

**Forest Biometry**  
**Prof. Mainak Das**  
**Dr. Ankur Awadhiya**  
**Department of Biological Sciences & Bioengineering & Design Programme**  
**Indian Institute of Technology, Kanpur**

**Lecture – 06**  
**Shape of a tree: Form and Taper**

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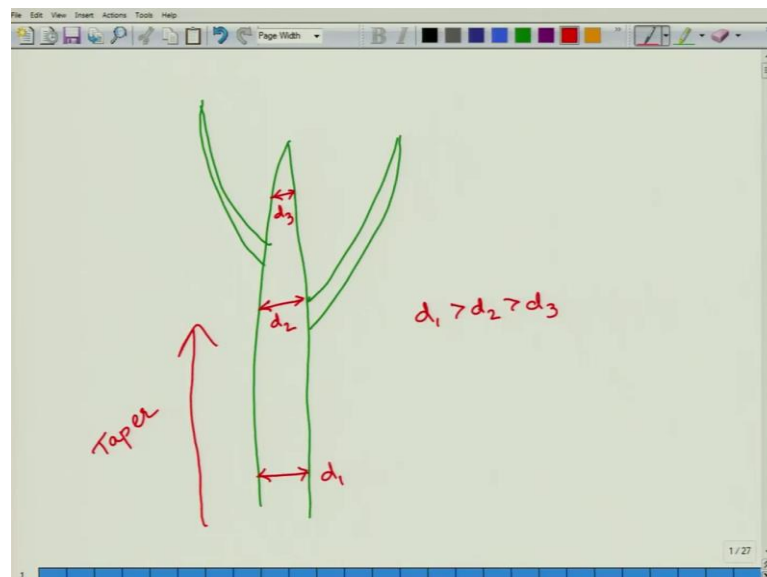
[FL]. In this week we shall explore the shapes of trees. Trees come in a variety of shapes and sizes look at this. Magnificent tree with lots of branches and shade the main stem is short, but wide branching begins at a low height. As we move up from the base the diameter of the branches reduce, why is that so, we will come to that in this week.

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This is another tree devoid of leaves. Trees that shed their leaves during dry part of the year are called deciduous trees. This is an adaptation to conserve moisture. Here the main stem is long and the branches begin at a greater height. If we concentrate on the main stem, we will observe that it tapers with height

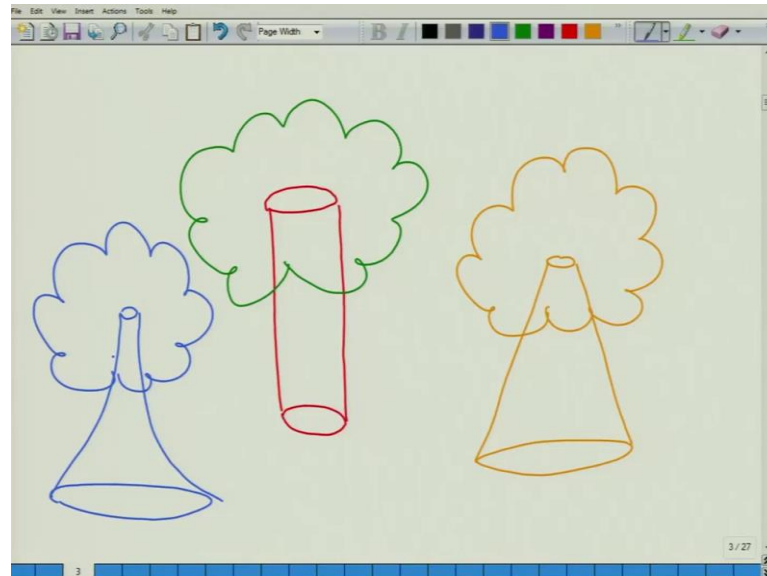
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So, what do we mean by taper? If you look at this particular tree, it would look something like this. So, essentially the diameter here let us call it  $d_1$  is greater than the diameters here it is call it  $d_2$  is greater than the diameter here  $d_3$ . So,  $d_1$  is greater than

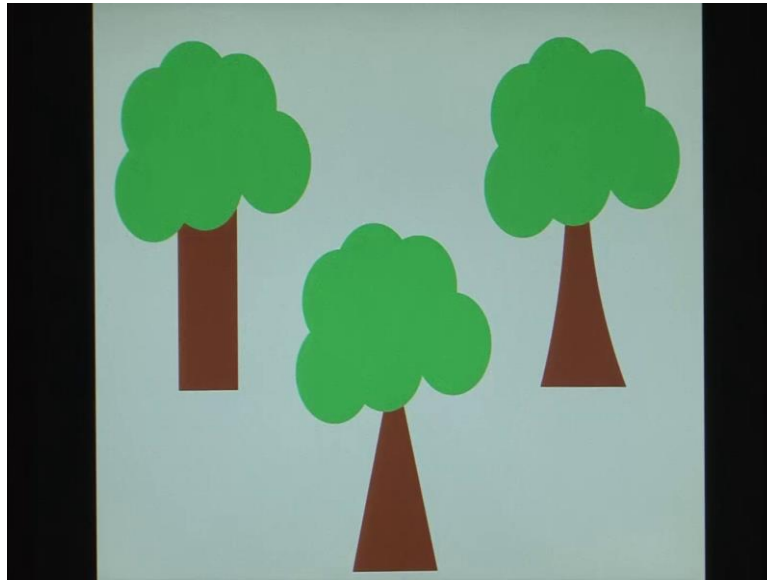
$d_2$  is greater than  $d_3$ . So, this reduction in diameter as you move from bottom to up is called as taper. So, how do we describe the shape of it of the three species main stem.

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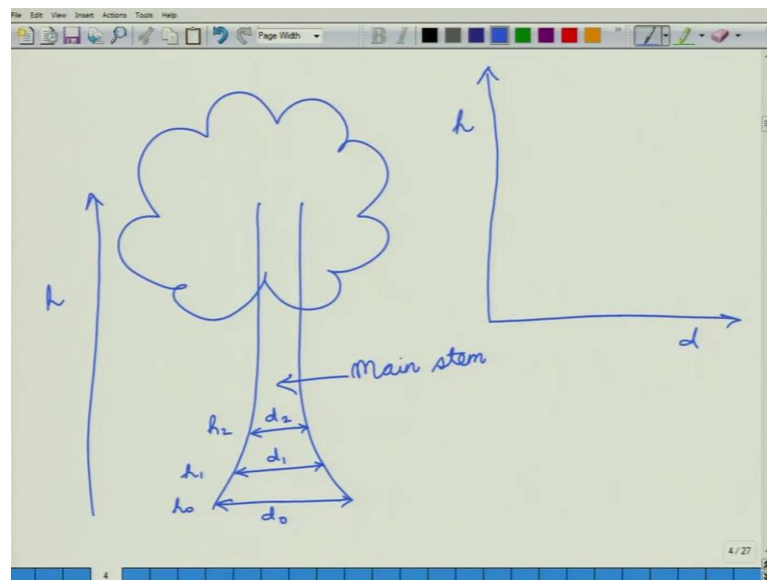
So, for example, can we describe a tree like this? So, we have a cylinder, and then we have leaves, can this describe the shape of a tree. Or for instance, can we have a cone, and on top of the cone we have the leaves is this a good description of a tree, or maybe some other shape say a shape like this the frustum of a paraboloid that could describe the shape of tree. So, which one of these is the best description of a tree shape. So, is it cylindrical, is it conical or is it maybe in some other shape.

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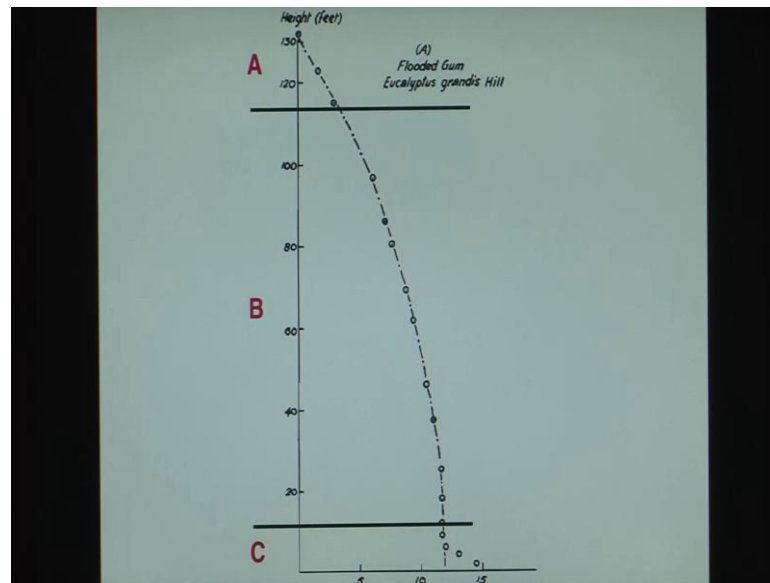
So, how can we figure out the shape of a tree? We do this by plotting its diameter at various points.

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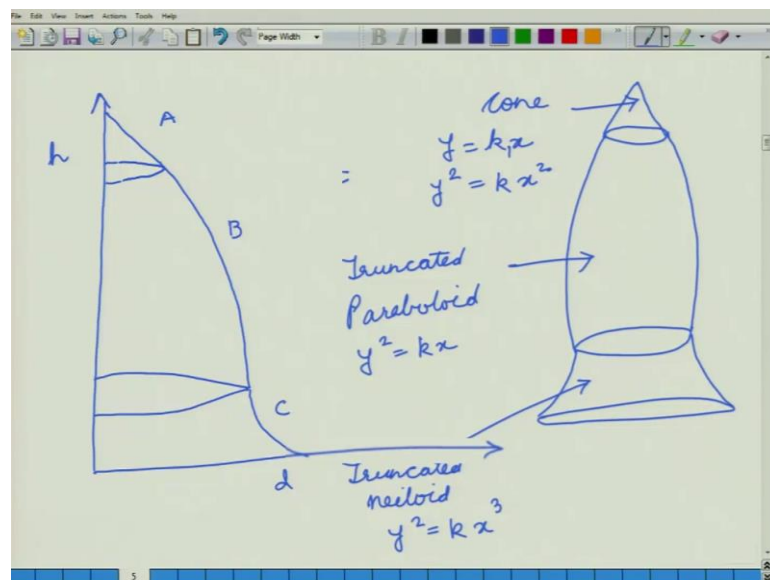
So, suppose you have a tree and this portion is the main stem. So, if we plot with height  $h$ , what are the various diameters? So, here we have a diameter  $d_0$  at the bottom then at height  $h_0$ , you have this, at height  $h_1$  you have a diameter of  $d_1$  at height  $h_2$  you have a diameter of  $d_2$  and so on. So, what will happen if we plot height versus diameter this is what we will get.

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So, on the x-axis here, we have diameter in inches; on the y-axis we have the height in feet. So, if we look at this curve this is for a eucalyptus species. So, if you look at this curve we can divide it into three parts.

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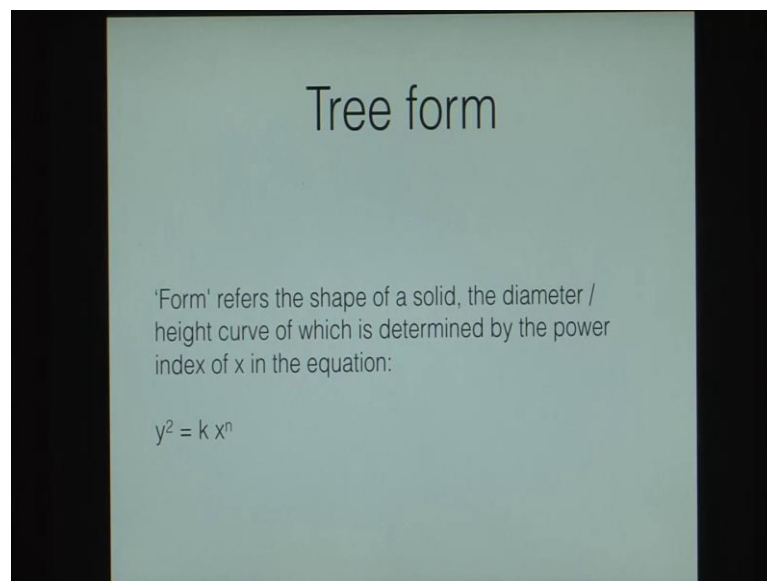


So, the topmost region which is depicted as section a, in this figure it looks like a cone. So, let me draw this curve for you. So, here we have the diameter in here we have the height. So, the topmost portion would look like a cone. So, this is a conical section then we would have a paraboloid section, this is the second section. And then a third section

that goes on like this. So, we call this section A, section B, and section C. So, the section A, it is a conical section or maybe it will go to the other side as well. So, it will be so this is like the half of the tree. And similarly we will have it on the other side as well. So, the topmost section is a cone followed by a paraboloid section followed by a neiloid sections.

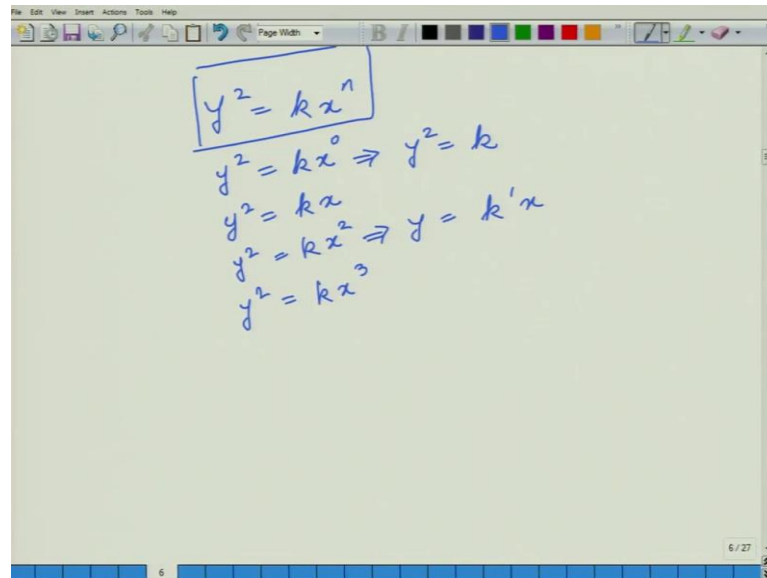
So, if we described the shape of a tree as this, we could derive equations for the various parts. So, this portion because it is a cone it will have an equation of  $y$  equal to  $kx$ . Now, traditionally we write these equations as  $y$  square. So, if this was  $kx$ ,  $y$  square equal to  $kx$  is the equation of the upper portion which is the cone. Now, here  $x$  is the distance from the apex and  $y$  is the diameter. Now, in the middle portion, which is a truncated paraboloid; here we will have the equation of  $y$  square is equal to  $kx^2$ . And in the lower portion which is a truncated neiloid, we will have the equation of  $y$  square is equal to  $kx^3$ .

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So, what do we mean by a tree form? The shape of a tree can be described in two parts the form of the tree and the taper of the tree. Form refers to the shape of a solid, the diameter or height curve of which is determined by the power of  $x$  in the equation,  $y$  square is equal to  $kx^n$ .

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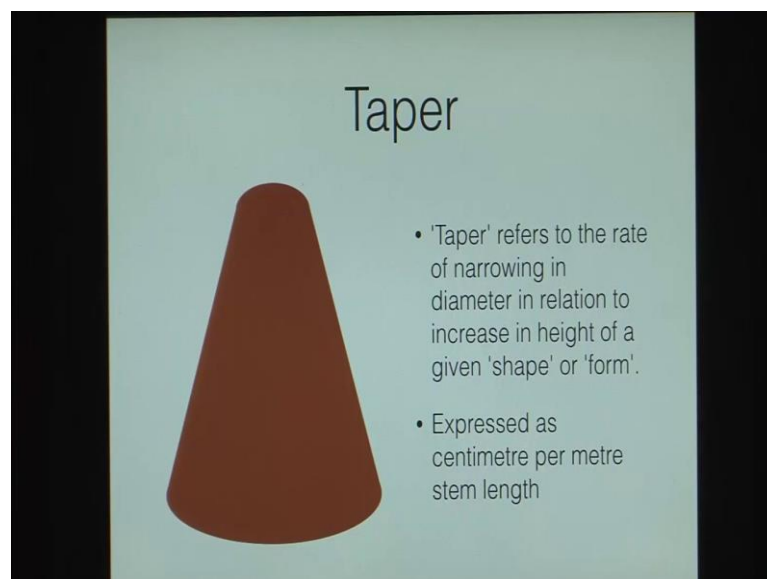


A whiteboard with a toolbar at the top and a page number '6' at the bottom. The equations are written in blue ink:

$$y^2 = kx^n$$
$$y^2 = kx^0 \Rightarrow y^2 = k$$
$$y^2 = kx$$
$$y^2 = kx^2 \Rightarrow y = k'x$$
$$y^2 = kx^3$$

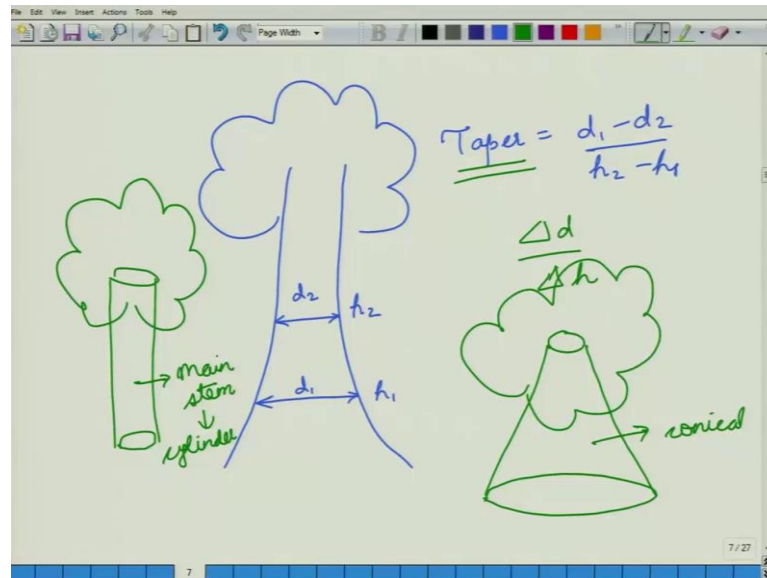
So, now when you have  $n$  equal to 0, you will have the equation  $y$  square is equal to  $k$   $x$  to the power 0, which will mean that  $y$  square is constant. If you have  $k$ , if you have  $n$  equal to 1, you will have  $y$  square is equal to  $k$   $x$ . When you have  $k$   $n$  equal to 2, you will have  $y$  square is equal to  $k$   $x$  square which can also be written as  $y$  square is equal to  $k$  prime  $y$  is equal to  $k$  prime  $x$ . And if you have  $n$  is equal to three you will have  $y$  square is equal to  $k$   $x$  cube. Now all these different shapes as defined by the equation  $y$  square is equal to  $k$   $x$  to the power  $n$ , they are known as form of the tree.

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So, what is the taper? A taper refers to the rate of narrowing in the diameter of a tree when you go from its bottom part to its topmost part. It is expressed as centimeters per meter of the stem length.

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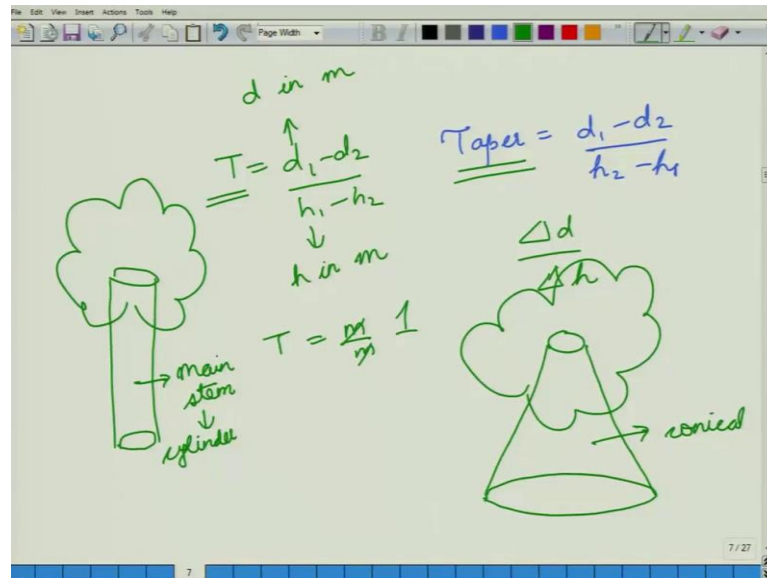


So, basically in a tree if you have the diameter is equal to  $d_1$  at height  $h_1$ , and the diameter is equal to  $d_2$  at height  $h_2$ . So, your taper is given by  $d_1$  minus  $d_2$  divided by  $h_2$  minus  $h_1$ . So, essentially we are taking the difference of the diameters, it is  $\Delta d$  divided by  $\Delta h$ . So, taper of a form refers to the rate of narrowing in diameter in relation to increase in height of the given shape or form. Now remember that the diameter for tree stem goes on decreasing as we move above the ground level as we saw in the in the very early photographs.

The rate of this decrease is called taper a tree with a large taper would resemble a cone. So, for instance, if you had a taper like this, the tree would resemble a cone. But if you had a very low taper something like this with taper nearly equal to 0, if you trees main stem would become a cylinder in this case it is conical. Now, if we look at this quantity taper, if you use another convention, and represented it in a negative form.

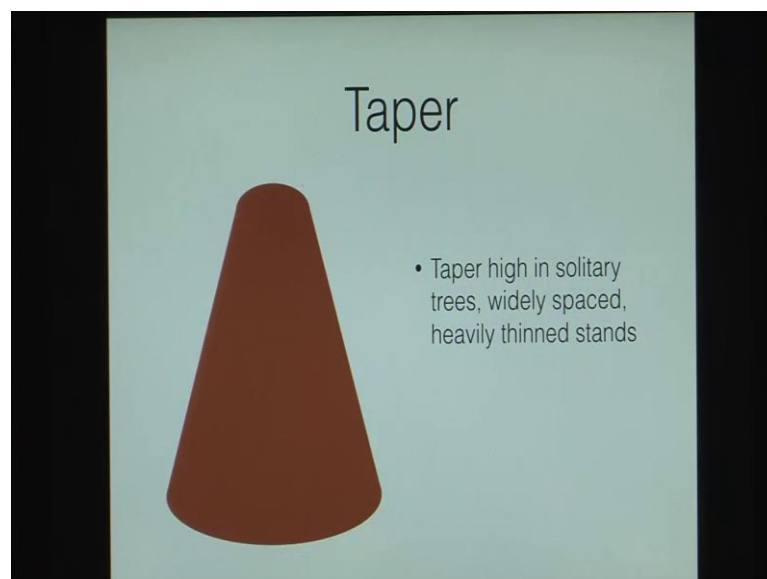


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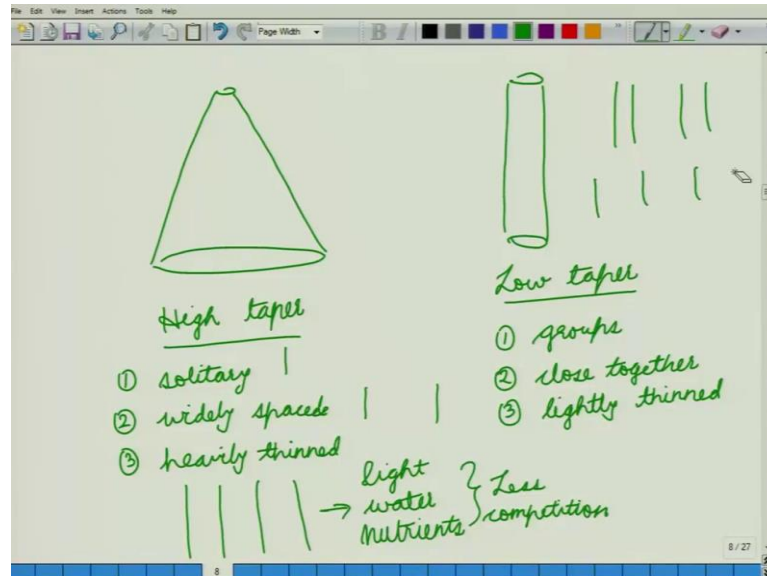
You could also write it as  $t$  equal to  $d_1$  minus  $d_2$  upon  $h_1$  minus  $h_2$  in which case a negative taper would show its diameter is decreasing with height and a positive taper would show that its diameter is increasing with height which is not seen in natural conditions. However, we can also note that if we express  $d$  in say meters and  $h$  in meters then what is the unit of taper. Taper would be meter upon meter. So, it will become a dimensionless quantity. So, taper is a dimensionless quantity.

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Taper is usually found to be high in solitary trees, and in widely spaced heavily thinned stands. Now, what do we mean by that?

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Taper is high when we say that taper is high it means that the tree is assuming a conical shape as against a cylindrical shape. So, this is high taper and this is low taper.

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Now, we say that a tree has high taper, if it is solitary. So, solitaire means that you have a single tree somewhere or it is widely spaced which means that if you have one tree here, the second tree is very far apart here you only have one tree or in heavily thinned. Now,

thinning is a procedure in the case of management of forest stands, where if you started with a very closely planted trees, when you perform thinning, you remove a few trees. So, for instance you are removing these trees. Now this is done so that these trees can have more of light more water, more nutrients. So, essentially they will be having less competition because there are less number of individuals fighting for the same number of resources.

So, essentially we have a high taper in the case of solitary trees, in the case of widely spaced trees and in the case of heavily thinned stands. Whereas, we have a low taper in the case of trees that are in groups that are close together and that are lightly thinned stands, in the case of a lightly thinned stand if you started with a stand like this. So, you removed one out of say every three individuals. So, this would become a lightly thinned. Whereas, if we started with the same stand and if you removed every second individual, it would become a heavily thinned stand.

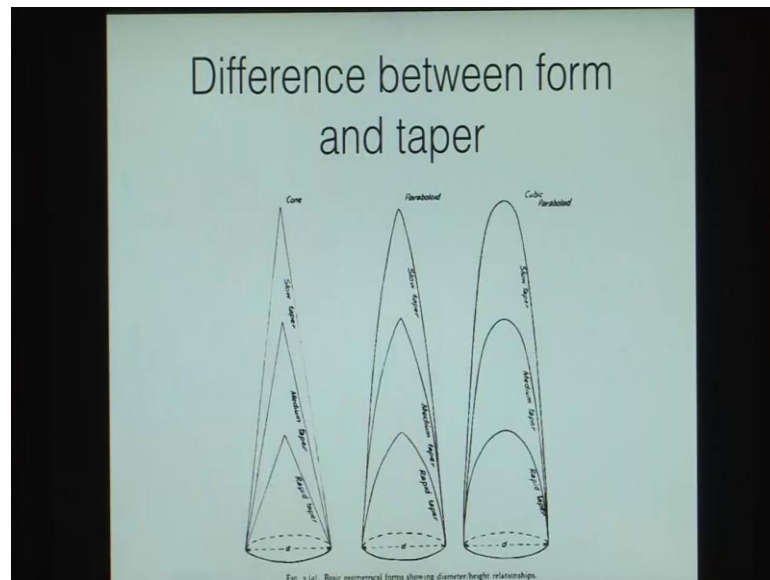
So, let us look at this picture. Here is a picture from a dense forest. So, as you can see there are so many trees in a very small area. So, can you observe here that, the three stems here appear cylindrical in shape. Now, again remember that when we say that trees have a cylindrical shape, it means that they have a low taper. So, when trees are having a low taper they are cylindrical in shape, which means that these are in groups, they are close together and they are lightly thinned. So, now let us move back to the image. So, here you can see that you have a stand that is very close together. So, these trees are very close together it is very lightly thinned. So, essentially you are having a very large amount of competition and these trees are together in groups.

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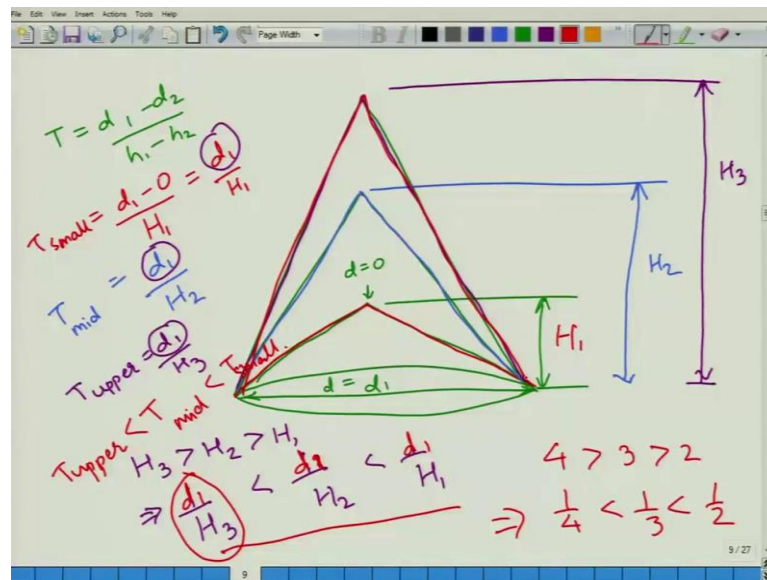
Now, let us look at another image. Let us look at this tree. Now, this is a solitary tree in a windy location and this happens to be a eucalyptus tree. Now, eucalyptus trees are generally very tall trees, but in this solitary location, we can see that it has become a very short tree, and its main stem is reducing rapidly in diameter as we move up. So, when we say that it is reducing rapidly in diameter, it means that it has a high taper. So, when we say that this tree has a high taper, it is either solitary or it is widely spaced or it is heavily thinned. Now looking back at the image this is the solitary tree which is why it is having a very high amount of taper. Now, how do we explain these differences in the tree forms, these differences are explained by the theories of tree form that we shall look into soon.

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Now, to sum up we have seen form and we have seen papers. So, what is the difference between paper and form? So, in these three images, we observe that we have what three different forms. Now, remember form is a shape. So, here we are seeing three different shapes. On the very left side, we are seeing a cone, so it is a conical shape or a clinical form. In the middle, we have a paraboloid shape; and in to the very right we have a cubic parabola shape. Now, whenever we are talking about shapes we are referring to the forms. Now, again looking back at the image we see that these forms are having different amounts of taper the longest figures. So, let us concentrate on the very left side figure which is conical figure.

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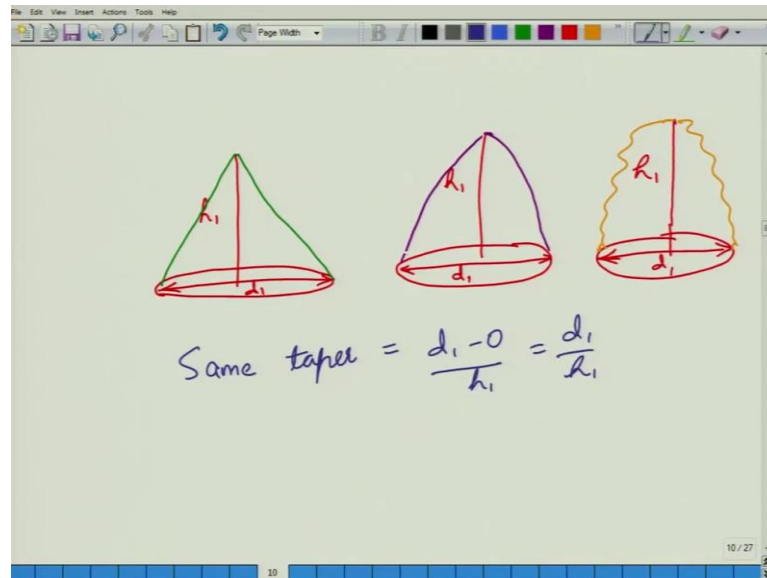
So, in this figure we are seeing three different figures. So, all these three are having the same base, but they are moving like this and like this. So, now let us consider that the diameter here is  $d$  and the diameter at this point would be  $d$  equal to 0, here it is equal to  $d$  equal to  $d_1$ . Now, consider these heights. So, in the in the smallest cone, we have a height of  $h_1$ . Now, the taper would be defined as  $d_1$  minus  $d_2$  upon  $h_1$  minus  $h_2$ . So, in the case of the bottom most cone stick another color. So, if you are considering this cone here we have taper is equal to  $d_1$  minus 0 divided by  $h_1$  minus  $h_2$  is this height. So, let us represent it by a capital  $H$ . So, this is  $H_1$ . So, the taper here becomes  $d_1$  upon  $H_1$ .

In the case of the middle cone, this cone we have taper of middle. So, this one was the taper of the small cone. So, in the case of the middle cone, similarly we have taper is equal to  $d_1$  upon  $H_2$ , where  $H_2$  is the height of the middle cone. And similarly in the case of the uppermost cone, we have  $T_{upper}$  is equal to  $d_1$  upon  $H_3$  where  $H_3$  is thus height. So, what can we say about these three tapers. Now, as you can see the numerator is the same in all the cases, but we have  $H_3$  is greater than  $H_2$  is greater than  $H_1$  which would mean that one upon  $H_3$  is less than one upon  $h_2$  is less than one upon  $H_1$ .

So, to look at it intuitively for instance if we have 4 is greater than 3 is greater than 2. So, we can say that 1 by 4 is less than 1 by 3 is less than. So, similarly in this case we have 1 by  $H_3$  is less than 1 by  $H_2$  is less than 1 by  $H_1$ . Now, if you multiplied all these three

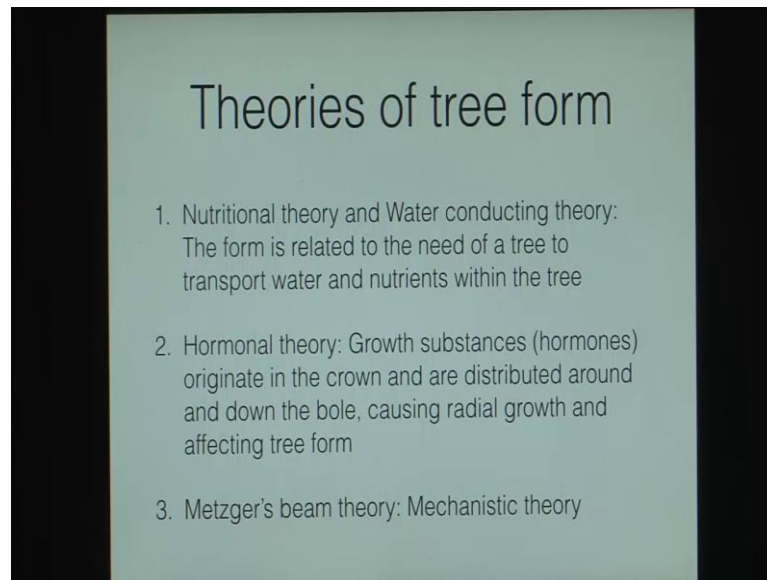
by  $d_1$  we would have  $d_1$  by  $H_3$  is less than  $d_1$  by  $H_2$  is less than  $d_1$  by  $H_1$  which would mean now  $d_1$  by  $H_3$  is your taper of the upper cone is less than taper of the middle cone is less than taper of the smallest cone or the lower most cone. So, the taper of this outer cone is the smallest. So, now, in this figure we are seeing three different tapers for the same form.

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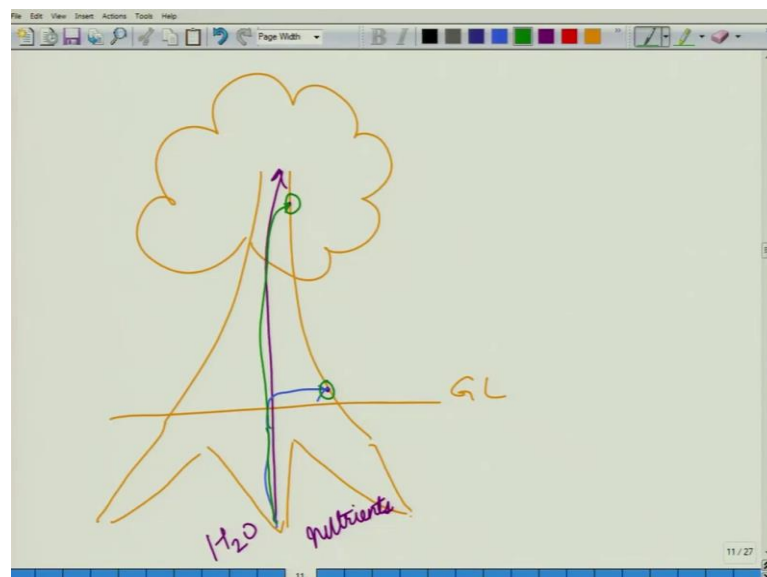
Alternatively, we could also have three different forms with this same paper. So, suppose we have this base. So, in all these three cases we have the same diameter  $d_1$  and the same height  $H_1$ . Now, we could connect these say in a conical fashion or we could connect it in a paraboloid fashion or we could even connect it in any arbitrary fashion. Now, in all these three cases we would be having the same taper which would be equal to  $d_1$  minus 0 upon  $h_1$  is equal to  $d_1$  upon  $h_1$ . So, all these three are having the same taper, but these are different forms. So, coming back to the slide, here also we can observe that we have different forms for the same taper and different taper for the same form.

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Now, what are the theories of tree form? We have three most important theories or rather four most important theories of tree form. The nutritional theory and the water conducting theories relate the tree form to the need of the tree to transport water and nutrients within the tree. So, if we tried to optimize water in nutrient conduction through pipes a tree form would appear to be the most optimized shape.

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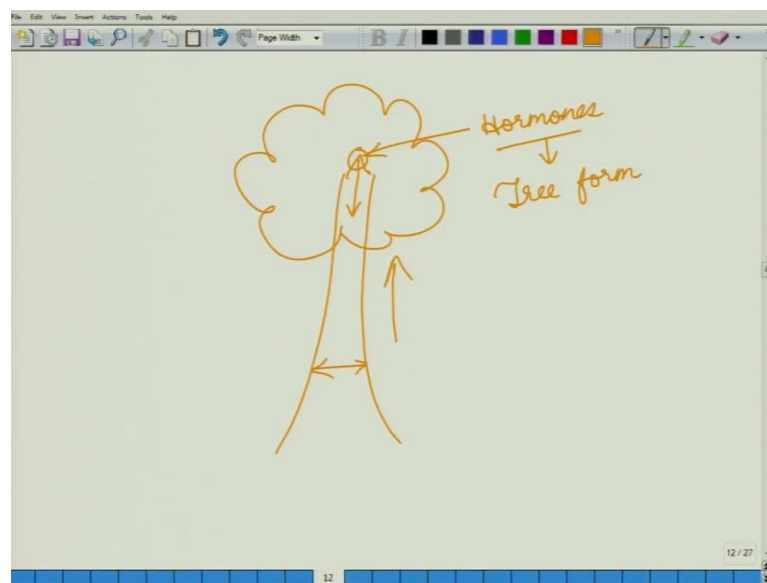


So, what do we mean by that? Consider the shape of a tree. Now that will be having roots below. So, suppose these are the roots and this is the ground level. Now, then the



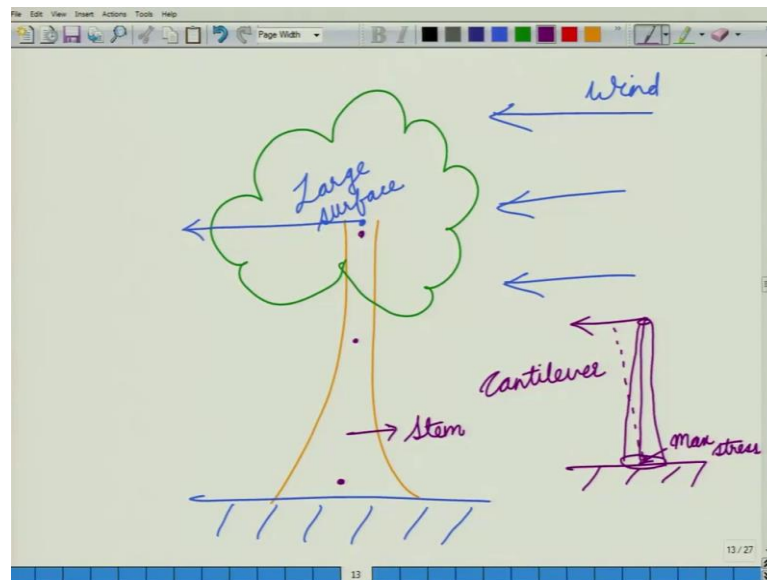
nutritional theories and the water conducting theories state that because this tree needs to take water and nutrients from the bottom to the top most portion. So, if it tried to optimize how much distance it has to carry thus water. So, suppose if you consider one point here and suppose one point here. So, if water needs to reach at this place it needs to go from here up till this point, whereas for this point it needs to go like this up to this point. So, if we try to optimize, the shape of a tree considering that water and nutrients need to reach at every point in the most optimal fashion then we would get the shape of a tree physiologically.

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The other theory is called the hormonal theory. Now, the hormonal theory states that if we have a tree then add the epochs that is at the tip we have some substances some growth inducing substances called hormones. Now, these hormones they are formed at the tip at the apex and then they move down. Now, the effect of the hormones is to stimulate the sideways growth as well as the growth in height. Now, because these growth substances called hormones because they originate in the ground or at the top end of the tree and then they are distributed down and around the bowl or the stem then the impact of these hormones would naturally give us the tree form. So, this is again a another physiological theory. Now, both of these theories though they have been there for quite some time they have not received very large credence because it is very difficult to prove these. On the other hand the third theory the Metzger's beam theory is a mechanistic theory.

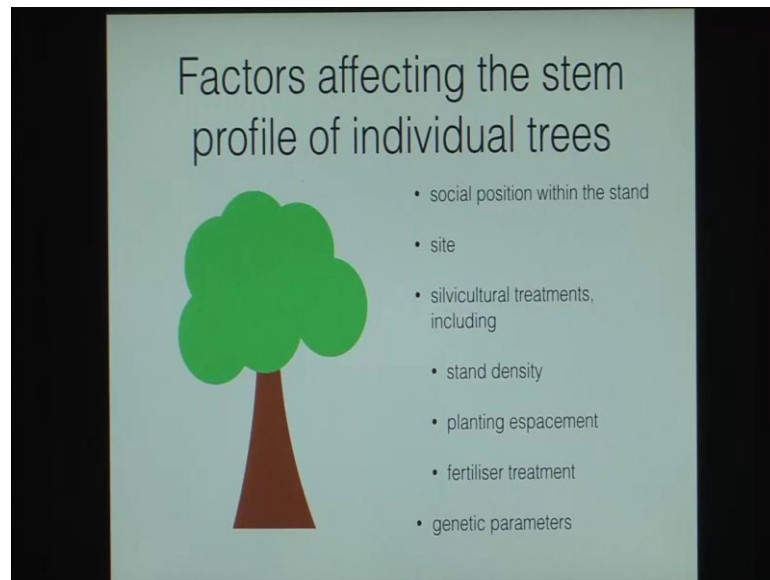
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So, will come to it in a later lecture, but essentially what it states is that if you consider a tree. Now, if this tree is standing on a ground, it will be facing wind pressure. Now, because the canopy has a very large surface area, so most of this wind pressure would result in a force that is acting on the canopy say on the center of the canopy now this tree is anchored at the bottom in the soil. So, essentially if we wanted to represent it in an engineering fashion, it would be something like this. So, you have a beam on which a force is being applied and this beam is anchored at the bottom. Now, this structure is called a cantilever.

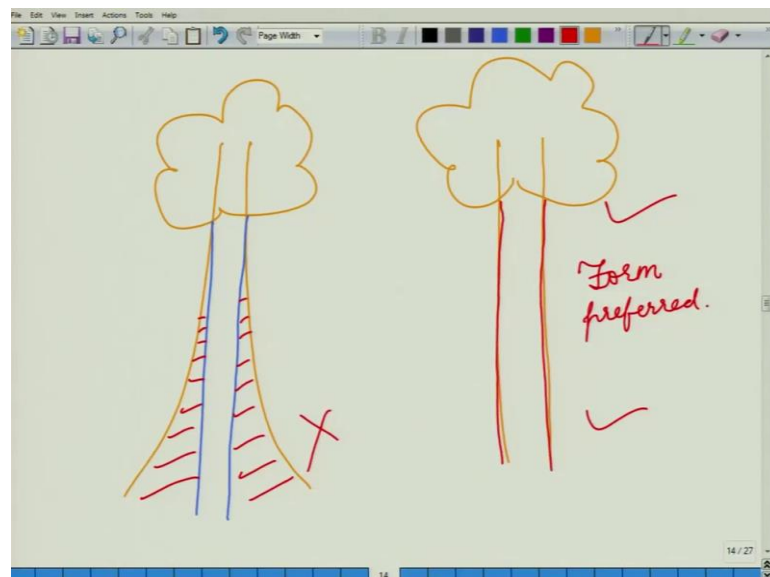
Now because the substance that is making the tree stem is the same whether you consider a point here or you consider a point here or somewhere in between. So, and also when this force is acting, the maximum amount of stress would be felt at this point because this is where this force is trying to topple that the tree. So, if this tree does not want to get toppled, so it would try to accumulate more mass at the bottom as compared to at the top where you have less amount of stress. So, this would give it a tapered structure is what the Metzger's beam theory states. So, we will come to it in much more detail in a later lecture.

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So, now, form is an important attribute of a tree especially when we wish to harvest the tree for economic purposes it considering economics is also consequential, when we wish to take forestry to the forms in the form of agro forestry say for carbon sequestration and in climate change mitigation purposes.

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So, if we wanted to engineer the shape of a tree which shape would be the most useful is this a more useful shape or is this a more useful shape. So, if you wanted to use the word for say sawing purposes, if you made a cut here because we essentially require very

straight sheets of wood. So, we would be losing this wood. Whereas, in this case nearly all the wood be would be utilized very easily, so this shape is preferred. So, if we know that we want this form. So, shape is also called as form, so this form is the preferred form we would like to know how we can engineer are trees to have this form and not this form So, this is why it is important to know the factors that affect the stem profile of individual trees.

So, as we can see in this slide, it is affected by a number of factors say the social position within the stand, the sight conditions the silvicultural treatments including stand density, planting espacement, fertiliser treatment and genetic parameters. So, when we know that these are the factors that impact the tree form can be engineered these yes we can. So, for instance, genetic parameters can be engineered by using tree breeding exercises and by selection of plus trees for seed collection. Field situations can also be altered in a stand by regulating the amount of thinning that we are doing to the stand by regulating the amount of nutrients that the tree is getting in this stand c by fertilization or otherwise and by other silvicultural treatments

So, in this lecture, we have seen what are the forms of trees, what are the tapers of trees and what are the factors that regulate a tree form and it is taper.

Thank you so much for your kind attention, [FL].