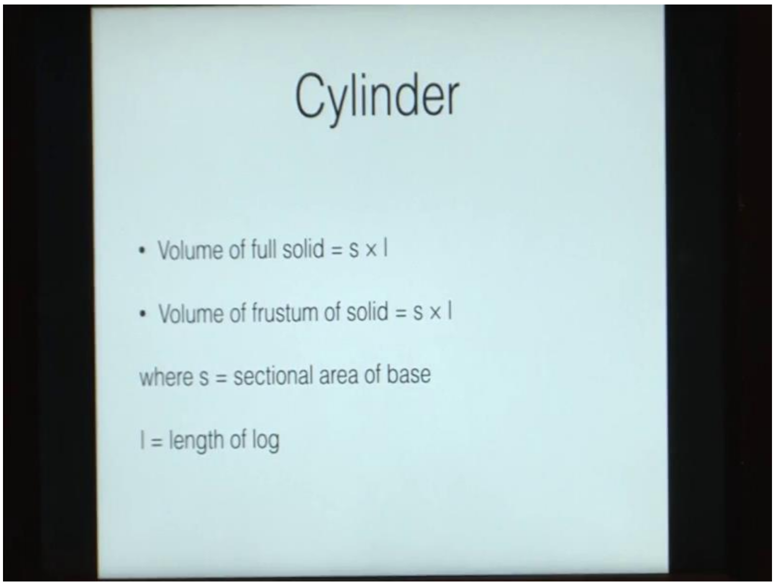

So, in a cylinder both the end sections are circular it has a length of l. So, it has and it has a radius of r. And so, we use to say that volume is equal to pi r square h.

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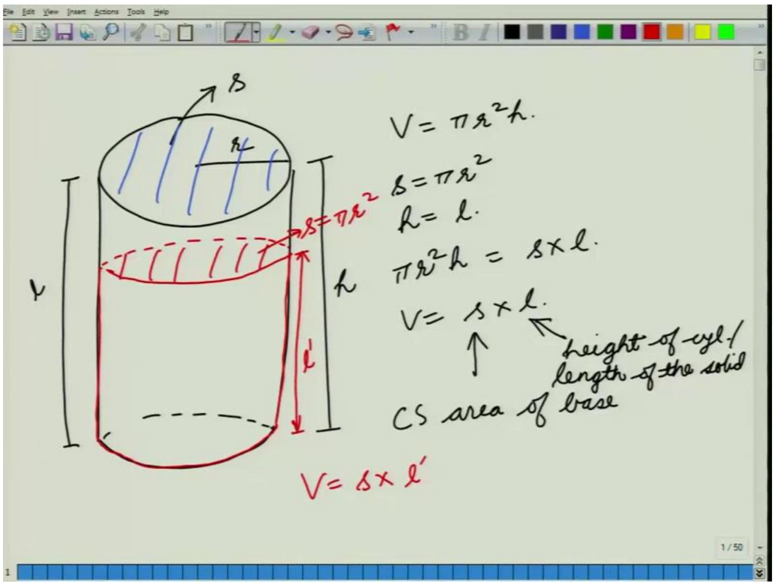

In today's lecture we will redefine these terms. So, we consider the area of a cross section to be s. And we consider this height as the length of the solid. So, in this case because the section is a circle. So, s is equal to pi r square h is equal to l. So, we can write that pi r square h is equal to s into l. So, that is a volume of a cylinder that is s into l. Where s is the cross sectional area of base, and l is the height of cylinder or we can call it as the length of the solid.

Now, if we take the section of this cylinder. So, let us take a section at this point. So, we have removed the top portion, and we are now considering this section. So, this part of the cylinder is left. So, if we wanted to calculate the volume of this frustum of a cylinder, then also we could use the same equation because this section and this section both are parallel circles. So, the cross sectional area here is will also be equal to pi r square. And now we can compute it for another length let us call it l bar. So, in this case the volume will be given by s into l bar. So, when we are talking about a cylinder.

So, coming back to the slides. The volume of the full cylinder is or the volume of the full solid is given by s into l, the volume of the frustum of a solid is also given by s into l where l now is the length of the frustum. And here s is the sectional area of the base and l is the length of the log or the length of the solid.

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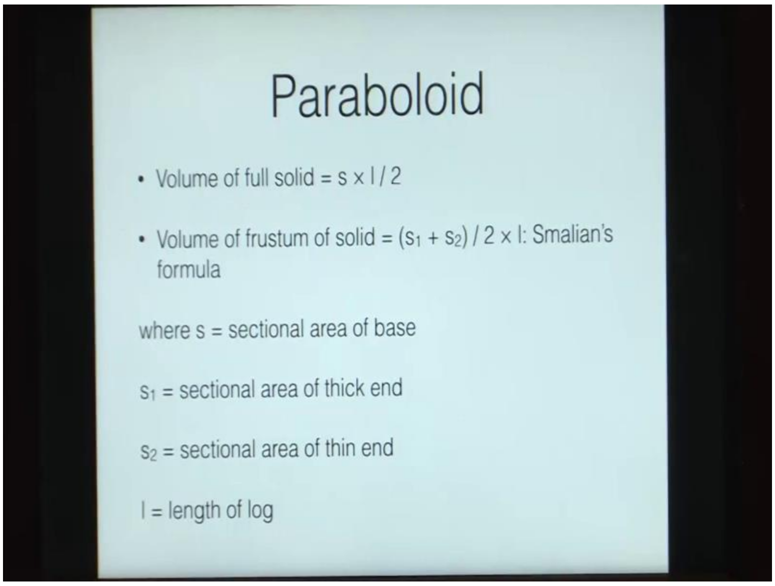

Now, we also looked at a shape called paraboloid. Now in the case of a paraboloid the volume of the full solid is given by s into l by 2. So, if you remember in the case of a cone the volume was 1 by 3 pi r square h in the case of a paraboloid, in the place of 1 by 3 it is one 1 by 2. So, the volume of a full solid is given by s into l by 2 and the volume of the frustum of the solid if he takes Smalian's formula it is s 1 plus s 2 by 2 into l.

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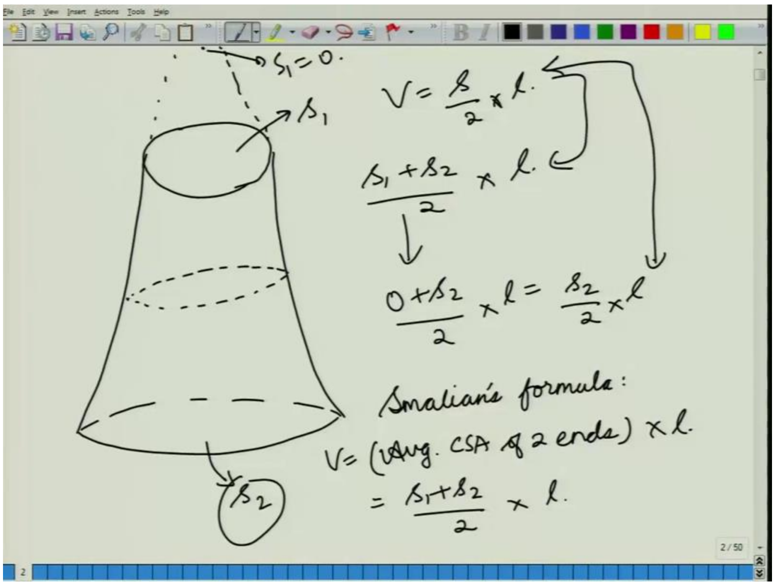

So, in the case of paraboloid Now, if this portion has a cross sectional area of s and this portion has a cross sectional area sorry, let us call it s 1 and s 2 then the volume of the frustum of the solid can be taken as the average of both these areas. So, that would be the area of the cross section nearly of the middle of the solid. So, it will be s 1 plus s 2 upon 2 into l. Now previously we saw that the volume is given by s by 2 into l. So, how can we correlate both of these? Because if we extend this solid, such that both these ends meet at a point.

So, at that point your s 1 will be equal to 0. So, then we will have if we use this equation it will be 0 plus s 2 where s 2 is the cross sectional area of the base upon 2 into l which will be s 2 upon 2 into l which is the same as this equation. So now, this formula is known as the Smalian's formula. Volume is the average cross sectional area of 2 ends into length of the solid. So, the average cross sectional area is given by s 1 plus s 2 upon 2 multiplied by the length of the solid. So, this is the Smalian's formula.

Now coming back to the slides. So, here we have volume is equal to s 1 plus s 2 upon 2 into l. Now in the top equation in this equation the s is the sectional area of the base, in this case s 1 and s 2 refer to the sectional areas of the ends when we are taking a frustum and l is the length of the solids. You must not confuse between s s 1 and s 2. These also another formula for paraboloid, which is known as the Huber's formula.

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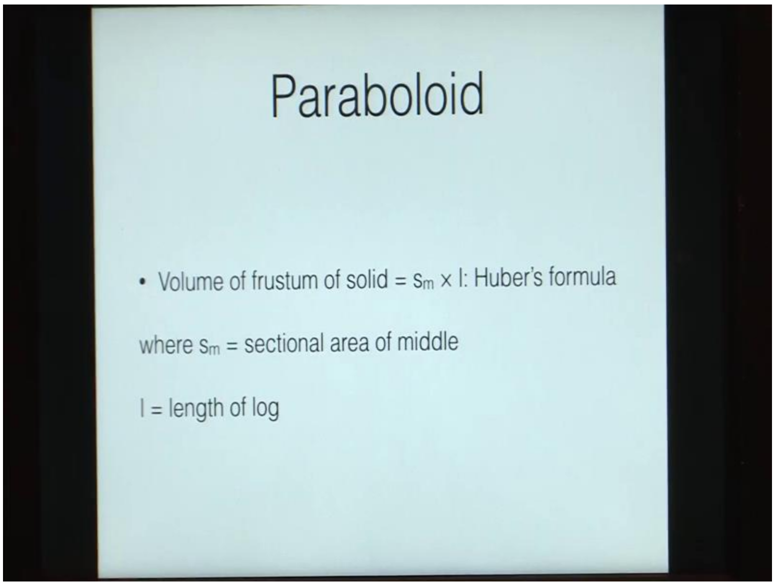

So, this talks about the volume of a frustum of solid and it is given by s m into l. Where s m is the sectional area of the middle of the solid and l is the length of the log.

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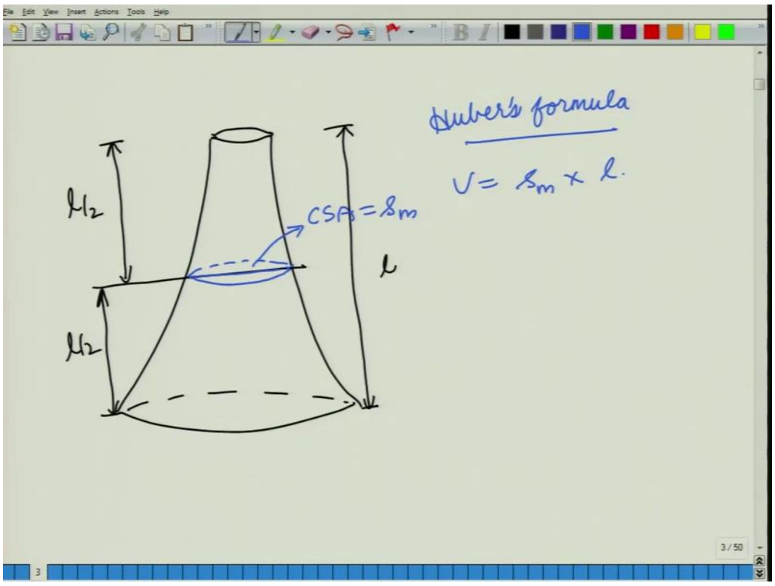

So, if we drew our paraboloid if the length is l. So, this much is l, if we take l by 2. So, we are dividing into 2 parts. So, this one is l by 2 and this much also is l by 2. So, this is the middle point. So, at the middle point if we take a cross section and we define this cross sectional area as s m. So, s m is the cross sectional area of the middle point of the solid, then our Huber's formula gives volume is equal to s m into l. So, s m here is the sectional area of the middle of the solid and l is the length of the solid. Now coming back to the slides. So, this is the Huber's formula for a paraboloid.

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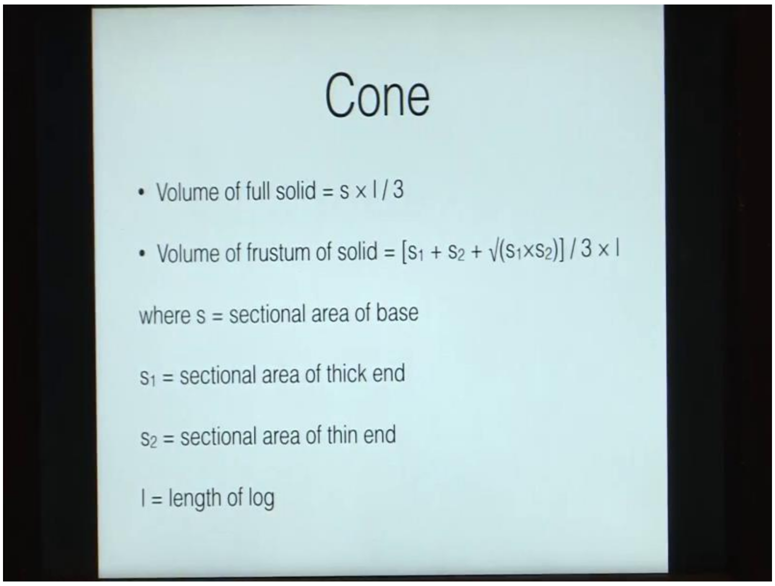

Now, let us consider a cone. So, in the case of a cone we had earlier defined the volume of the full solid as 1 by 3 pi r square h.

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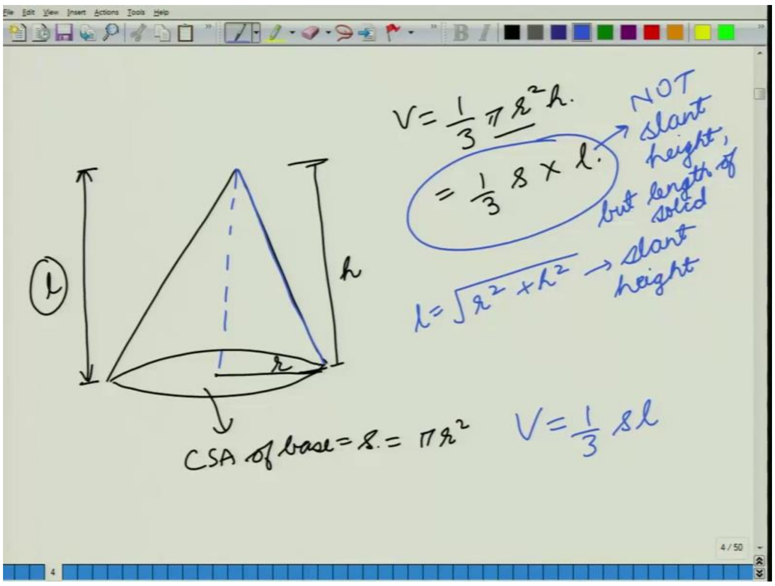

Now pi r square is the sectional area of the base of the cone. So, if we drew a cone. So, if this is a cone with a height h and the bottom with a radius of r, then we define volume is equal to 1 by 3 pi r square h. Now changing the notations we can call the length of the solid as l and the cross sectional area of base as s. So now, cross sectional area of base is given by pi r square. So, in this formula we are replacing. So, 1 by 3 pi r square can be written as s and h in this case can be written as l.

So now in this case l is not the slant height. So, earlier we had defined the slant height as the site So, this length. So, we had defined l is equal to square root of r square plus h square which is the slant height. Now remember that when we are defining this equation this l is not the slant height, but length of solid. So, you must not confused between this l and the previous l. So now, we can write the wall the volume as 1 by 3 s into l. Now what will be the cross what will be the volume of a frustum of a cone?

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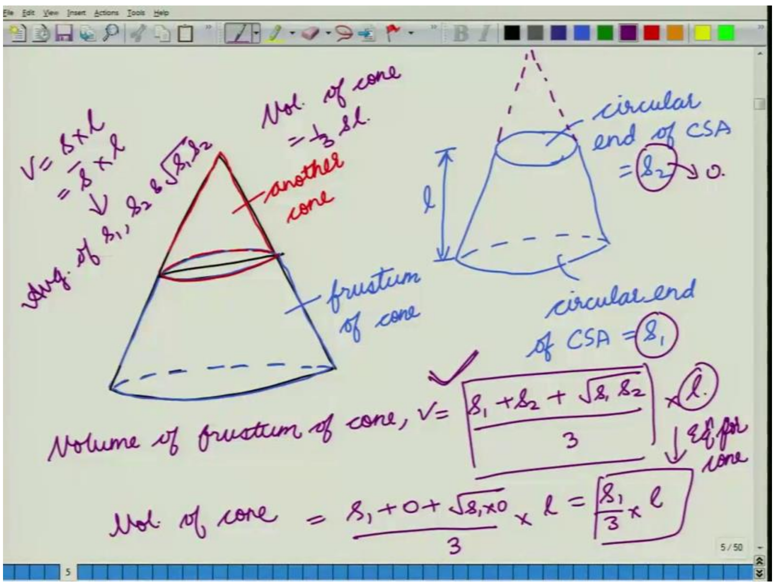

So, what is the frustum of a cone? If we drew a cone again and we are taking a section here. So now, this portion So, the top again is a cone, but this portion is the frustum of a cone.

So, in this case we are removing the top portion, the top portion incidentally is another cone. So, the frustum of a cone is us a portion of a cone in which it is top has been cut to give it 2 ends. So now, if you drew the frustum of the cone again. So, it is top would be circular, the sites will be straight and the bottom edge will also be circular. So, this is a circular end this again as a circular end. Now we can write this now both these ends have their own sectional areas. So, this sectional area is So, this is crop circular end of cross sectional area is equal to s 1 and here it is the circular end of cross sectional area is equal to s 2.

Now, if l be the length of the solid. So, if this height is equal to l, then the volume of the frustum of the cone frustum of cone is given by v is equal to s 1 plus s 2 plus square root of s 1 s 2 pull upon 3 into l. So, we have taken both these cross sectional areas s 1 and s 2, you add them up then you multiply s 1 and s 2 and then take a square root. Then you take all 3 of these take the average. So, this is the average of s 1 s 2 and square root of s 1 and s 2. So, this is the average cross sectional area multiplied by the length. So, for instance earlier we had define the volume to be s into l in the case of a cylinder here it is s bar into l, where s bar is the average of s 1 s 2 and square root of s 1 s 2.

Now, incidentally if we extended this frustum to make a cone again. So, in that case your s 2 will be 0. So, what will be the volume now? Volume will be s 1 plus 0 plus square root of s 1 into 0 which is 0 upon 3 into l which will be s 1 by 3 into l. So, this will be the volume of cone. So, the volume of the cone is 1 by 3 s into l, volume of cone is 1 by 3 s into l where s is pi r square. So, by using this equation for the frustum we can derive the equation for cone as well. So, this is one formula that you need to remember the volume of a frustum of a cone is given by the average of s 1 s 2 and square root of s 1 s 2 multiplied by the length of the solid. Now coming back to the slides again.

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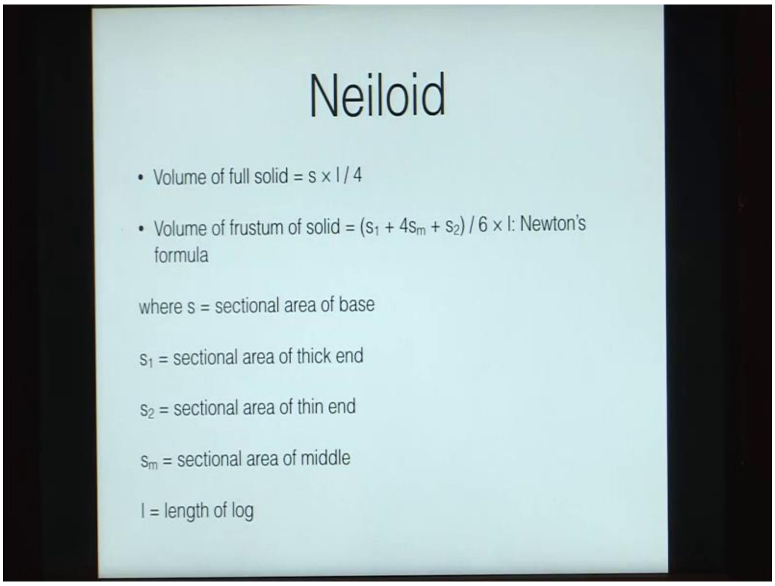

When we talk about a neiloid. So, neiloid is also a shape that we encountered before. So, here we define the volume of the full solid as 1 by 4 s into l.

So, in the case of cone it was 1 by 3 in the case of a neiloid it becomes 1 by 4. And the volume of the frustum of the solid is given by Newton's formula by the average of s 1 s 2 and 4 times the middle sectional area divided by 6 into l.

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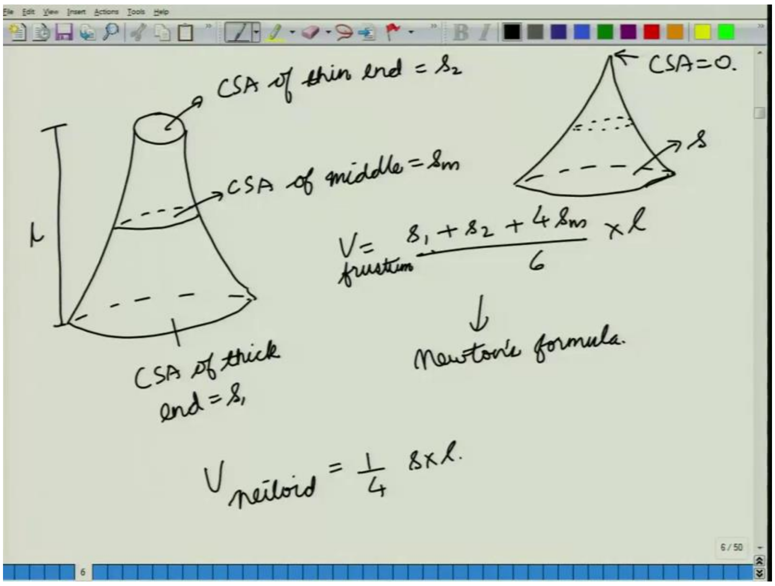

So, in the case of a neiloid So s 1 is taken to be the cross sectional area of thick end is s 1 cross sectional area of thin end is s 2. Then we take a middle point and we take the cross sectional area of middle is s m the length of the solid is l. So, volume is given by s 1 plus s 2 plus 4 times of s m divided by 6 into l. In this thing is known as Newton's formula.

This is the volume of frustum. Now if we extended this frustum such that it became a full solid with one end to be circular by the given by the cross sectional area of s. And the other being a point with a cross sectional area of 0. Then what would be the volume of the neiloid complete neiloid. So, in this case at around half of it will have the s m will be roughly equal to half of s and then will give the volume of the neiloid as 1 by 4 s into l. So, this is the volume of the full solid. Now coming back to the slides. So, this was the Newton's formula.

So, the Newton's formula if you generalized it says that the volume of the frustum of a solid is given by the 2 cross sectional areas s 1 plus s 2 plus 4 times the cross sectional area of the middle portion So, 4 into s m divided by 6 into l.

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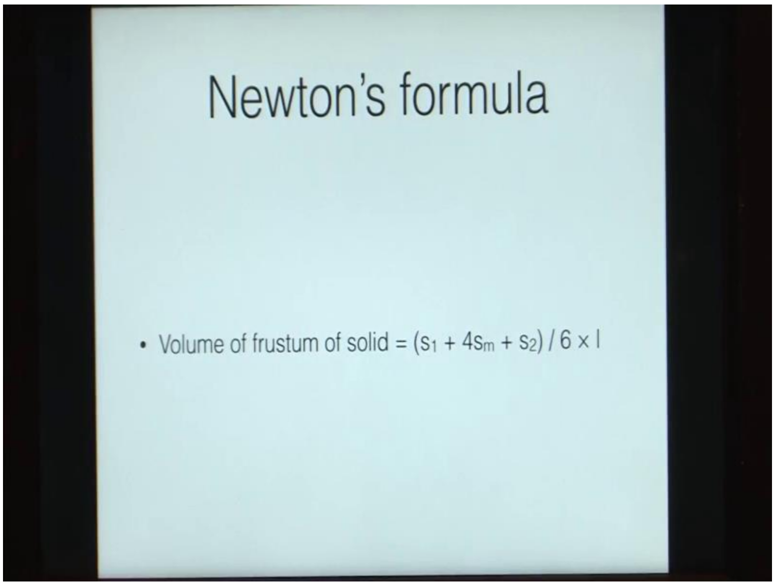

So, if you look at this formula in greater detail this formula is one of the most accurate formulae which can be which is used to find errors in the other formulae.

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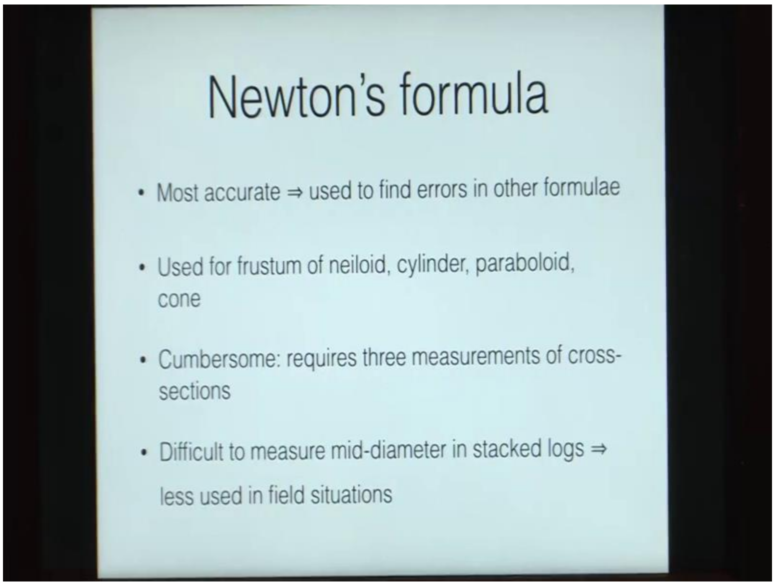

So, the other formulae are Huber's formula and the Smalian's formula. So, Newton's formula is considered to be the most generalized and the most accurate of all these 3. It can be used for the frustum of a neiloid cylinder paraboloid and a cone.

So, for all of these cases you can use Newton's formula; however, this formula is more cumbersome then the other 2 formulae, because here you are require you are required to take 3 measurements of cross sections of both the ends and the middle and in case your logs are stacked.

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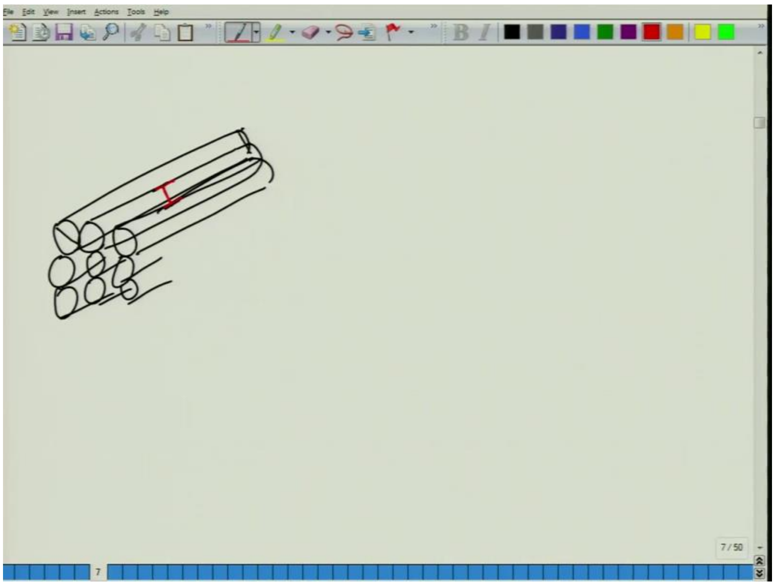

So, when we say stacked logs we mean that this is your first log, then you have the second log in continuation then you have your third log. Then there is a log below this then a log below these and so on. So, when you have a this stacked stack of logs. So, in this case taking the diameters of the ends is easy because they are accessible, but taking the diameter of the middle portion of a log is very difficult.

So, taking a diameter here or measuring the cross section here, becomes difficult because you do not have access to the central portion of the log. So, coming back to the slides, it is difficult to measure the mid diameter in stack logs which is why Newton's formula even though it is the most accurate of all the 3 formulae it is less used in the field conditions.

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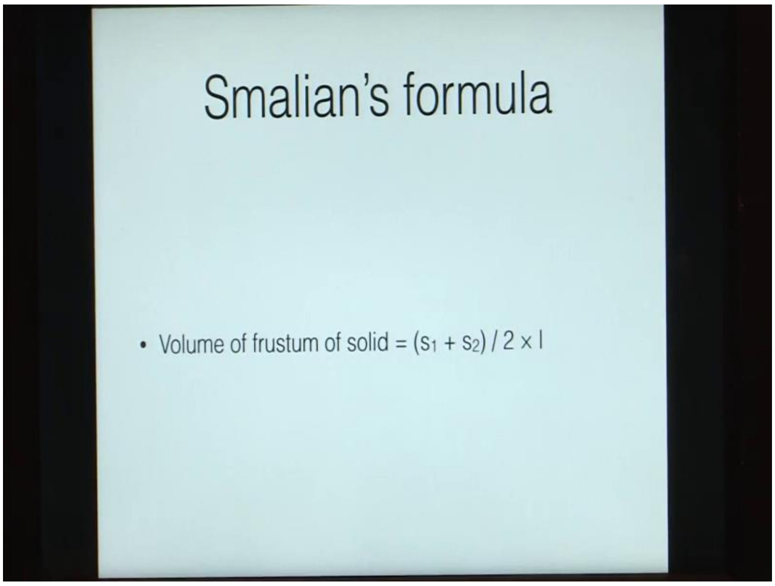

Now let us look at Smalian's formula and try to generalize it. So, it states that the volume of a frustum of a solid is the average of the end cross sectional areas multiplied by the length of the solid.

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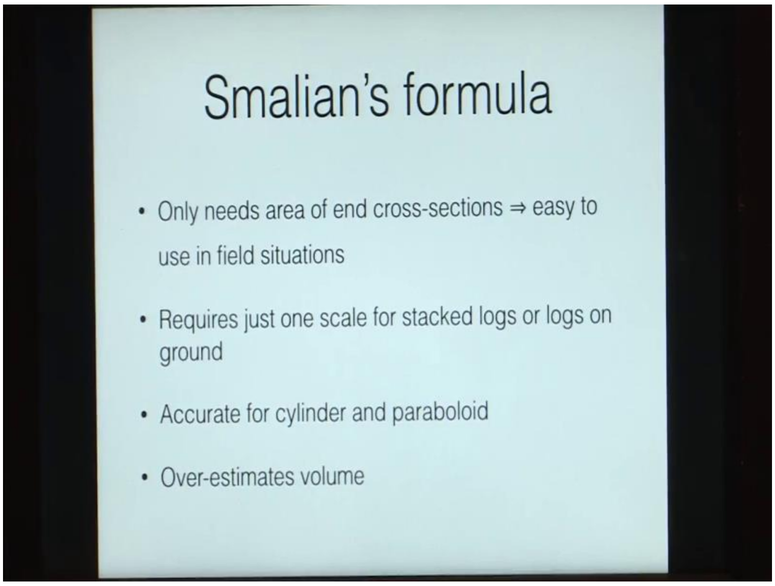

So, here in place of taking 3 readings you are only required to take 2 readings of the end cross section. So, it is easy to use in the field conditions. And when you have a stack of logs because you only are required to calculate the end cross sectional area. So, you can just use a scale or a ruler to measure the diameter of the ends and to get these cross sectional areas.

Smalians formula is accurate for cylinder and paraboloid, but it overestimates the volume by a small bit. That the formula that we looked at today is the Huber’s formula.

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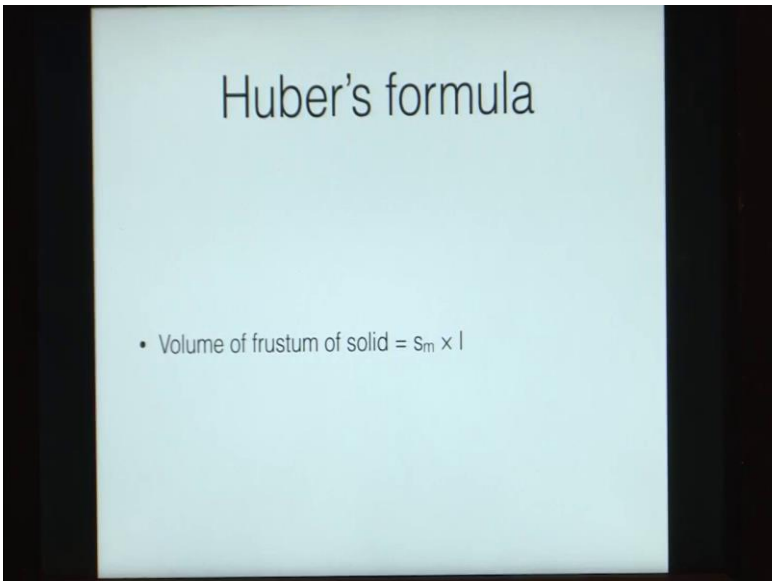

So, in the case of Huber’s formula it is defined as the cross sectional area of the middle of the solid multiplied by the length of the solid.

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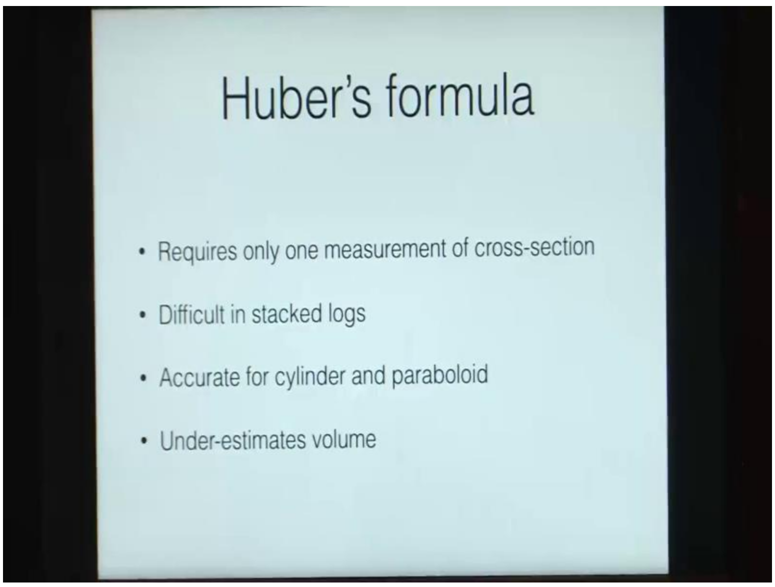

So, in this case you are required to take only one cross sectional area measurement at the middle, but again in the case of stacked logs stack becomes difficult because it is difficult to access the central part of a stacked log. It is accurate for cylinder and paraboloid and unlike Smalian's formula that overestimated the volume Huber's formula underestimates the volume.

So now that we know that the Newton's formula is the most accurate formula, what is the amount of overestimation or underestimation in the other 2 formulae?

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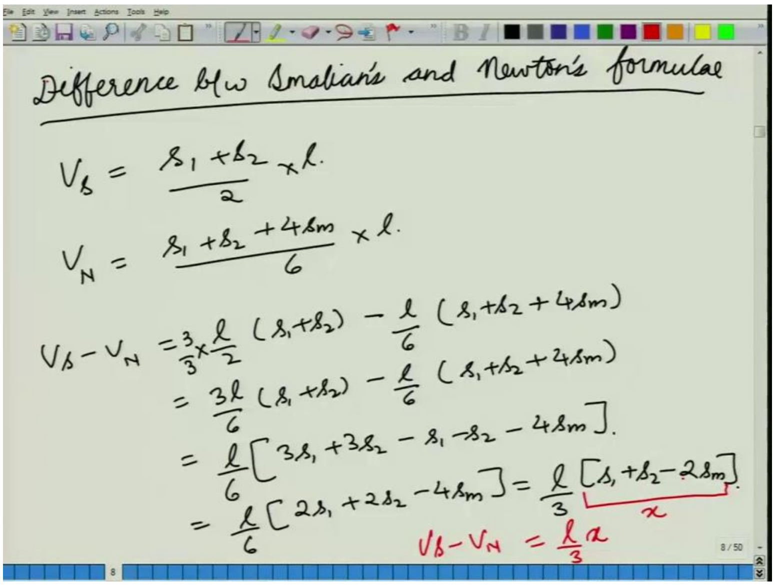

So, let us find out the difference between Smalian's and Newton's formulae. So, if you remember the volume in the case of the Smalian's formula. So, v s is given by s 1 plus s 2 upon 2 into l. In the case of Newton's formula the volume is given by s 1 plus s 2 plus 4 times of s m whole divided by 6 into l.

So, what is the difference between both of these? So, if you take v s minus v n we get l by 2 into s 1 plus s 2 minus l by 6 into s 1 plus s 2 plus 4 s m. Or if you multiplied by 3 by 3 that is multiplying get by unity we get 3 l upon 6 s 1 plus s 2 minus l upon 6 s 1 plus s 2 plus 4 s m, or taking l by 6 common. So, we have 3 s 1 plus 3 s 2 minus s 1 minus s 2 minus 4 s m, which is equal to l by 6 2 s 1 plus 2 s 2 minus 4 s m. Which is equal to l by 3 s 1 plus s 2 minus twice of s m.

So, this is the difference between the smalian and the Newton's formula. Now if you calculated the difference between Huber's and Newton's formulae, what would that be?

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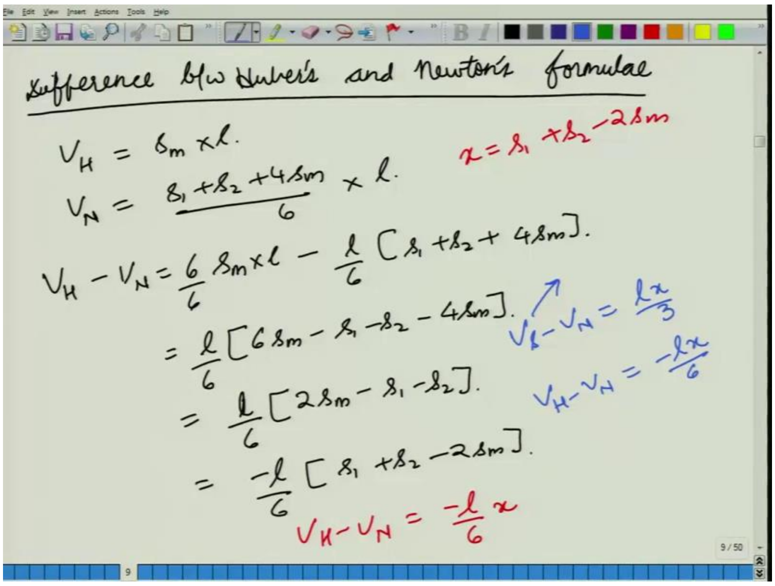

So, volume by the Huber's formula is given by s m into l. Volume by the Newton's formula is given by s 1 plus s 2 plus 4 times of s m by 6 in to l. So, if you calculated volume by Huber's minus volume by Newton's, what do we get? We get s m into l minus l by 6 s 1 plus s 2 plus 4 times of s m. Multiplying it by 6 by 6 and taking l by 6 common we get 6 times of s m minus s 1 minus s 2 minus 4 times of s m. Which is equal to l by 6.

So, we have twice of s m minus s 1 minus s 2. Or we can write it as minus l by 6 s 1 plus s 2 minus twice of s m. Now if you looked at the previous page here we had calculated the difference as l by 6 as l by 3 s 1 plus s 2 minus twice of s m. So, let us write this portion as x. So, here we have the difference v s minus v n is equal to l by 3 into x. So, x is equal to s 1 plus s 2 minus twice of s m right. So, s 1 plus s 2 minus twice of s m is x.

So, putting it here we have v h minus v n is minus l by 6 into x. So, what do we get here? We have v s minus v n is equal to, So here it first l by 3 x l x by 3 and v h minus v n is minus l x by 6. So, if this formula if the first formula is overestimating the a volume as compared to the Newton's formula and remember the Newton's formula is considered to be the most accurate formula. So, if the first one is overestimating then the second one would be underestimating, and if the first one is under estimating then the other one would be over estimating because in the first case it is plus and in this case it is minus.

So, today we had a look at the volume computation by the direct method. And we also looked at Huber's formula Smalian's formula and Newton's formula for different kinds of solids in frustums of solids. And we saw that if we considered Newton's formula to be the most accurate formula the Smalian's formula and the Huber's formula would is would bring out different estimations of the volume, which would be one if one is overestimation then the other one would be an underestimation.

So, we can use these formulae to calculate the volumes of different sections of logs or of complete logs by themselves.

Thank you for your attention. [FL].