Forest Biometry
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Lecture – 08
Form Factors and Form Quotients

[FL]. Now, we know about the form of trees that is the shape of trees. We also know about the taper of trees which tells us the rate at which its diameter is reducing as the height increases. Now, if we wanted a single factor that could tell us about both the form and the taper, what would that be? Remember that the aim of forestry is to grow more and more wood to generate more and more biomass and to harvest more and more biomass, not only because this biomass is going to help the society fulfill its various needs, but also because this biomass is a form of sequestered carbon which helps in the mitigation of climate change. So, in this lecture, we shall look at form factors and form quotients as a rapid way of discerning or trees taper.

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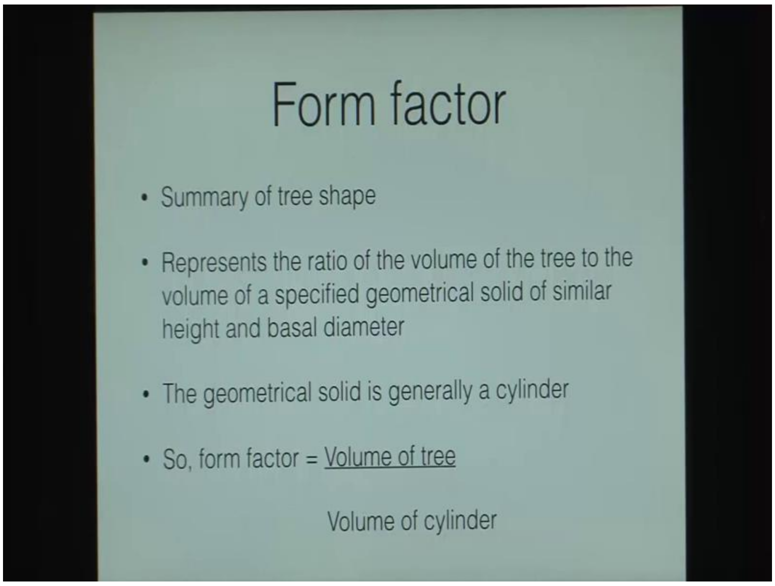

So, let us come to the form factor. A form factor is the summary of trees shape. It summarizes the shape of a tree. Basically it represents the ratio of the volume of a tree to the volume of a specified geometrical solid of similar height and base diameter which is generally a cylinder.

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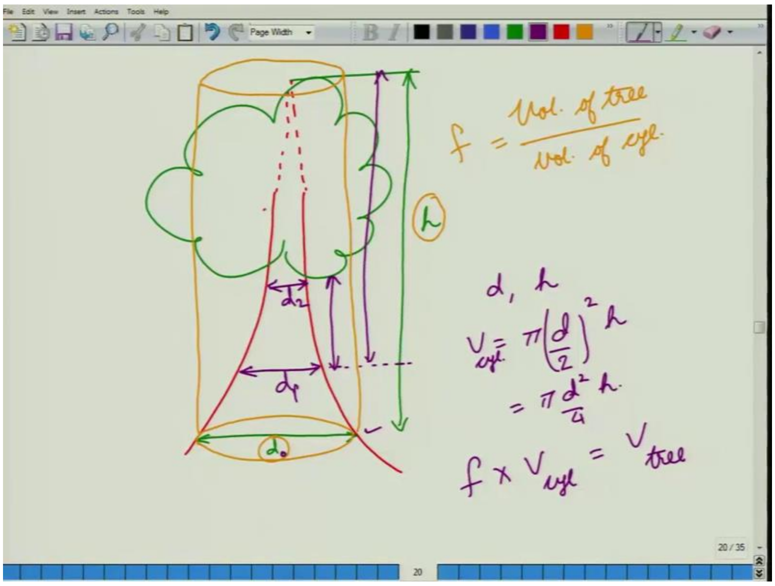

So, what do we mean by this if we had a tree? So, suppose we take this as our reference plane. Here we have a diameter of d and consider the height of the tree to be h. You are considering these branches to be growing to that height. So, if we made another solid figure that had the same diameter and we generally take it to be a cylinder, so consider this to be a cylinder. So, this is a cylinder with the same diameter as that of the base and the same height as that of the tree; now, if we calculate the volume of tree and divide it by the volume of cylinder that would give us the form factor f.

So, basically the form factor of a tree or stem is defined as the stem volume expressed as a proportion of the volume of a cylinder of the same height, and with a diameter equal to the stem diameter at the selected reference point.

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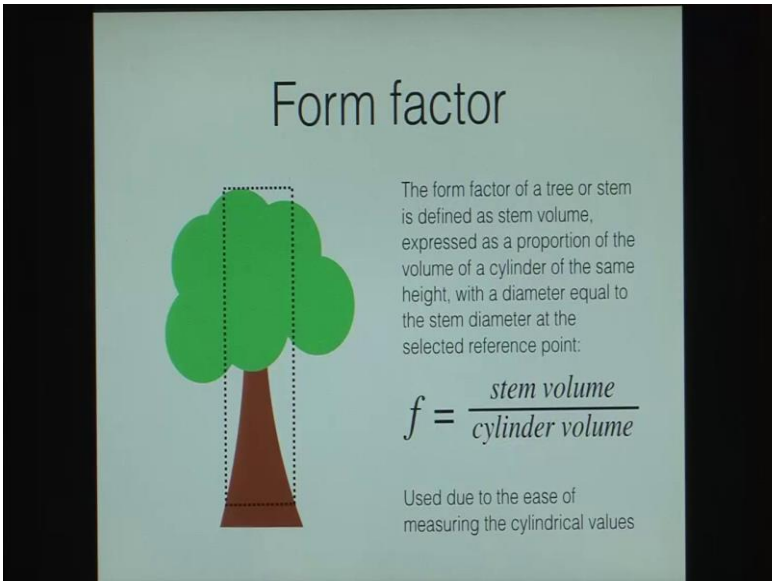

Now, why do we choose a cylinder to measure this tree? The answer is because it is very easy to calculate the volume of a cylinder if we knew its diameter d and if we knew its height h. We have volume is equal to pi r square h or pi d square by 4 into h. So, we can very easily calculate the volume of the cylinder and if we knew the form factor, we could just multiply f multiplied by volume of cylinder and it would give you the volume of the tree, but now the question is where do we measure the diameter of the cylinder. Do we measure it at this point? Do we measure it at this point? Do we measure it at this point? Because remember that the diameter is going to go on decreasing as we move up the tree. So, which is the difference point at which we should be measuring the diameter? Similarly, what is the height of the tree? Suppose if we measure the diameter from at this point, do we take this height? Which is the height that we should be taking? So, the difference is that we choose give us different form factors.

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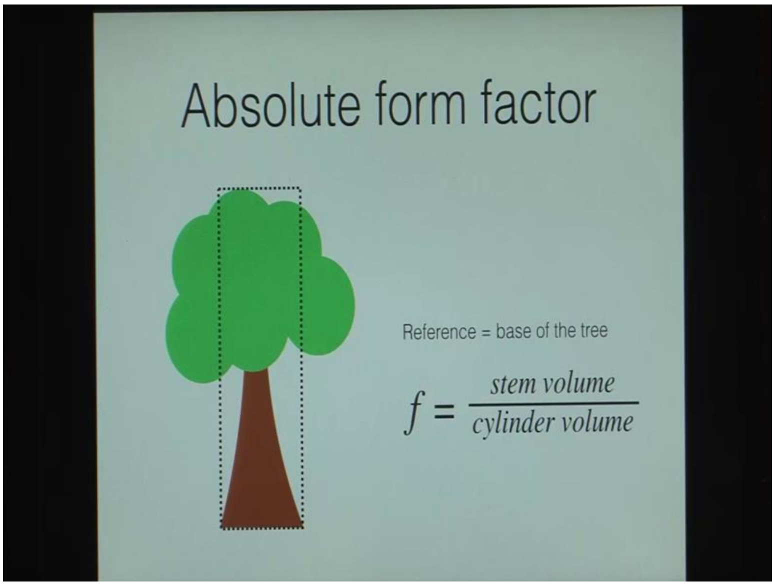

Let us look at the first one, the absolute form factor. If we take the base of the tree as the reference, we get the absolute form factor. So, let us draw a tree.

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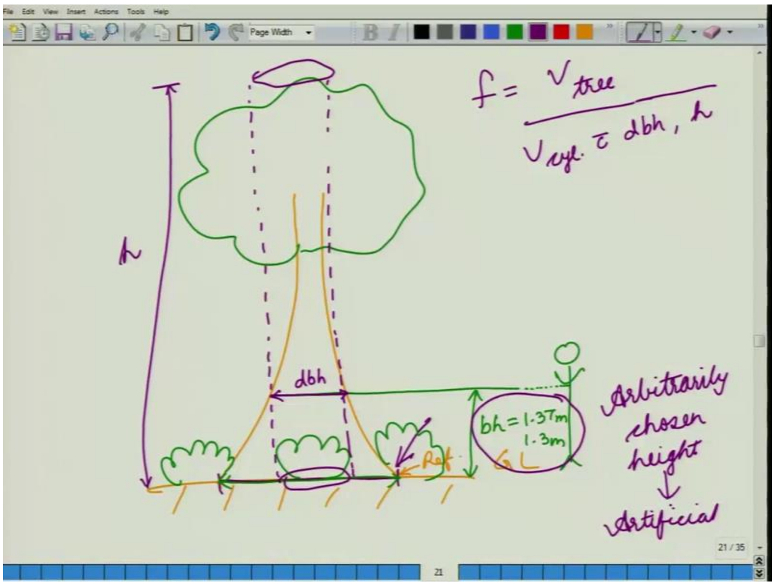

Now, suppose this is the ground level. If you took this as the reference, we would get the absolute form factor. So, essentially here the diameter of the tree of the cylinder is the diameter that this tree has at its base and the height is the total height of the tree. If you did that and if we calculated the form factor by dividing the stem volume by the cylinder volume, we would get the absolute form factor.

Now, this form factor is intuitive to use. However, it is hardly used in practice. Why? It is because at this reference, we might be having some small shrubs around here which would make it extremely difficult to get this diameter correctly. At the same time in the case of forestry, we have a convention of measuring the height at a height known as h and we measure diameter at the height known as the breast height.

So, what does breast height mean? Suppose we have an individual who stands like this. So, the height of his or her chest is called the breast height. In our country, breast height is usually taken to be as 1.37 meters. In the case of certain European countries, it is taken as 1.3 meters. So, this is our standard and we regularly measure our trees diameter at this height. So, now if we took this diameter which is called the diameter at breast height, if we took this as the difference and we took the complete height of the tree and made a cylinder, this would be the cylinder. So, if we took this cylinder and if we calculate the form factor, this would give us the false or artificial or breast height form factor. So, it is equal to the volume of the tree divided by volume of cylinder with diameter at breast height as its diameter and the height of the tree as the height of the cylinder.

We also have another form factor called the true or the normal form factor. Here we take a reference say 10 percent of height. So, in the case of the breast height or the artificial form factor, we were taking 1.37 as an arbitrarily chosen. So, this is arbitrarily chosen height which makes it artificial, but suppose we did not choose this height and suppose we took something as say 10 percent of height.

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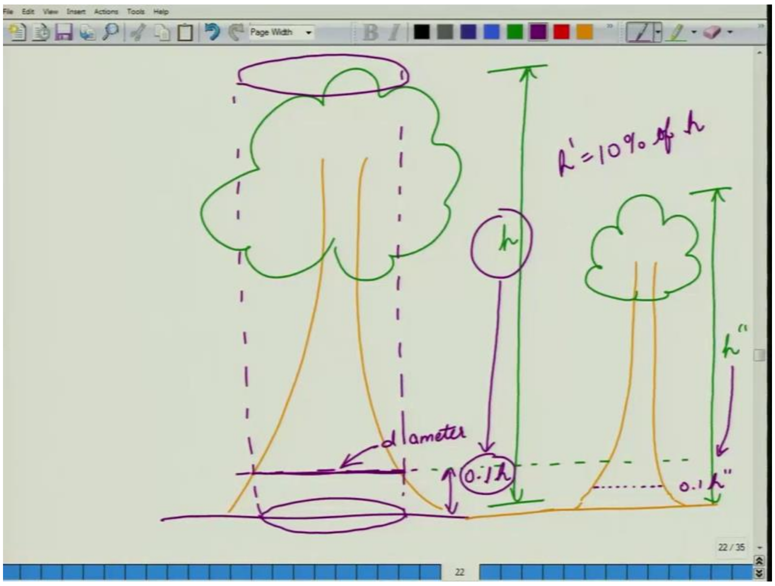

So, let us take another tree. So, suppose this height is h and suppose we said that we are only going to measure its diameter at a point that has a height h prime which is 10 percent of h. So, we are not fixing this height arbitrarily to be any fixed figures for all the trees, but in the case of this tree, we will take a height that is 0.1 h and here we are going to measure the diameter and then, we are going to construct a cylinder.

So, how does it make it more normal because suppose we had a smaller tree at the same base because this height is lesser. The reference point will not be the same as that of the larger tree, but we would take another reference that is 0.1 of h double prime that is this height. So, essentially in the case of a true or normal form factor, the height that we take is not fixed, but the percentage that this height has in reference to the total height of the tree is taken as constant.

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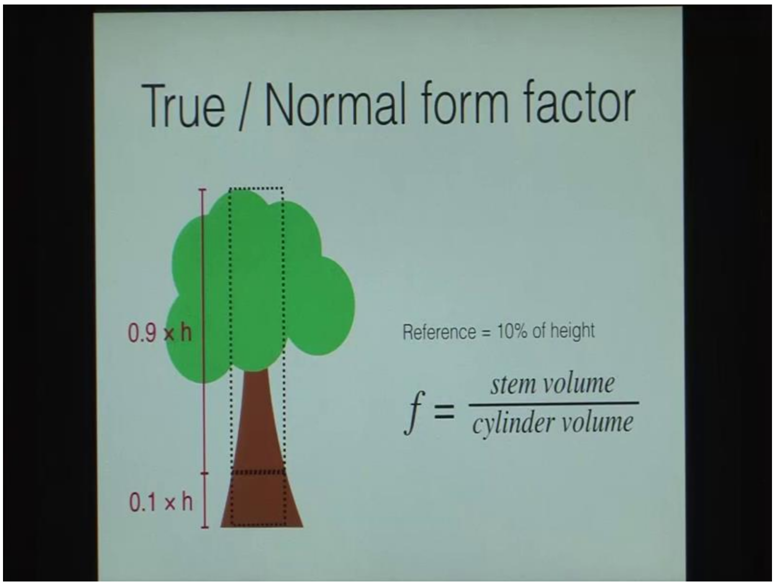

So, this makes this procedure much less arbitrary as in the case of the breast height form factor. However, in this case, we first need to measure the height of the tree and from that we need to determine what 0.1 h is and then, we need to take this reference and then, we need to do the measurements. So, this makes it a bit more intricate as compared to the breast height form factor. So, it makes it also a bit more demanding than the breast height form factor.

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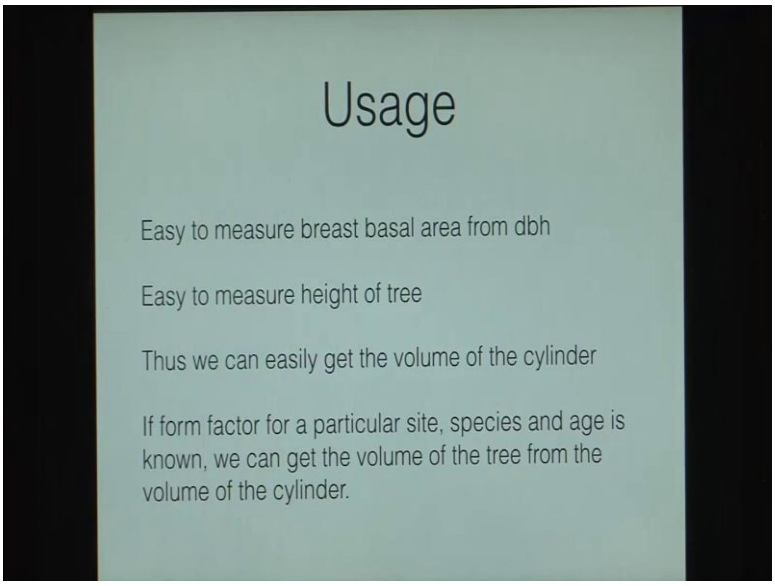

So, the breast height form factor is the most commonly deployed form factor in the case of forestry. So, what is the usage of this form factor? How does it help? Well, it is easy to measure the breast basal area from diameter at breast height and it is easy to measure the height of a tree.

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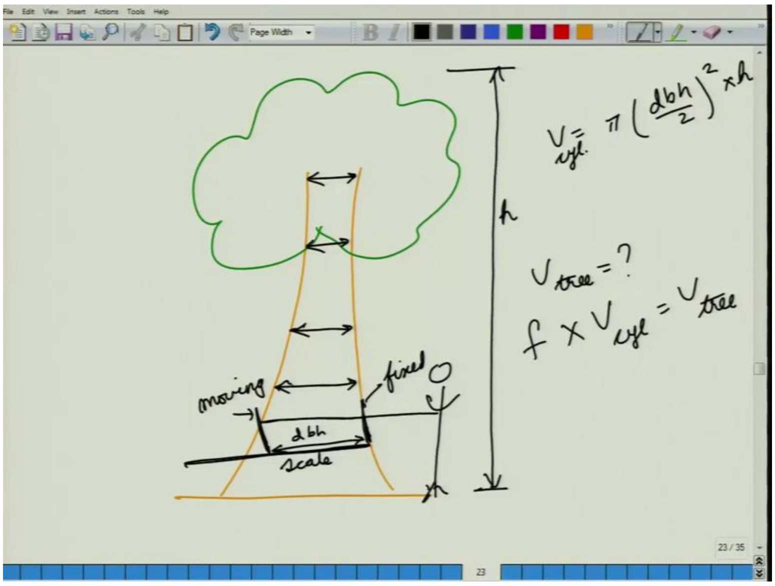

So, if we consider the tree, if you stood at this point, it is easy to take its diameter at this point. We could use a tape or we could use calipers. The calipers are very similar to the vernier calipers that you have used in your school days. So, basically you have a scale and you have a fixed arm and moving arm and you have a scale. So, this reading would give you the diameter at breast height. So, it is very easy to the centered dbh of this tree. Similarly, it is easy to measure the height of the tree. So, we could use our trigonometric principles. So, we could use a number of other equipments to measure the height of the tree. So, we can very easily get the volume of the cylinder.

So, the volume of the cylinder would be pi into db h by 2 whole square multiplied by the height of the tree. So, we can very easily get dbh, we can very easily get the height. So, we can very easily get the volume of the cylinder. Now, if the form factor for a particular site species and age is known, we can very rapidly get the volume of any tree from the volume of the cylinder. Now, it is difficult to measure the diameters at this point, this point this point and so on to get the actual volume of this of the tree. So, volume of the tree is difficult to measure, but if we had the form factor, we could very easily multiply form factor by the volume of the cylinder to get the volume of the tree without the need to measure its form or it is taper. This form factor acts as a rationalizing factor in the calculation of tree volume.

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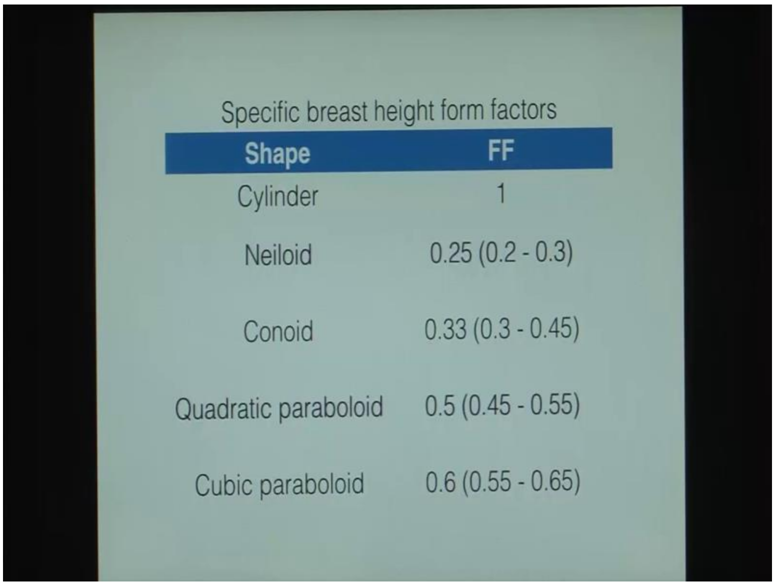

Specific breast height form factors

Shape	FF
Cylinder	1
Neiloid	0.25 (0.2 - 0.3)
Conoid	0.33 (0.3 - 0.45)
Quadratic paraboloid	0.5 (0.45 - 0.55)
Cubic paraboloid	0.6 (0.55 - 0.65)

So, let us now look at some specific breast height form factors for some different shapes. Consider the shape of a cylinder.

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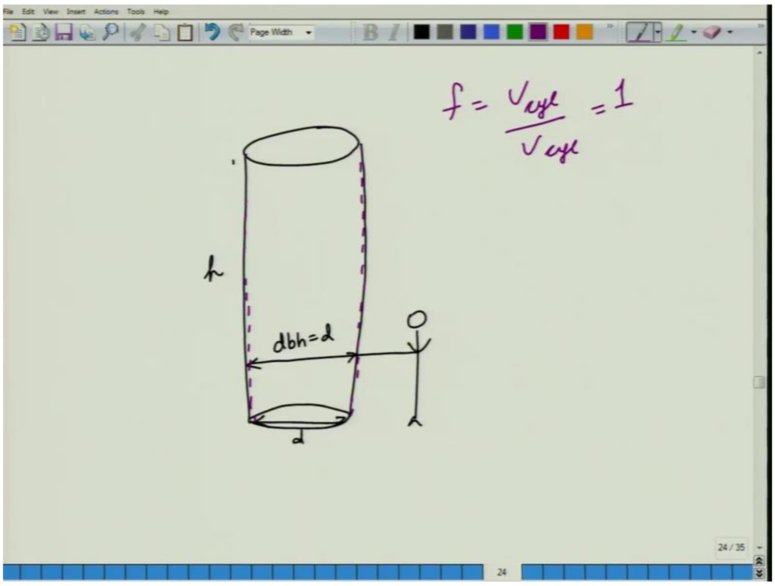

So, here we have a cylinder. What would be its breast, height form factors? Suppose you stood here and took it is diameter at the breast height. So, its dbh will be equal to d, where d is this diameter. Now, the height of the cylinder that you would be plotting is the same as the height of your original cylinder. So, your form factor would be volume of the cylinder divided by volume of cylinder which will be equal to 1. So, the breast height form factor in the case of a cylinder is 1.

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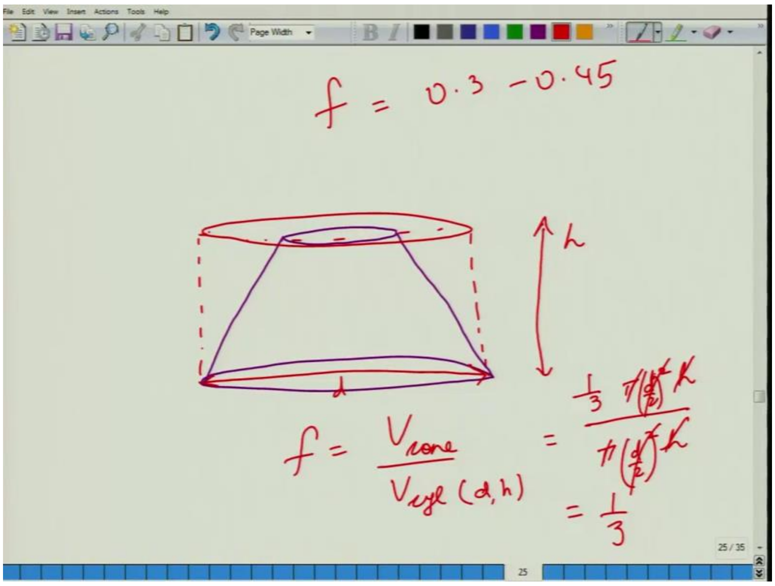

Now, let us calculate the breast height form factor in the case of a cone. So, let us first calculate the absolute form factor. So, in the case of an absolute form factor, you would have the diameter of the cylinder which would be the same as d, the height of the cylinder is the same as the height of this cone which is h.

Now, what is your form factor? Form factor is the volume of the cone divided by the volume of the cylinder with the same dia and the same height. The volume of a cone is 1 by 3 pi r squared h, the volume of the cylinder is pi r square h. Now, r is your d by 2. So, we replace it by d by 2. Here will now h and h cancel out, d by 2 square cancel out, pi and pi cancels out. So, the form factor becomes 1 by 3. Now, if we calculate the form factor for a conoid or say for a frustum of a cone, suppose your cone did not go till the end, but it went till here. Suppose you wanted to calculate the form factor for this figure. So, here you will have the volume of the cylinder would be the volume of the cylinder with the same height as that of the conoid. What would be the form factor? It turns out that in this case, the form factor would be in the range of 0.3 to 0.5.

So, now looking at this slide, we can see the specific form factors. The specific breast height form factors for the number of different factors in the case of a cylinder, it is 1. In the case of a neiloid, it goes from 0.2 to 0.3. We can take a middle value of 0.25. In the case of a conoid, it goes from 0.3 to 0.45. We can take 0.33 as a representative figure. In the case of a quadratic paraboloid, it goes from 0.45 to 0.55 with 0.5 as a reference and in the case of a cubic paraboloid, it goes from 0.55 to 0.65. So, we can take 0.6 as a difference value. So, after the form factor, we now come to form quotients.

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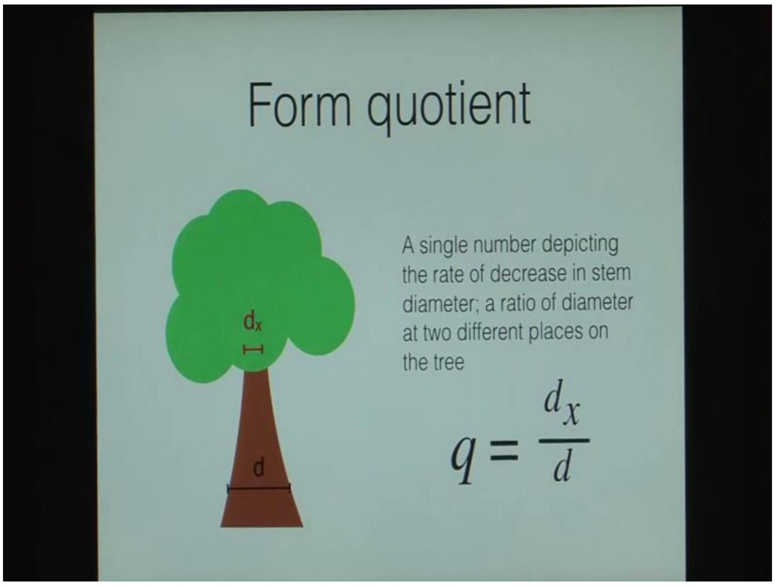

A form quotient is defined as a ratio of diameters at two different places on the tree. So, it is a single number that depicts the rate of decrease in the stem diameter or a ratio of diameter at two different places on the tree.

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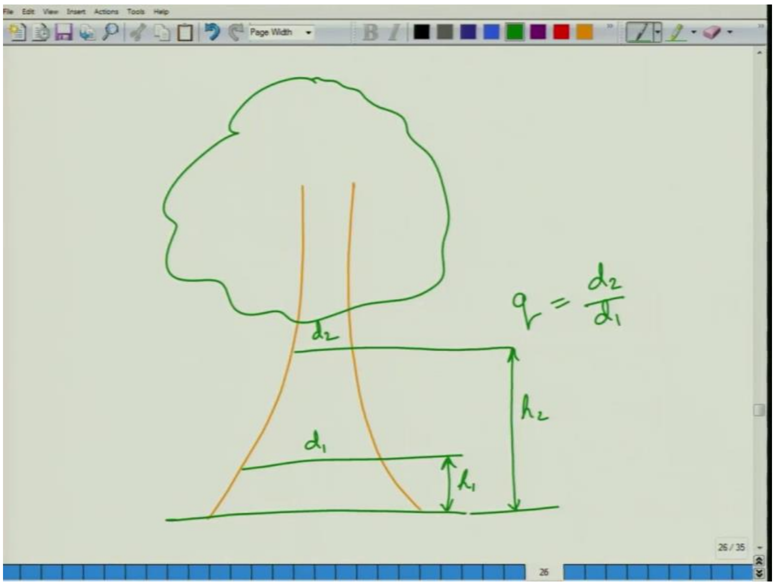

So, now let us draw a tree. So, now, suppose we took two references and we said that we are going to measure the diameter at the site h 1 and we are going to measure the diameter at this height h 2. So, here we have d 1 and here we have d 2. Now, if we chose these as the references and if we defined that d 2 upon d 1 is a ratio, so we can define this ratio as the form quotient. Now, as in the case of a form factor here to the choice of the reference points gives us different form quotients.

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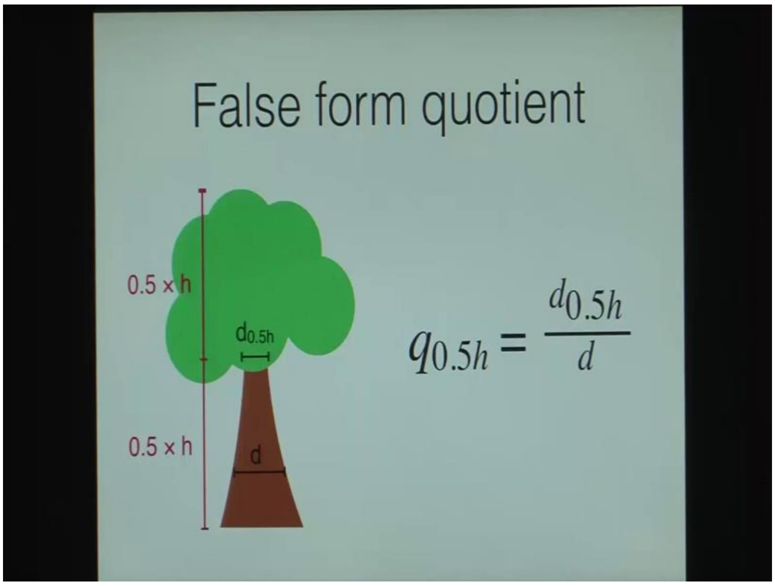

So, the first one is called the false form quotient. A false form quotient takes the diameter at 50 percent height; add as the numerator and the diameter at the breast height to be the denominator.

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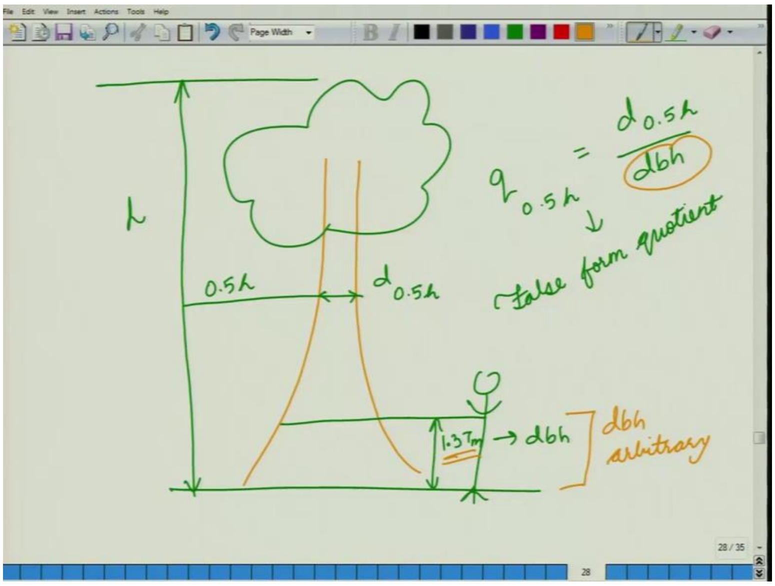

So, in the case of a false form quotient, if we took a tree, we need to take two diameters. Now, 1 diameter is when you stand here and measure it at the breast height which is at 1.37 meters. So, that is your dbh. Then, the next thing that you do is to measure the height of the tree which is if it comes to h then you take a point that is at 50 percent of the height. So, this is at 0.5 h and you measure the diameter here. So, we write it as d 0.5 h. So, it is diameter at 0.5 height. So, your false form quotient which is written as q 0.5 h is equal to d 0.5 h divided by the dbh. So, this is the false form quotient.

We also define true form quotients. Now, a true form quotient is if it takes diameter at 50 percent height in the numerator and the diameter at 10 percent height in the denominator, so in the previous case we had arbitrarily taken this dbh which is arbitrary in the denominator.

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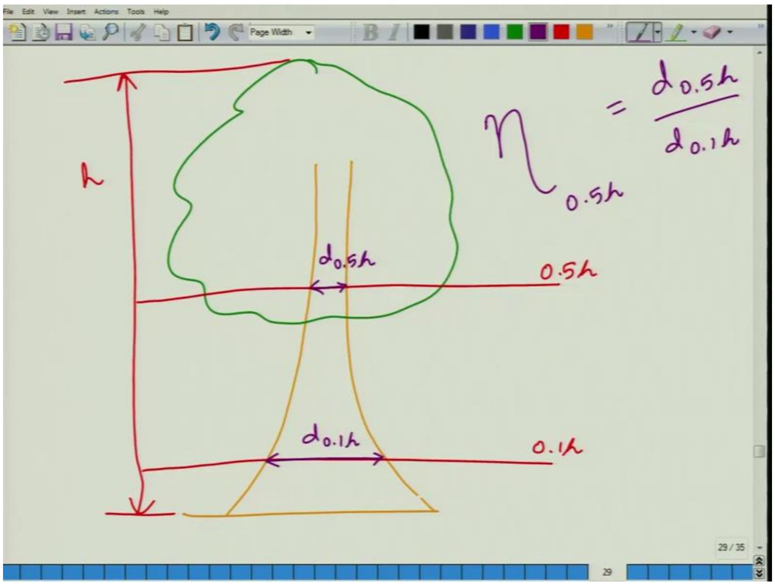

Now, if we did not choose this figure 1.37 meters and if we took it as a fraction of the tree height, so in the case of a true form quotient, we take a tree and the steps of calculating a true form quotient would be the first step is to calculate the height of the tree and then, to take two locations i.e. one is at 0.5 h and the second location which is at 0.1h. Now, we measure the diameters at these two locations. So, this is d 0.1 h and this is d 0.5 h.

Now, the true form quotient which is represented by eta 2 eta 0.5 h is equal to the diameter at half of the height divided by the diameter at 10 percent of the height.

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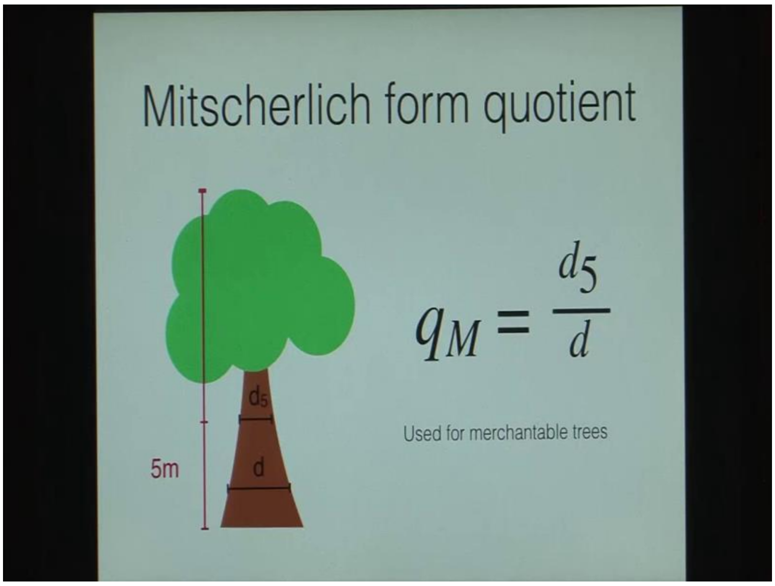

So, this is the true form quotient. The Mitsccherlich form quotient is a term that is used for merchantable trees. It takes the diameter at 5 meters height in the numerator and the diameter at the breast height in the denominator.

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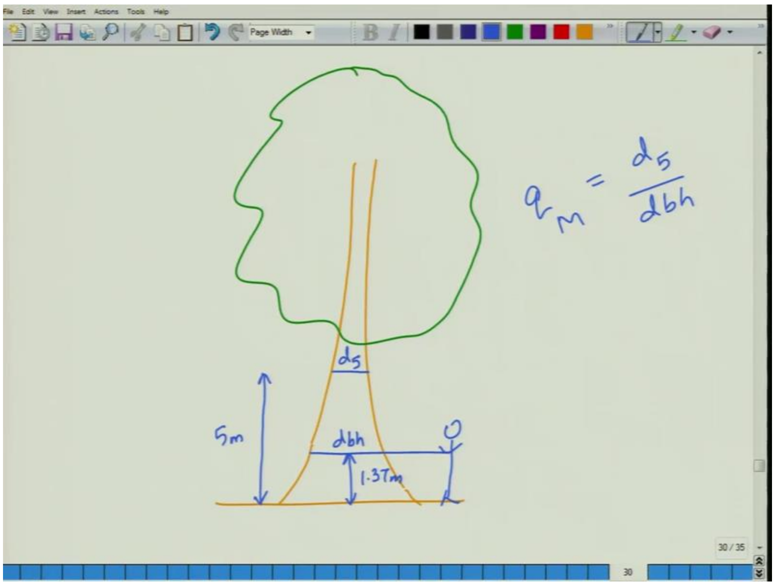

So, how do we find Mitsccherlich form quotient? You take a tree and then in this case, you do not need to find out the height of the tree. So, this has pure economic values. So, first of all you stand in front of the tree and you take its diameter at the breast height. So, this is 1.37 meters and this is the dbh. Then, you take another diameter that is at 5 meters. This is d 5.

Now, Mitsccherlich form quotient is given by d 5 divided by dbh. So, why is this important because in this case we do not need to measure the height of the tree. So, it simplifies the calculation a lot. We also have another form quotient called the Hohenadl's form quotient.

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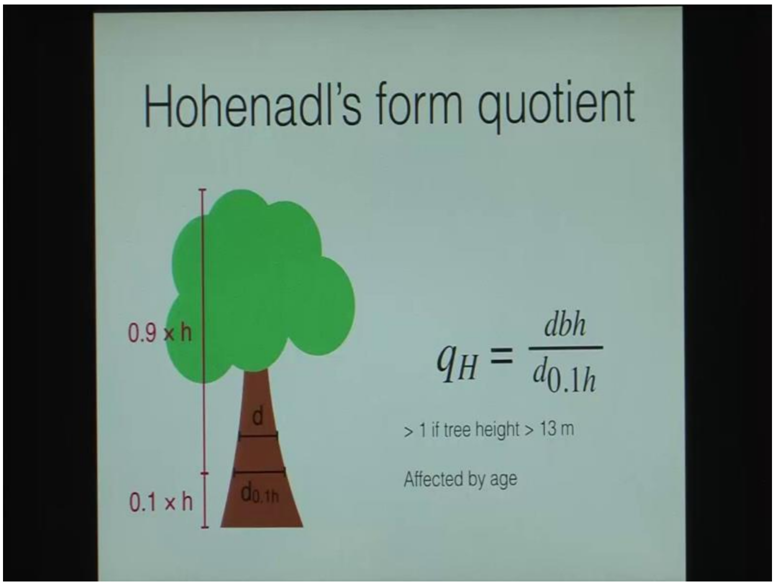

In the case of Hohenadl's form quotient; it takes the diameter at breast height in the numerator and the diameter at 10 percent of the height in the denominator. So, how do you calculate the Hohenadl's form quotient?

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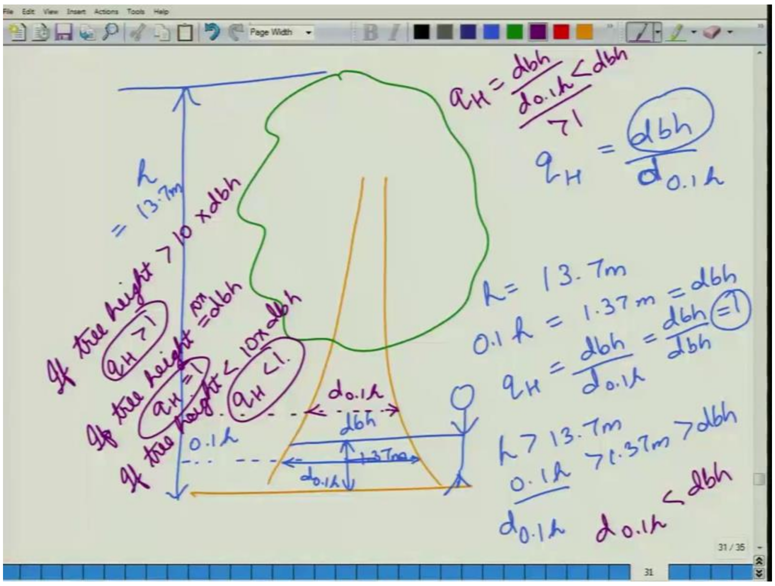

With this tree, first of all you go and stand in front of the tree and measure its dbh at 1.37 meters and then, you measure the height of the tree. Then, you take a location that is at 0.1 of the height and you measure this diameter which is d 0.1 h. So, the Hohenadl's form quotient is given by the dbh divided by d 0.1 h. So, the diameter is at 10 percent of the height.

Now, remember that in the previous form quotients, we had taken dbh in the denominator. So, this is the only form quotient that is taking deviation. The numerator and this form quotient is highly affected by age. So, suppose the height of the tree is 13.7 meters. What would be the Hohenadl's form quotient in this case? So, h is equal to 13.7 meters. So, h 0.1 h is equal to 1.37 meters which is equal to dbh. So, the Hohenadl's form quotient would be dbh divided by d 0.1 h is equal to dbh divided by dbh is equal to 1. So, if the height of the tree is 1.37 meters, the Hohenadl's form quotient is 1.

Now, what happens if the height of the tree is greater than 1.37 meters? So, if h is greater than 13.7 meters, 0.1 h is greater than 1.37 meters is greater than dbh, so what would be d 0.1 h? Now, because 0.1 h is greater than dbh, this line would go above the dbh. So, in this case we have because this tree is tapering towards the top will be having d 0.1 h is less than the dbh. So, what will happen to the Hohenadl's form quotient? It is dbh divided by d 0.1 h. Now, because d 0.1 h is less than dbh, this figure would be greater than 1.

So, if tree height is greater than 10 times dbh, then Hohenadl's form factor is greater than 1. If tree height is equal to dbh, Hohenadl's form factor is equal to 1 and if tree height is less than 10 times dbh will have q h is less than 1. Now, the height of a tree goes on increasing with its age. So, basically whether q h is greater than 1 equal to 1 or less than 1 and what would be its exact value would be dependent on its age.

So, this is one form quotient that is highly dependent on the age of the tree.

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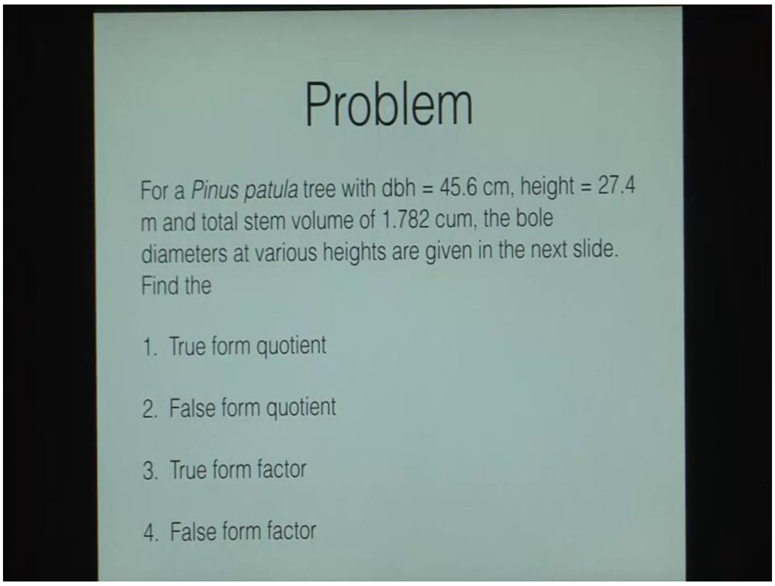

So, now let us do one problem. It is calculate the form factors in form quotients for a tree.

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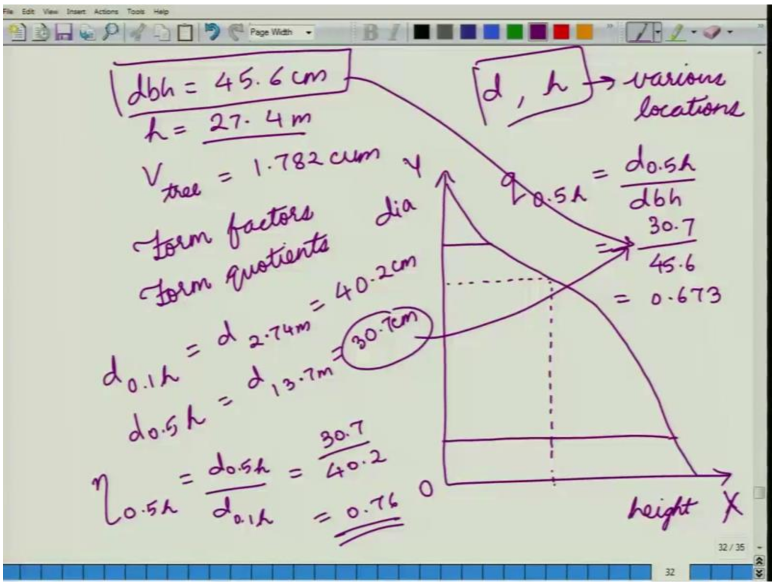

For a Pinuns patula tree with dbh is equal to 45.6 centimeters, height is equal to 27.4 meters and the total stem volume that is the volume of the tree is equal to 1.78 cubic meters. The bole diameters at various heights are given on the next slide and we need to find out the true form quotient, the false form quotient, the true form factor and the false form factor.

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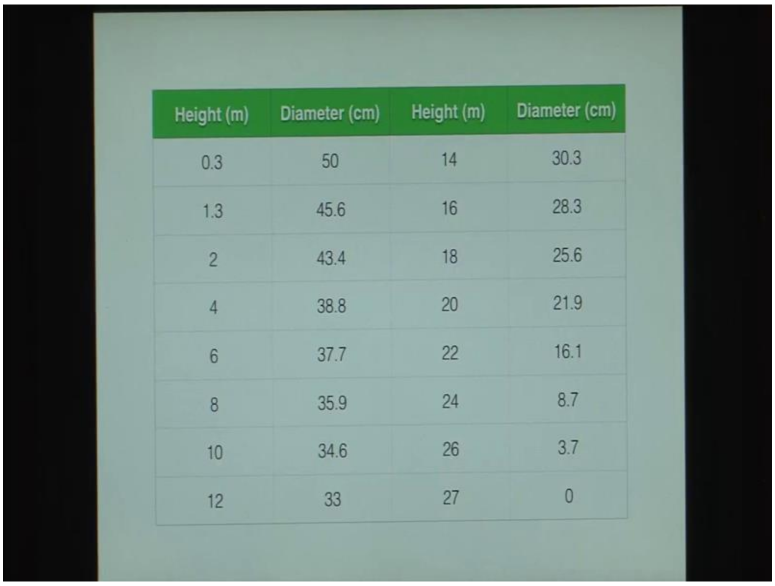

Height (m)	Diameter (cm)	Height (m)	Diameter (cm)
0.3	50	14	30.3
1.3	45.6	16	28.3
2	43.4	18	25.6
4	38.8	20	21.9
6	37.7	22	16.1
8	35.9	24	8.7
10	34.6	26	3.7
12	33	27	0

So, here are the heights and in the diameter set those heights. So, as you can see as the height goes on increasing, the diameter goes on decreasing telling us that this is a tapering tree and we need to calculate the form factors and the form quotients.

So, how do we do that? We are given the diameter set and height at various locations.

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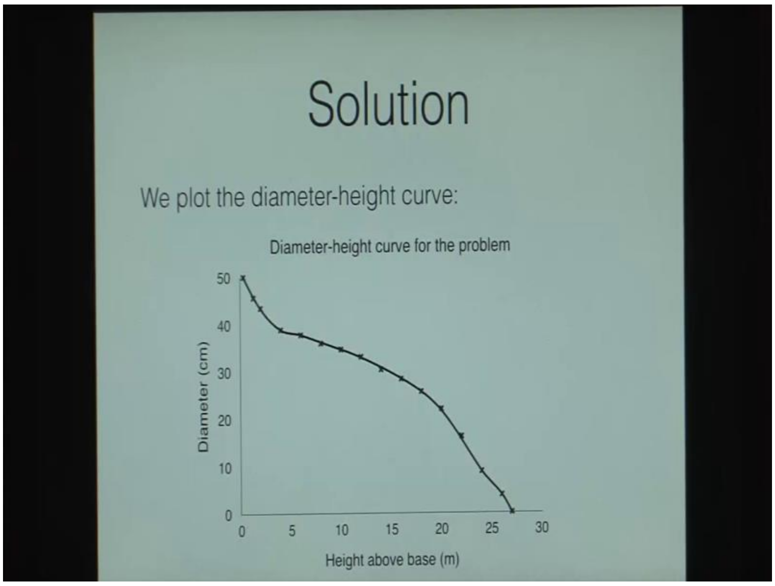

Now, the way of solving this problem is to first of all plot the diameter height curve. So, as we can see in this figure, this is similar to what we did in the very begining. We take dia on the y axis and the height on the x axis and we plot these various values that is given to us. We would get a curve that goes something like this. This would be the conical portion, we would be having a parabolic section and we would be having a niloid section, but once we have plotted this curve, we can find out the diameter at any height and the height corresponding to any diameter.

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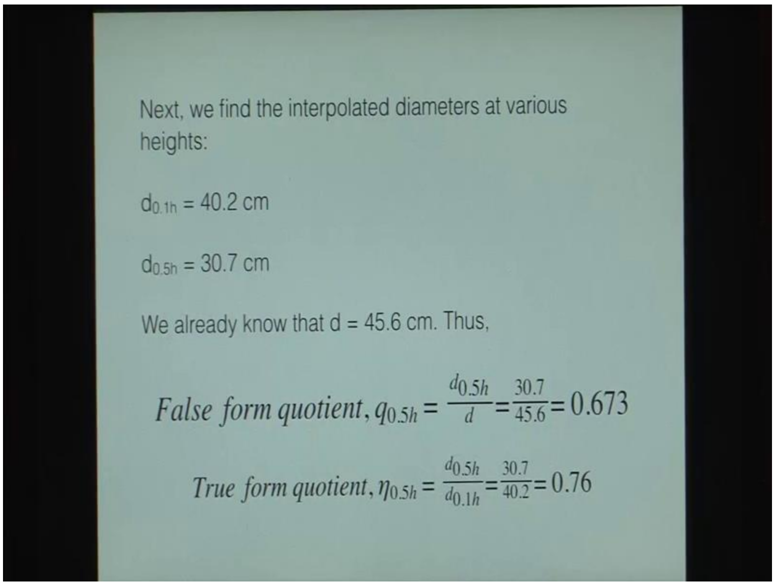

So, once you have plotted this curve, next we will need to find out the interpolated diameters at various heights. So, what are the pertinent heights? Here we need d 0.1 h, d 0.5 h. Now because the height is known, so d 0.1 h will be equal to d at 2.74 meters and this would be equal to d at 13.7 meters from the curve. We can calculate the interpolated value and this comes to 402 centimeters and this comes to be 30.7 centimeters. Now, we are given that dbh is 45.6 centimeters.

So, what is the false form quotient? The false form quotient is given by q 0.5 h is equal to d 0.5 h divided by the dbh. Now, d 0.5 h we have found from here. So, that is 30.7 divided by 45.6 is equal to 0.673. Now, what is the true form quotient? True form quotient as given by eta 0.5 h is equal to d 0.5 h divided by d 0.1 h which comes to 30.7 divided by 40.2 is equal to 0.76.

Next we need to calculate the volumes of the cylinders.

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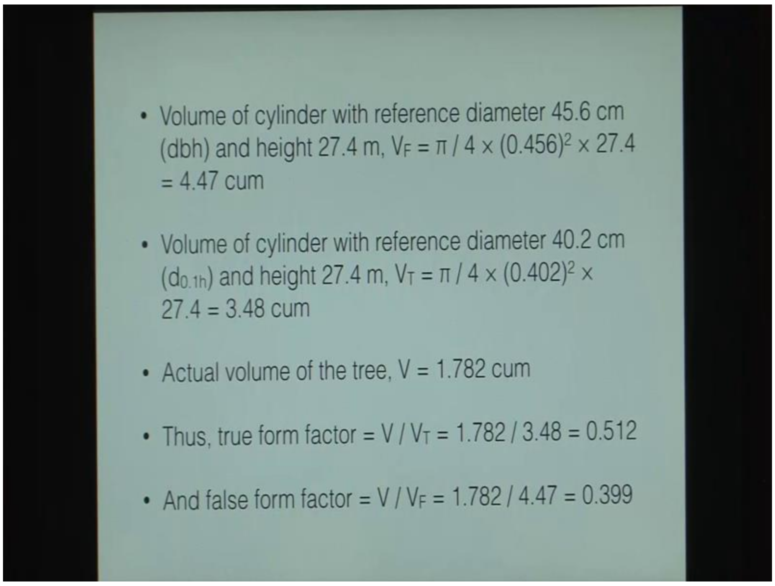

So, now the volume of a cylinder with a reference diameter of 45.6 centimeters which is dbh and a height of 27.4 centimeters, it comes to be pi by 4 d squared h is equal to pi by 4 into 0.456 square because we need to convert 45.6 centimeters into meters into 27.4 is equal to 4.47 cubic meters and the volume of a cylinder with a reference diameter of 40.2 centimeters that is d 0.1 h and the height of 27.4 meters.

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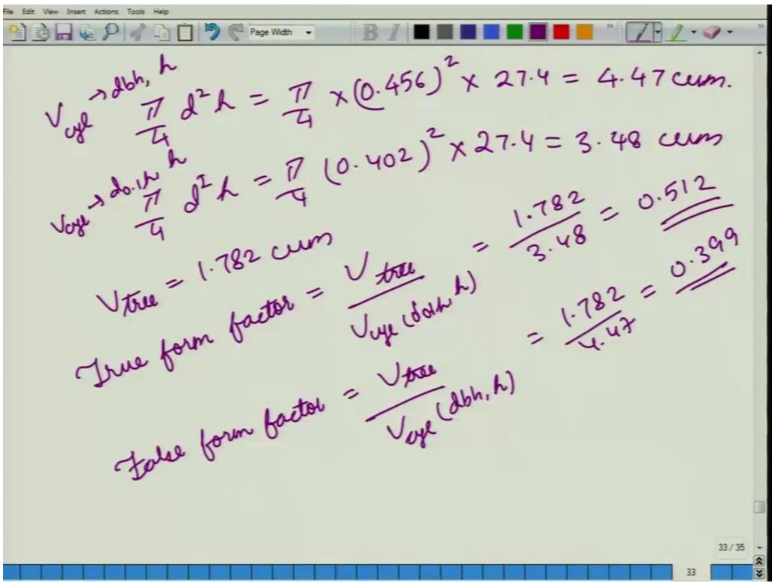

So, essentially we want this diameter and this height. So, if we calculate the volume of the cylinder, it would be pi by 4 d squares h is equal to pi by 4. We took this diameter d

0.1 x that is 40.2 centimeters. So, that becomes 0.402 meters square into the height of the tree that is 27.4 meters. 27.4 it comes to 3.48 cubic meters. So, this is the volume of a cylinder with dbh and height and this is the volume of a cylinder with d 0.1 h and height.

Now, we are given in the problem that the volume of the tree is 1.782 cubic meters. Now, we need to find out the true form factor. Now, true form factor is given by the volume of the tree divided by the volume of the cylinder with d 0.1 h and height which becomes 1.782 divided by 3.48 cubic meters is equal to 0.512 and the false form factor is equal to the volume of the tree divided by the volume of the cylinder with dbh and height. So, this becomes 1.782 divided by 4.47 is equal to 0.399. So, these are true and the false form factors. So, as you can see in this example, it is very easy to calculate the form factors and the form quotients.

Thank you for your attention. [FL]