**Forest Biometry**
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**Lecture – 07**
**Metzger's theory**

[FL]. In this lecture, we shall try to explore Metzger's mechanistic beam theory of tree form. This theory is one of the most accepted theories of tree form, and this theory tries to explain the shape of a tree as the most efficient design to counter wind forces.

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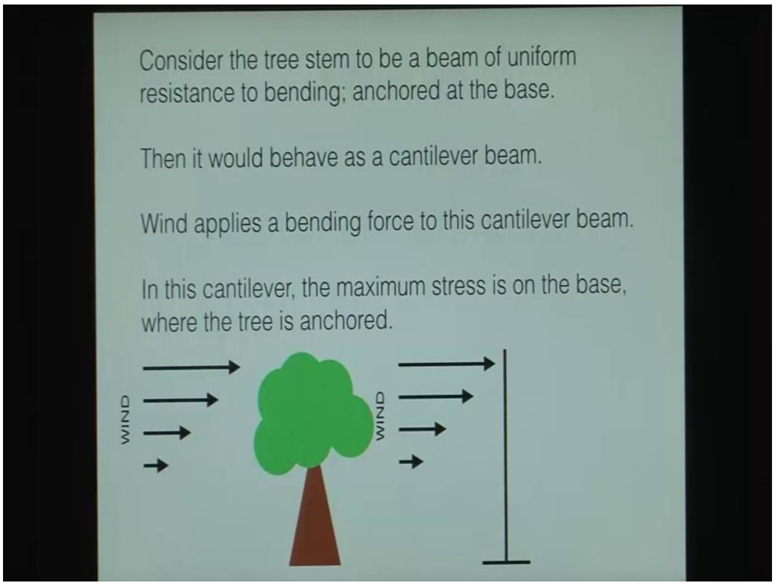


So, consider the tree stem to be a beam of uniform resistance to bending, anchored at the base.

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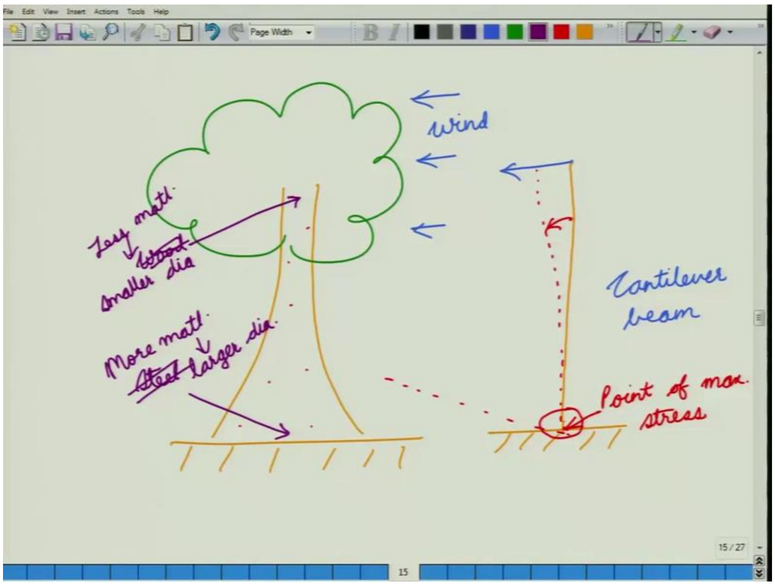


So, as before we have a tree that is anchored to the soil, and we are trying to consider this as a beam of uniform resistance to bending which is anchored at the base. Now, this beam is also known as a cantilever beam. Now, wind applies force to the tree; and in the case of cantilever, we apply this force at the very top end. In this cantilever, the maximum stress is on the base where the tree is anchored. So, when this wind pressure is trying to bend this tree to its side, the maximum stress point is here point of maximum stress. So, if this tree did not have sufficient resistance here, it would snap at this point and maybe it will fall down.

So, a tree needs to reinforce this point by adding more materials now because the material at any point of the tree is the same. So, for instance if we considered in an engineering position, we could have say steel here and we could have say wood here, but because a tree does not have these options, the only option that it has is to accumulate more materials at this point. So, because steel is not an option, so it will try to add more material here. So, giving it a larger diameter at this point; and at this point it will add less materials giving it a smaller dia. So, this should intuitively result in a tapered form of the tree stem with more materials added downwards given larger diameters at the bottom and lesser materials added upwards giving smaller diameters at the top.

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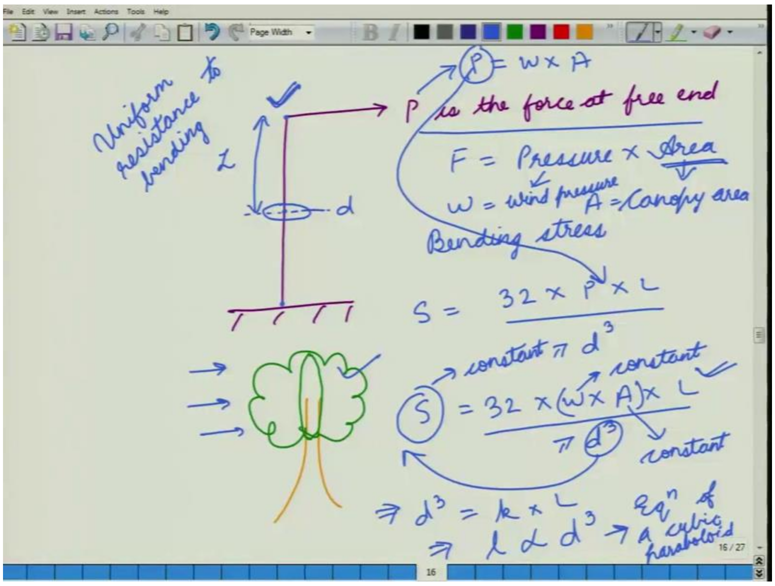


But can we show it mathematically? So, let us now consider a cantilever beam on which a force P is being applied. So, P is the force at free end. Now, let us consider a point somewhere here. So, here the diameter is d and this point is at a length L from the top. So, L is the distance of this cross section, suppose this is the cross section. So, L is the distance of this cross section from the point of application of force, and d is the diameter of the beam at this point. Now, mechanics tells us that the bending stress is given by S is equal to 32 times P times L upon pi d cube, where P is our force at the free end, L is the distance from the point of application of force, and d is the diameter of the cross section at this point.

Now, so this is how the bending stress is calculated. Now, P is given by this p which is the force at the free end. So, force is equal to pressure into area. Now, here the area is the area of the canopy. So, for instance if we had this tree then the area of this cross section that is facing the wind forces is this area and p is the wind pressure. So, we represent this wind pressure by W, and if area by A which is the canopy area; and w is the wind pressure. So, in this case, we will have P is equal to W times A. So, now, replacing this P here in the equation will have s is equal to 32 times W times A times L. So, W by times A is P upon pi d cube.

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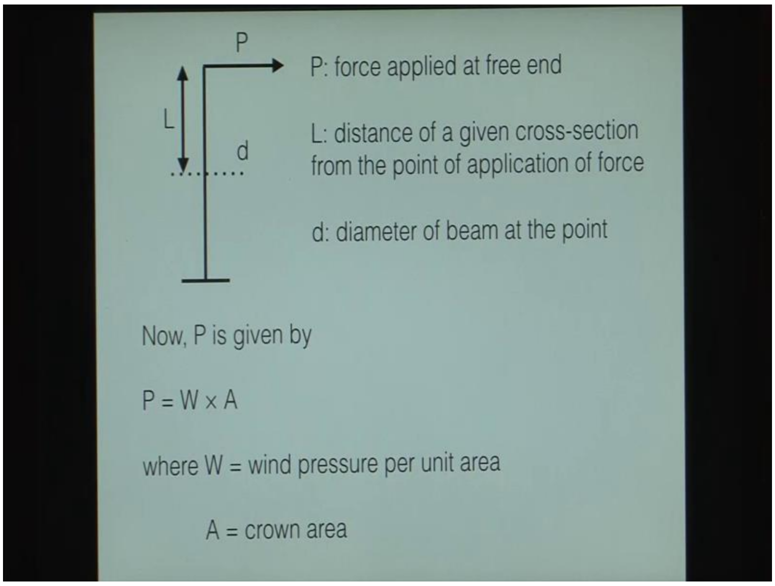


Now, since we have considered this cantilever beam to be a beam of uniform resistance to bending, so we can say that this S is constant throughout the beam. So, essentially the stress that the beam is experiencing at this point distress, that the beam is experiencing at this point or at this point are all the same now. So, S is a constant, W is the wind pressure at that particular area. So, this is also a constant, A is the canopy area. Now, the area of the canopy is not going to change suddenly. So, A is also a constant. So, if we rearrange this equation, we would get if I put d cube here, we will have some d cube is equal to all these constants times L, which would imply that L is proportional to d cube. Now, this is the equation of a cubic paraboloid.

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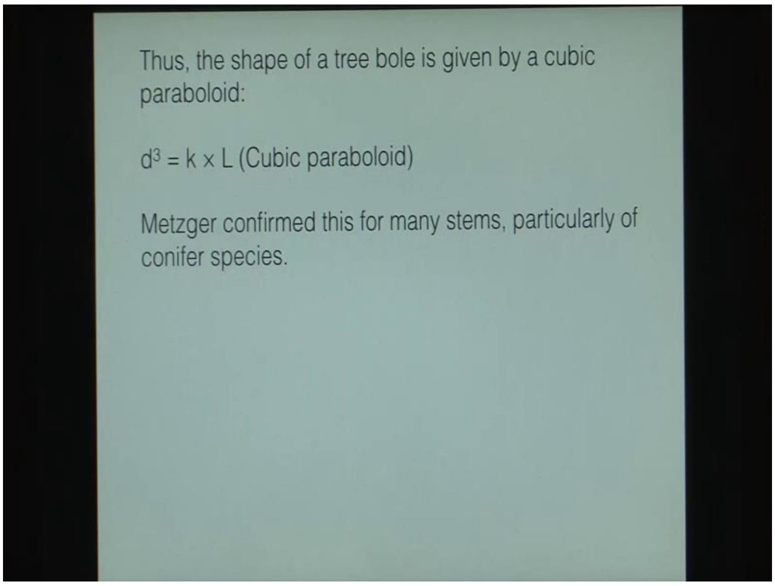


So, what Metzger's stated was that all trees have a form that is regulated mechanistically. So, in the form of a beam so which is why we call it the Metzger's mechanistic beam theory.

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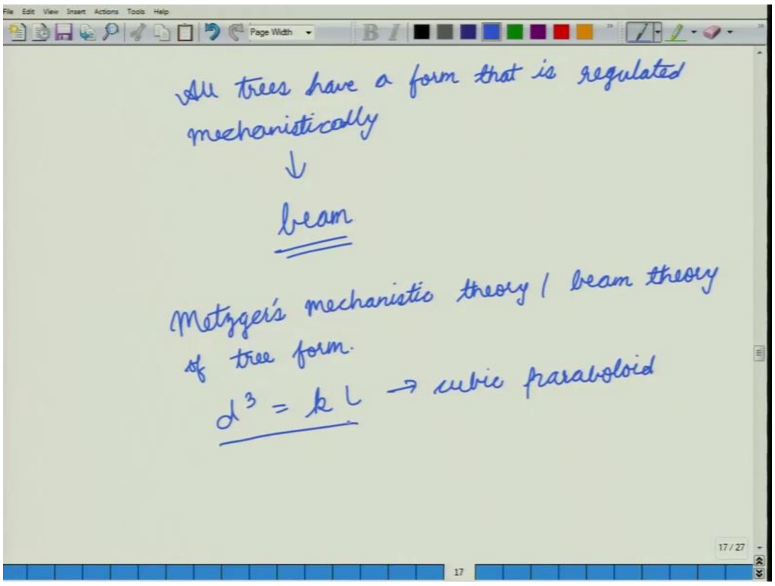


So, it is Metzger's mechanistic theory or the beam theory of tree form. And he also stated that the equation of the tree is d cube is equal to k times L, which is the equation of a cubic paraboloid. Now, you can derive these equations, but you also need to prove it. So, Metzger's confirmed this theory for many stems particularly in the case of conifer species of Europe.

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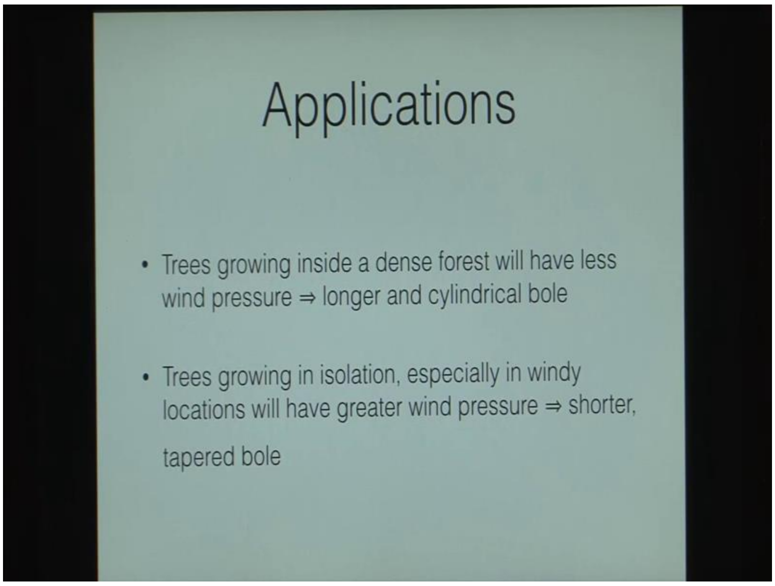


Now, how does this theory help us in our forestry applications? So, trees that are growing inside a dense forest will have less wind pressure, why because let us consider trees in a dense forest.

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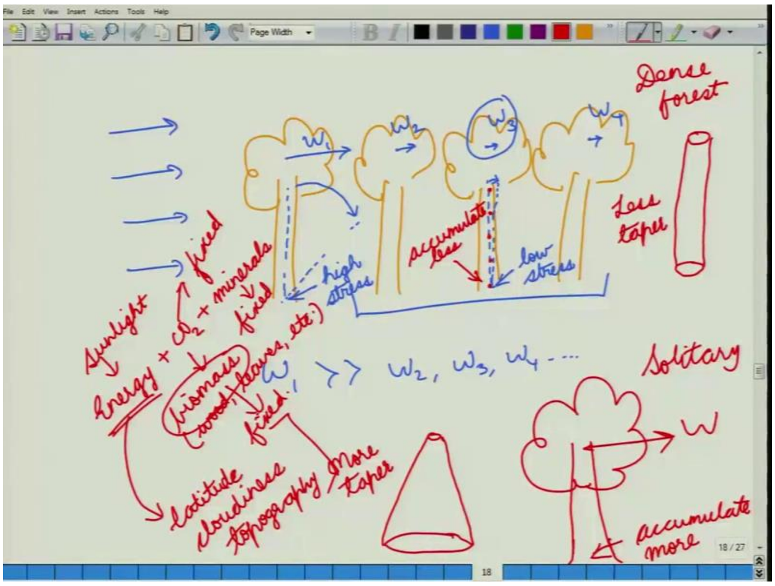


Now, suppose we look at the wind pressure. Now, the outermost tree will face a lot of wind pressure W, but this pressure is then shielded. So, this tree will face a very little wind pressure and so will all the other trees w 1, 2, 3, 4. So, essentially w 1 will be very much greater than w 2, w 3, or w 4 or so on. Now, because the trees that are growing inside the forest because these trees face very little wind pressure, so there is very little stress at the bottom. So, essentially if this tree could topple to a large extent, this tree would have a very less pressure here. So, we will have low levels of stress at this point as compared to the very outer trees.

Now, because we had considered trees to be made of materials with giving it a uniform resistance to bending, so this tree will have a low stress here, low stress here low stress here low stress here and low stress here. So, there is hardly any incentive for this tree to accumulate more resources at the bottom portion; whereas, a tree that was growing in a solitary fashion it was experiencing a very large wind force, so it had to accumulate more material. And here you have a very less amount of accumulation. So, what would this accumulation do? In the case of the solitary, because you have more accumulation at the bottom, you have a thicker diameter at the bottom and because you have a lesser accumulation on the top, you have a thinner diameter at the top. So, it would give it a more tapered structure; but in the case of a dense forest, you would have very little accumulation here and you will have a very little accumulation here. So, essentially you will have a more cylindrical bowl. So, this will have less taper and this will have more taper

Now, there is one other point, a tree is converting energy of sunlight plus carbon dioxide plus minerals into biomass which can be in the form of wood leaves etcetera. Now, the amount of energy that a tree is receiving at any point on the earth is governed by the latitude of that place by the cloudiness of that place and by the topography of that place. Now, because a tree cannot move, so it cannot change its latitude, cloudiness or topography. The amount of carbon dioxide that is there in the atmosphere is also fixed; and the amount of minerals that are there at a site is also fixed. So, essentially the amount of biomass that a tree can create at any point is also more or less fixed.

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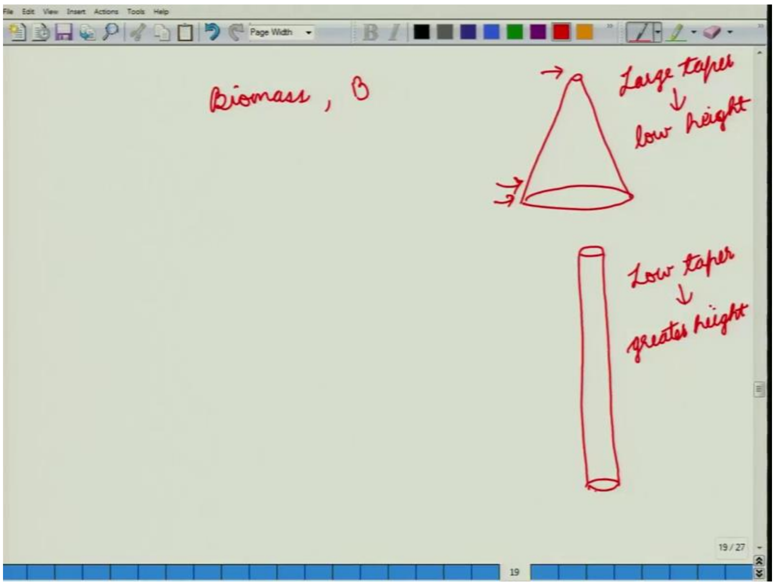


So, now if a tree has to distribute that biomass, so suppose we consider the biomass B. Now, suppose a tree has to deposit lots of biomass here at the bottom and because the amount of biomass that it has is fixed. So, if you have a large taper, you will also be having a low height, because all the biomass has been deposited at the bottom. So, essentially you have very less biomass that can go to the top, whereas when you have a low taper the trees can become to greater heights.

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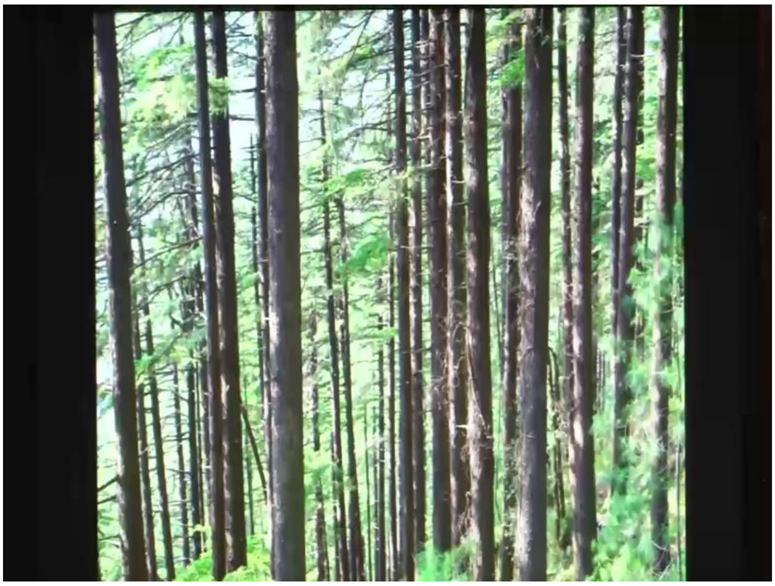

So, now let us come back to the images. So, we considered this forest again. So, here you can see that the trees are close by. So, in the centre where the trees are surrounded by other trees on all sides with the trees be facing high wind speeds or will they be shielded by the surrounding trees, well more or less they will be facing very less wind speeds because they have been shielded by the surrounding trees.

Now, since the wind speeds here are low does the tree need to reinforce the base? Well, the answer again is no, because it does not have any great amount of pressure at the bottom it is not facing a risk of being snapped at the bottom. So, there is hardly any need to reinforce the base, but it can produce large amounts of biomass. So, where would that biomass go, it will grow to the topmost portions and it will grow to greater heights. So, can we now explain why these trees stems are cylindrical in shape? Well, yes, Metzger's theory can very easily explain why these trees are cylindrical in shape and why they are having a great heights.

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Now, consider similarly this loan tree on a windy slope. So, what can we say about the wind pressure faced by this tree, will it be facing high wind speeds or low wind speeds, is there any shielding to protect it from high wind pressures? No, right because there are no trees surrounding this particular lonely tree that could shield it from wind speeds. Now, since the wind speeds are large when the bottom of this tree face greater stresses will it be more prone to snapping, yes, it will be more prone to snapping. Now, if it is facing greater stresses at the bottom does it need to reinforce the base? Yes, it needs to, because remember if it does not it is going to snap at the bottom.

So, can we now explain why this tree stem has a large taper? Well, Metzger's beam theory can very easily explained by this tree as a larger taper. But can you also explain why this tree has a shorter height? Well, yes, because the amount of biomass that it can produce is limited by a number of factors and because it has expended most of its biomass at the bottom locations. So, it cannot grow to very great heights. Now, will such trees have good economic values or will trees with a cylindrical bowl have better economic values? The answer is that we always preferred trees with a cylindrical bowl they are easy to work with and there is very little wastage.

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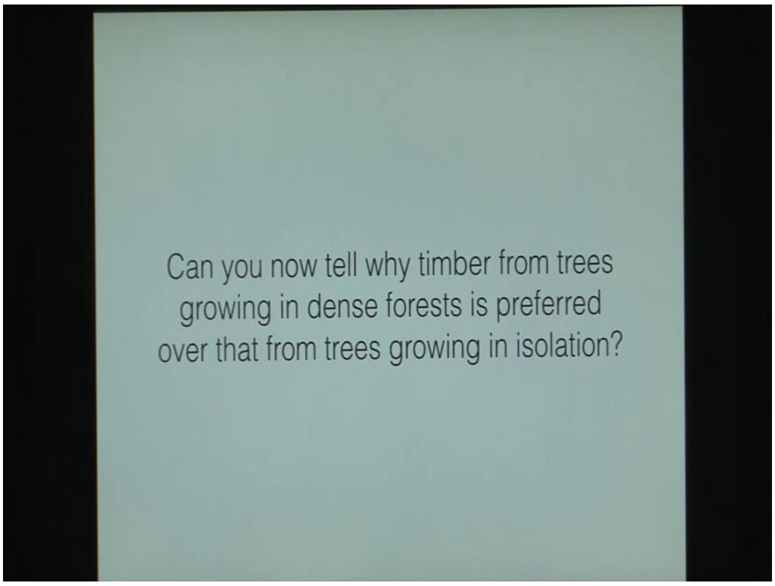


So, can you now tell why timber from trees growing in densest forests is preferred over that from trees growing in isolation using Metzger's theory? Yes, you can tell because in a dense forest your trees will be having lesser amount of tapers and they will be having greater heights as compared to tree is growing in isolation. So, Metzger's theory is a very important theory which has huge ramifications in the science of forestry.

Thank you for your attention, [FL].