Forest Biometry
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Lecture – 09
Taper Equations

[FL]. Today we will further build upon the topic of Taper look into forms and look at Taper equations.

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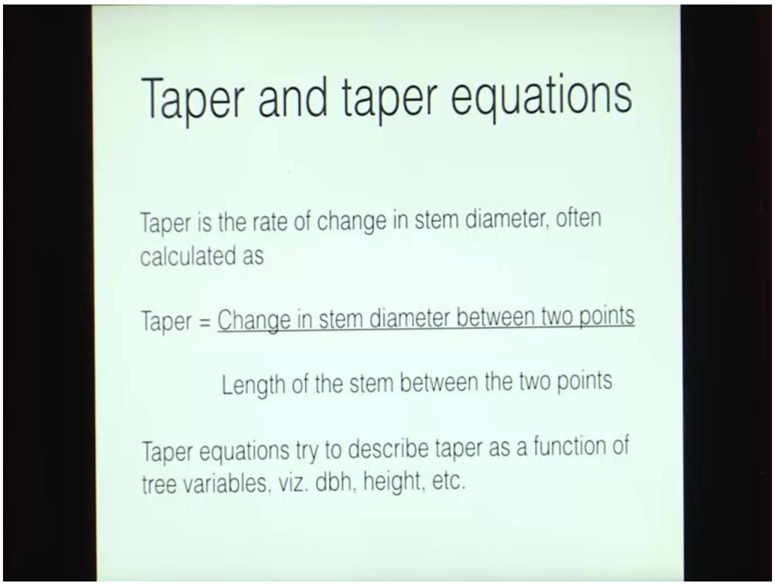

As we saw in the previous lectures, Taper is defined as Taper is equal to change in the diameter upon change in the height.

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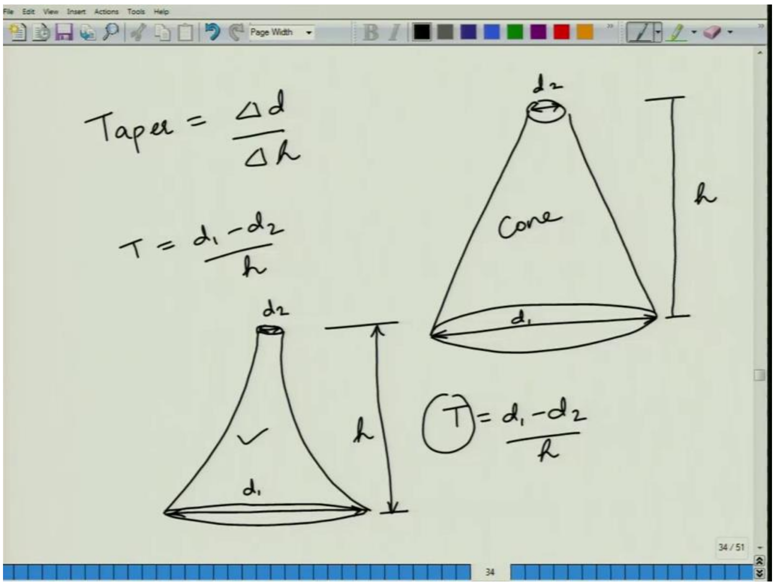

So, for instance if you consider this portion of a cone. If the diameter here is d 1, diameter here is d 2 and the height is h, we will have Taper is equal to d 1 minus d 2 divided by h.

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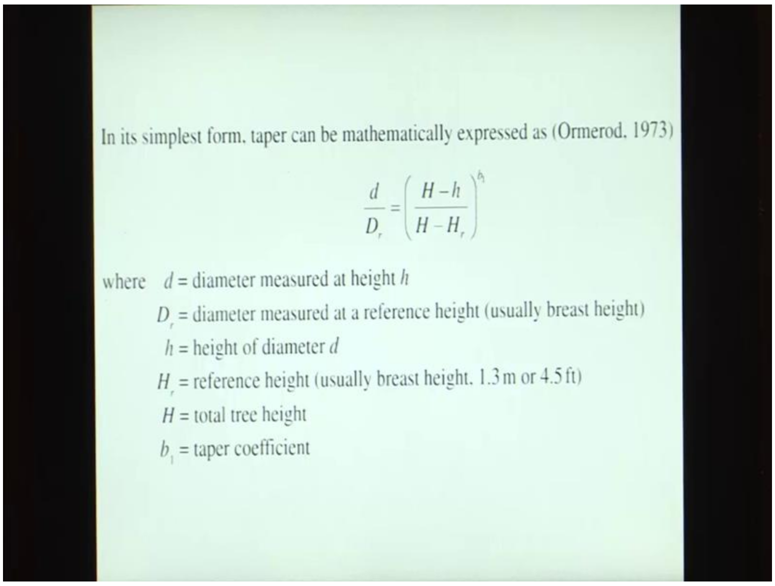

Now, this is for a very simplified solid, such as the cone. What if we had a solid like this? So, here as well the diameter at the bottom is d 1, the diameter at the top is d 2, the total height is h. So, here we will have T is equal to d 1 minus d 2 divided by h. However, we can see from these two figures that a cone or a portion of a cone is very different from this figure which is a portion of a paraboloid. So, we can improvise upon this equation of Taper to look at these details and to express these details mathematically. So, let us have a look at that. Now, in its simplest form, Taper can be mathematically expressed as follows.

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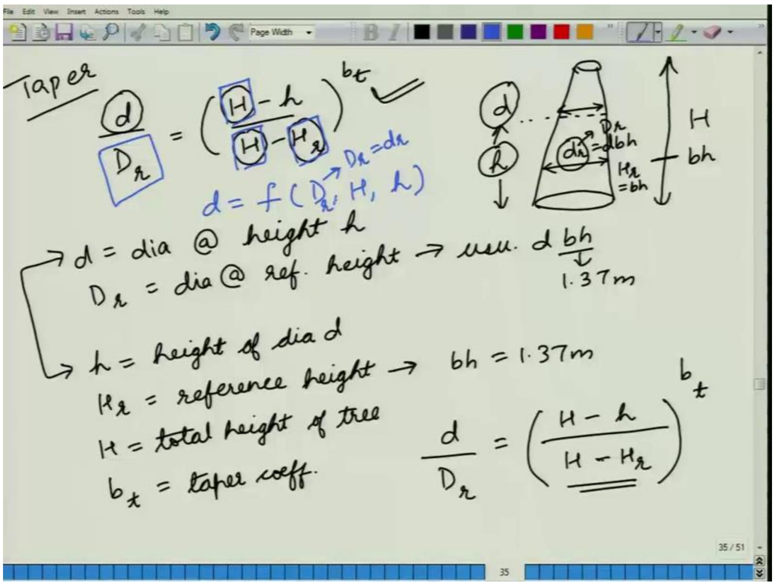

So, we have d upon D reference is equal to H minus small h divided by H minus H r whole to the power of b t. If you wrote this as the equation of a Taper, what would it denote? So, here we will have d is equal to dia at height h, D r is dia at reference height which is usually the diameter at breast height or breast height is 1.37 meters in the case of Indian conditions h. Similarly is the height of dia d. So, we are getting these two equations are equivalent. H r is the reference height which is in our cases the dia, the breast height or 1.37 meters, h is the total height of tree and b t will be t is the Taper coefficient. Now, this equation is same as our previous equation.

So, let us have look at this equation. So, we have a tree with a total height that is capital H. We are interested in the diameter at a height of small h. So, here we have a diameter of d, we also have a reference height which is the breast height and here we have the diameter reference which is our dbh. So, what does this equation now tell us, we are trying to see how this small d is wading as r reference diameter as. So, we can also refer to it as capital D r. So, we are trying to say how much is this small d in terms of our reference diameter as we go above the base of the tree. So, what is the numerator here?

So, here we are having a reference height in both of these. So, the reference height is the total height of the tree and our second reference is H r.

So, now if you look at any point, we are saying that r reference is this reference height of the breast height and if we look at total height minus this breast height, that comes in the denominator and then, in the numerator we have this total height minus the height at for which we are interested whole to the power of b t which is the exponent also known as the Taper coefficient.

Now, why would we be interested in knowing this small d in terms of this complete formula? Now, in this formula we can say that this D r is a constant for any tree, capital H is constant for any tree. So, this is also a constant, H r is also a constant because we have taken it as the breast height. So, essentially what this equation is doing is that it is referring this small d as a function of capital D, capital H and small h because H r and D r are known. So, this capital D is the diameter. This we can also refer to it as the small d r. So, why would this equation be useful to us? Well if we have d as a function of the diameter at the breast height, the total height and the height of any reference point, then we can figure out the volume of this tree.

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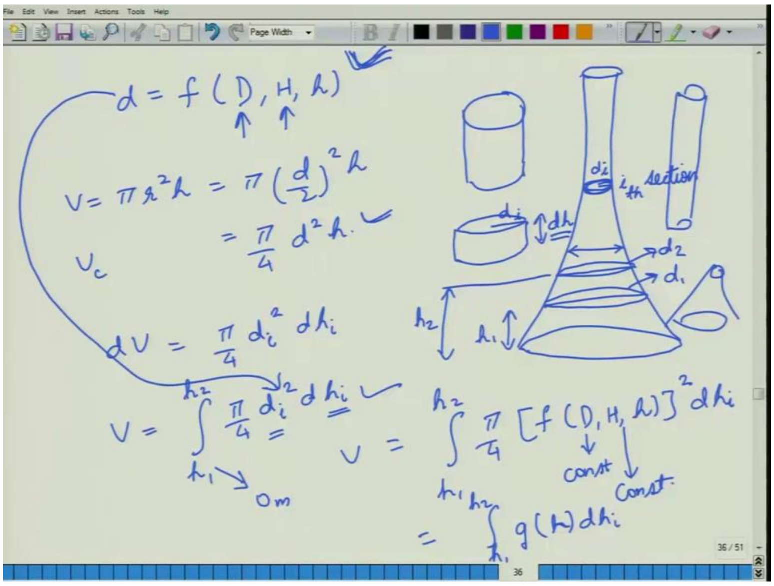

So, this is essentially the equation of the form of the tree. So, whether the shape is like this or the shape is cylindrical or the shape is conical, what this formula is doing is that it is telling us what the diameter of this form would be at different heights in terms of the diameter at the breast height and the total height of the tree.

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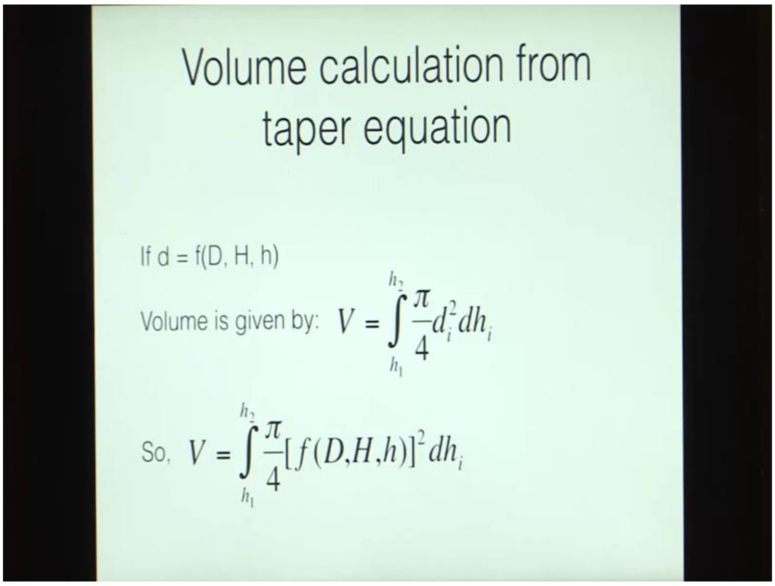

So, if we have this equation, can we figure out the volume? Now, let us consider a section that is a cylinder. Now, in the cylinder we know that the volume as pi r square h or pi d by 2 square h or pi by 4 d square h. Now, that is the volume of the cylinder.

Now, let us consider a very small section that has a height dh. Now, dh is a very small height. It has a diameter that is d at i. So, we are looking at the i th section of any shape. So, this is the i th section. So, we are considering this section to be having a very small height that is given by dh, the diameter of this section is di. So, can we find out the volume? So, the volume of this section also referred to as dv would be using this formula because this section is considered to be a very small section. It has a very small height. So, we can consider it as a cylinder. So, in that case we will have V is equal to pi by 4 d i square dh i. So, if this is r small volume of the section, what is the total volume of the tree? We can write it as an integral pi by 4 di square dh i, where the height goes from a height h 1 to height h 2.

Now, if we are considering the complete tree, then h 1 will be equal to 0 meters, but if we are considering any portion of the tree, suppose we are considering this portion, then we want to figure out the volume of this section that goes from. So, this height is h 1, this height is h 2, here you have the diameter of d 1, here you have the diameter of d 2. So, can we figure out the volume of this section? It will also be given by this formula. So, the volume is the integral. The height is taken from h 1 to h 2 pi by 4 di square dh i.

Now, in this equation we have two variables i.e. the height and the diameter. Now, if we use this formula for the diameter, we would get volume is equal to integral h 1 to h 2 pi by 4. Now, di will be taken from this equation. So, we have function of capital D capital H and small h square dh i. Now, because capital D is a constant, capital H is a constant for this particular tree. So, what we are getting here is that this becomes integral of h 1 integral from h 1 to h 2. So, this f can now be written as g as a function of h dh i.

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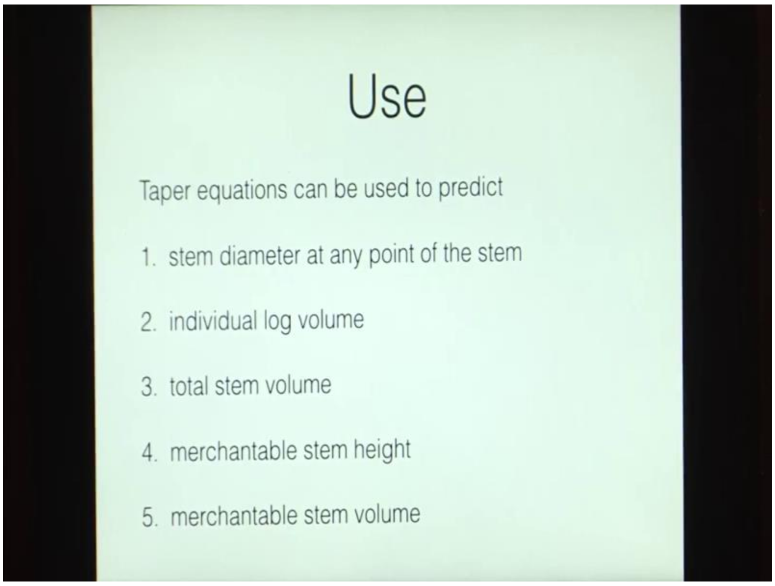

So, now this equation can be integrated. So, we can figure out the volume, once we know this equation, the diameter as a function of the height.

So, if we use this Taper equation, it can be used to predict a number of things. So, when we are writing d as a function of D, H and h, we can predict the stem diameter at any point of the stem.

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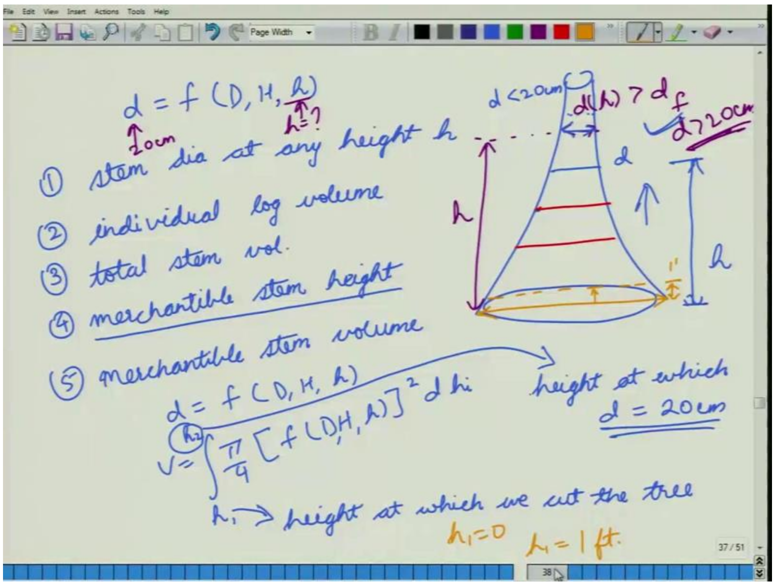

So, in this tree using this equation because this is the equation of the whole form of the tree, we can figure out the diameter d at any height h. So, the first thing that we can figure out is the stem diameter at any height h. We can also figure out the individual log volume. So, log volume would be the volume of any portion of this tree. So, as we saw in the previous slide, we can also figure out the total stem volume.

Now, the total stem volume as we saw in this previous slide, this total stem volume would be when we have h 1 is equal to 0 and h 2 is equal to the complete height of the tree. If we integrated for the whole tree will get the volume of the total stem. We can also figure out the merchantable stem height. Now, remember that the aim of forestry is to produce maximum amount of biomass and to extract or harvest this maximum amount of biomass. Now, this biomass has to be used for some purpose say for making a furniture or for making of doors or windows or anything.

So, for that we need a merchantable stem height. So, what is the merchantable stem height? It is that height at which using this same figure, it is the height at which the diameter d h is greater than a fixed 9 diameters say d f. So, for instance there are a number of equipments that are only able to process wood that is having a diameter say greater than 20 centimeters. So, if that is your equipment, you have such an equipment that can only process till 20 centimeters. Using this formula you can put d equal to 20 centimeters and you can find out what h is.

So, for any stand we can calculate what is the merchantable stem height what we can also do is to calculate the merchantable stem volume. So, what would be the merchantable stem volume? If we have d is equal to function of D, H and h, the merchantable stem volume V. So, taking this previous equation, it will be pi by 4 function of D H and h whole square dh i integrated from h 1 to h 2.

Now, in the case of the merchantable stem volume, your h 2 would be the height at which d is equal to 20 centimeter. Taking this example that your merchantable diameter is greater than 20 centimeter, so any height above this because this tree is tapering upwards; so, any height more than this would give you a diameter that is less than 20 centimeter. So, this height which you are calling as a merchantable stem height is the height at which you have d is equal to 20 centimeters. H 1 would be the height at which we cut the tree. Now, if we are able to cut the tree at the base, you will have h 1 is equal to 0, but in most cases it is very difficult to reach the base. So, we generally cut the tree at some height, it is generally 1 feet. So, in that case, we will have h 1 is equal to 1 foot.

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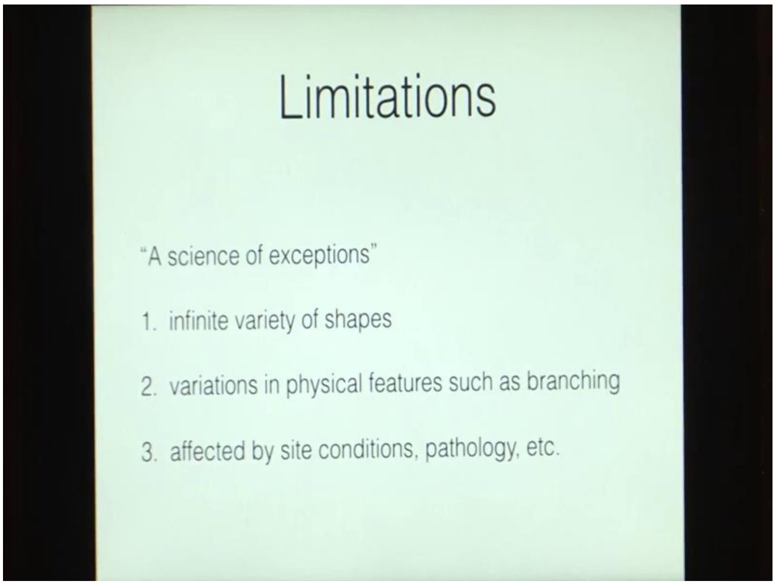

Now, so far so good if we knew d as a function of H D and h, we can calculate so many things.

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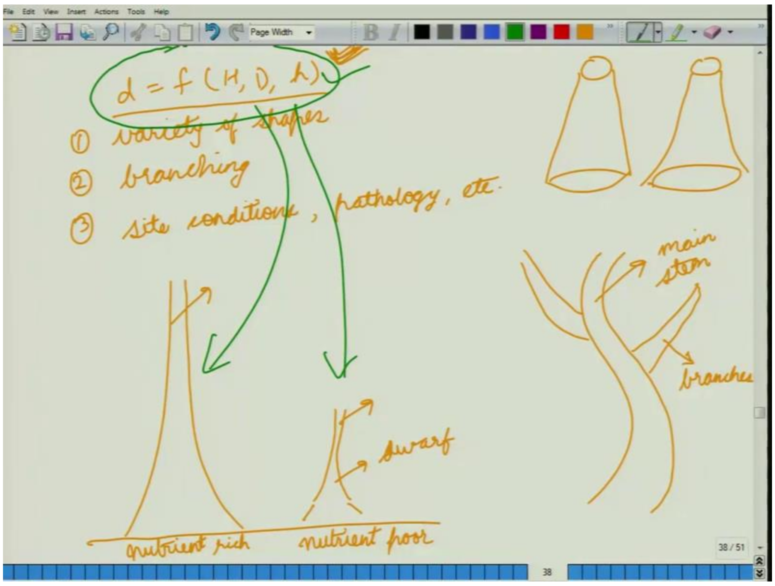

However, still we are having a few limitations. One limitation is that of the variety of shapes. Now, we can calculate all these volumes and the merchantable height only if we are able to get to this equation. Now, getting this equation is easy. If you have the shape of a tree which is say a regular shape like a cone or say a portion of the paraboloid, in that case it is very easy to get this equation. However, if you have a tree that is say crooked, it also has a few branches and then, getting to this equation would be extremely difficult. The second limitation is as we said before branching. So, when we are using this equation, we are only calculating the volume of the main stem. However, the branches in many cases are also having such a large diameter that they can also be used in an economic fashion, but our equation which is d is equal to function of capital D capital H and small h, it is not taking into account the branches.

The third thing is that if we wanted to figure out this equation forest stand, we would be able to do it, but in the case of a few trees, they would be affected by sight conditions, pathology etcetera. So, for instance in the case of one stand, you could have a tree that is having a very large height, but at the same time you could also be having a small tree. Why did this tree remain a dwarf? This could be because in the early stages of its life, it was affected or say infested by some insects or from some disease or may be at this location, we have the soil that is nutrient rich, but at this point we have a soil that is nutrient poor. So, we can have n number of conditions that would result in this tree remaining dwarf, but this tree becoming tall in the same stand. So, these could be just trees standing next to each other, but still you will find a number of differences in these conditions. We will not be able to deploy our equation, this equation to both these trees because this is an average equation.

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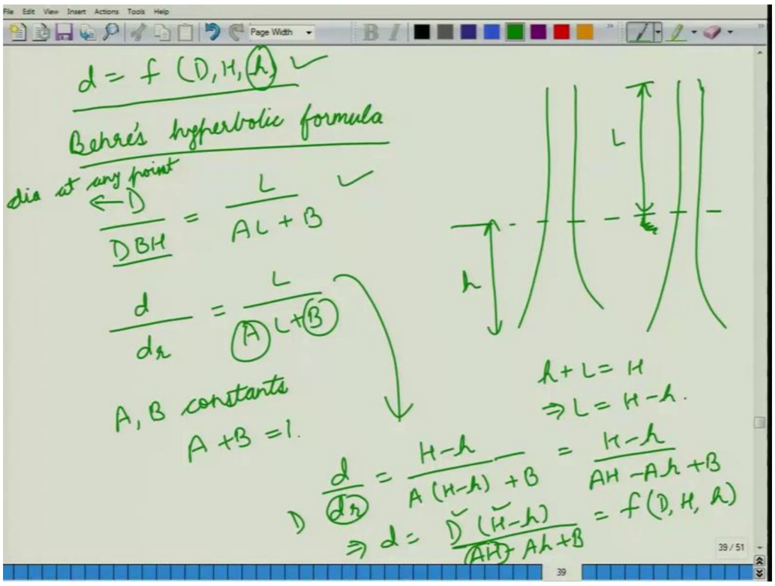

Now, the equation of the tree that is given by this general equation has been computed by a number of authors. So, once that equation is called Behri's hyperbolic formula. So, this equation is the generalized equation, but then how does this equation look practically. So, Behri found out one formula that goes like this. D upon DBH is equal to L upon AL plus B. Now, here that D is the dia at any point. So, this is the way in which this equation is written, but if we wanted to write it in this term, this would become D upon DBH are reference diameter is equal to L.

Now, this equation talked about H. H is the height from the ground, but various equation talks about L which is this distance. So, for the same point this is L. So, L is the distance from the tip whereas, H is the distance from the bottom. So, in any case you have h plus L is equal to capital H. So, you can always find out L as H minus h. So, this equation talks about L upon AL plus B. Now, here A and B are constants such that A plus B is equal to 1.

So, now if we put all these things into the main equation, we would have d upon d r is equal to H minus h divided by A H minus h plus B which would be h minus h divided by AH minus A small h plus B. So, now here d r can also be written as d. So, you will have d is equal to D H minus h divided by AH minus Ah plus B which is a function of capital D capital H and small h because this is a constant. H is a constant.

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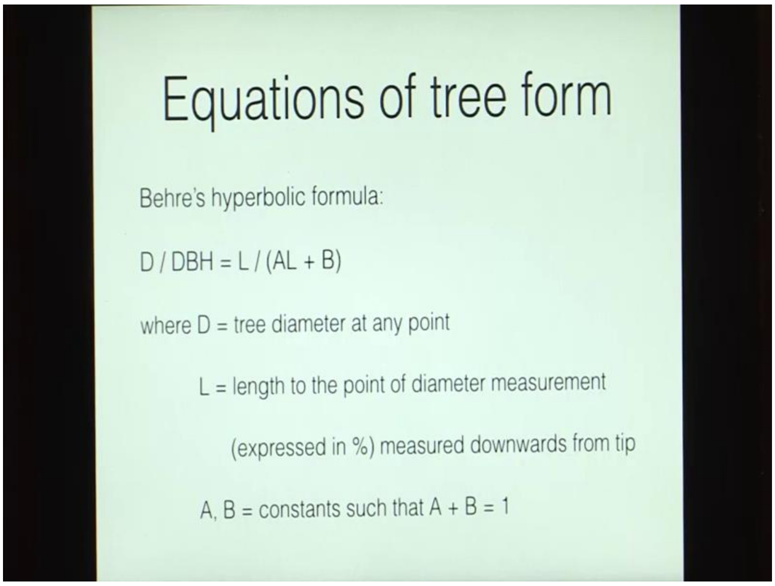

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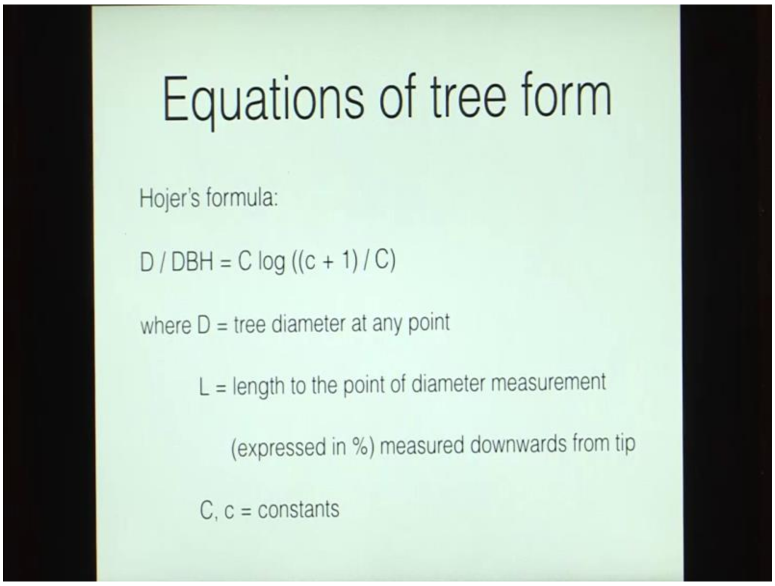

Similar to Behri's formula, we have also another formula which is called as Hojer's formula.

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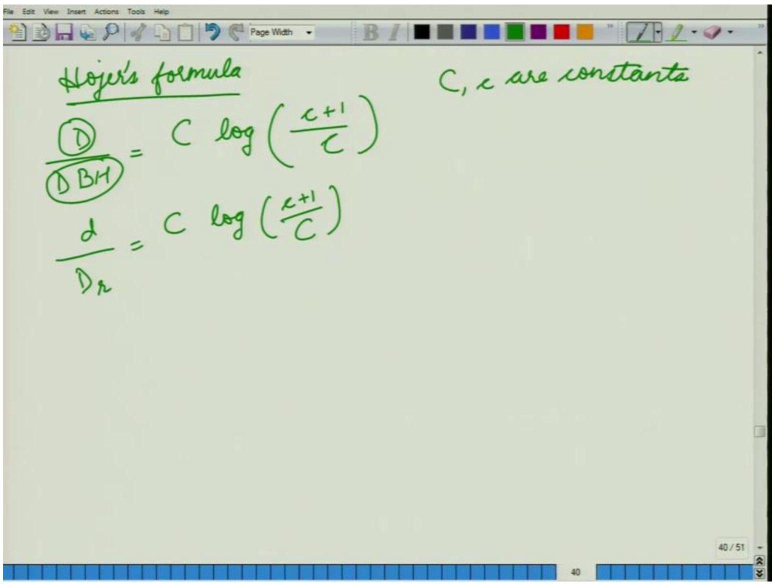

Now, Hojer's formula is written like this D upon DBH is equal to capital C log of small c plus 1 upon capital C. Here as before D is diameter at any point, DBH are reference diameter, capital C and small c are constants. So, this is log small c plus 1 upon capital C. You will write it as capital C. So, this is another formula that is generally used.

Now, we can write Taper equation as we saw is a function of D H and h, but once you have derived a very complicated equation, how are you going to use it in the field that becomes difficult.

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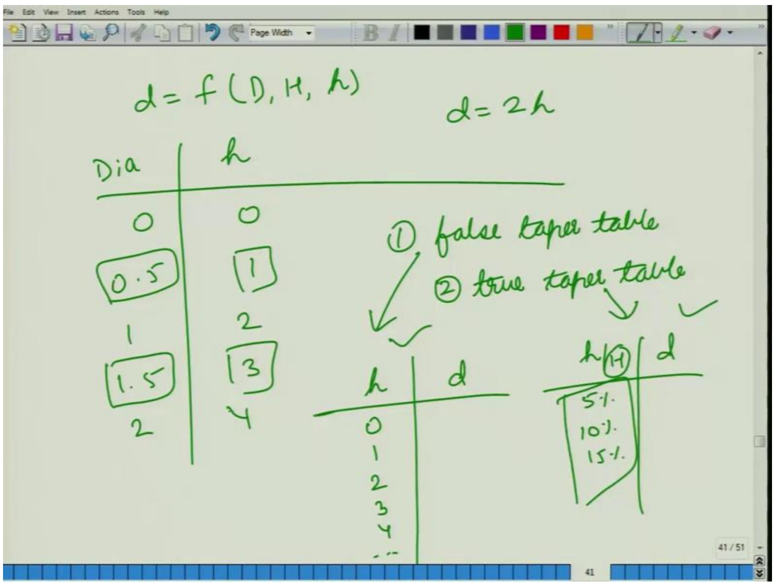

To solve that problem, we devised something that is known as a Taper table.

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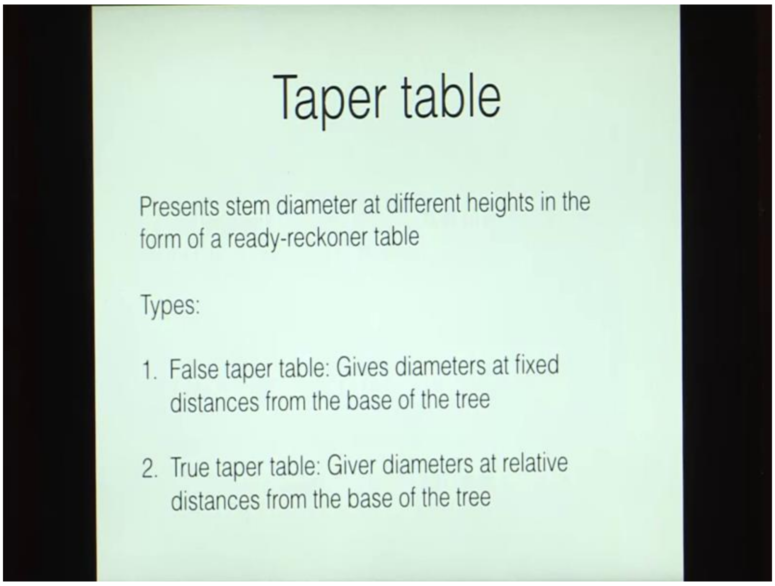

A Taper table is a table that presents the stem height, stem diameter at different heights in the form of a ready Reckoner table. So, for instance you would have the diameter for all the height h. So, if it is given something like this if you have a table like this, so instead of writing that your diameter is equal to 2 h, you can write it in the form of a table. So, when you have this table, if you wanted to figure out what the diameter at say a height of 1 meter is, you can get 0.5 is as the diameter or if you wanted to figure out what is the height at which the diameter becomes 1.5, you can get the height is 3.

Now, there are two kinds of Taper tables. One is a false Taper table. Now, a false Taper table gives diameters at fixed distances from the base of the tree. So, for instance if you had height and diameter and say x is distance say 1 meter. So, you will have 0 1 2 3 4 and so on. So, this would be a false Taper table. The second thing is a true Taper table which gives that the diameters at relative distances from the base of the tree. So, in the case of a true Taper table, in place of h, you might have because this is relative distance. You might have h by h versus d; for instance at a height of say 5 percent, 10 percent, 15 percent and so on.

Now, why is this called a false Taper table and why is this called a true Taper table? In the case of a false Taper table, it does not consider the complete height of the tree, where as in a true Taper table, it considers the complete height of the tree and gives all the heights as a relative fraction which is why it js called a true Taper table because there is less amount of arbitrariness here.

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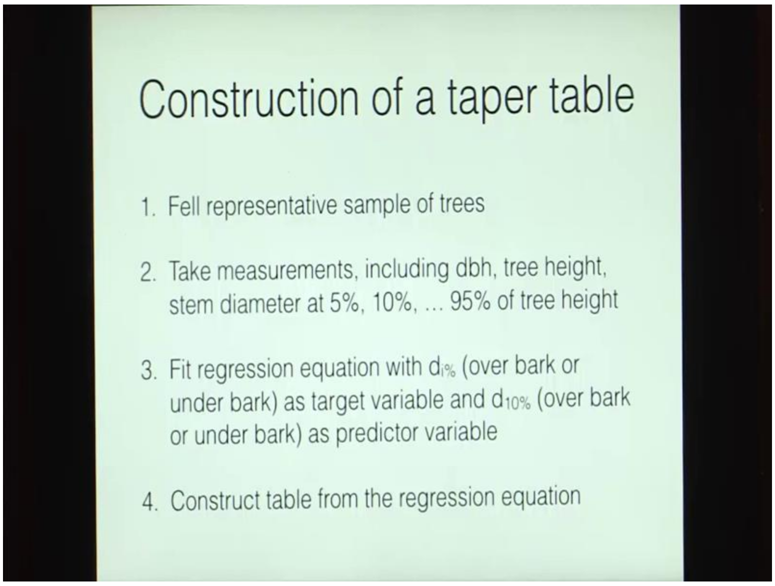

So, how do we construct a Taper table? The construction of a Taper table consists of four steps. One you fill a representative sample of trees.

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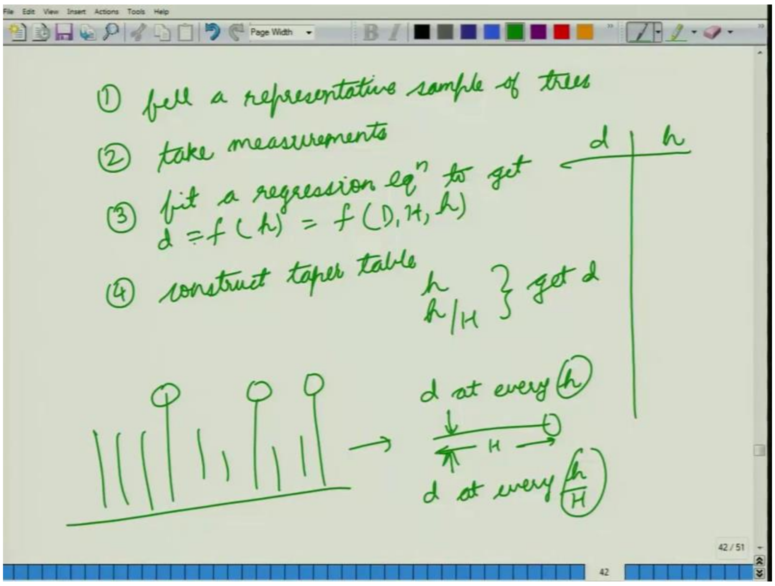

So, if you have say a forest stand which you have trees of different heights and if you wanted to construct a Taper table, you would select some trees say all those trees that have a height nearly this much. So, you will select these trees and then, will fell those trees. Once you have felled these trees, you take measurements. So, what are you going to measure? You are going to measure diameter at every small h. So, now because this tree is lying horizontal, you can now measure its diameter at different heights or you could measure D at every H upon h with capital H is the height of the tree. So, you can either measure it in fix terms, in terms of height or you can measure it in percentage terms 3, when you have this data of diameter and height, you can now fit an equation. So, fit a regression equation to get d as a function of height or as we wrote it before as a function of capital D capital H and small h.

We could either use the over bark diameter or the under bark diameter to fit this regression equation. Now, once you have the equation, you can now construct the Taper table. So, once you have the representative equation, you can choose any height or you can choose any representative height and get d for those heights or for those representatives height and then, represent it in the form of a table.

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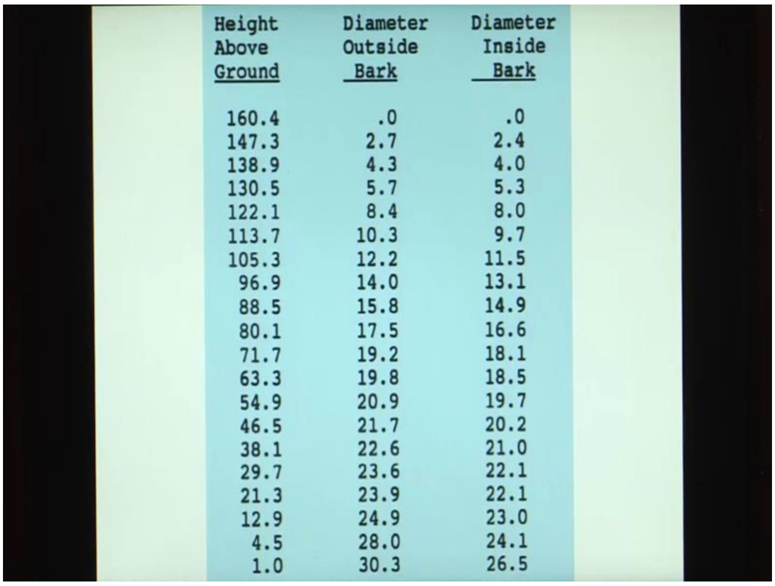

Height Above Ground	Diameter Outside Bark	Diameter Inside Bark
160.4	.0	.0
147.3	2.7	2.4
138.9	4.3	4.0
130.5	5.7	5.3
122.1	8.4	8.0
113.7	10.3	9.7
105.3	12.2	11.5
96.9	14.0	13.1
88.5	15.8	14.9
80.1	17.5	16.6
71.7	19.2	18.1
63.3	19.8	18.5
54.9	20.9	19.7
46.5	21.7	20.2
38.1	22.6	21.0
29.7	23.6	22.1
21.3	23.9	22.1
12.9	24.9	23.0
4.5	28.0	24.1
1.0	30.3	26.5

Now, if you look at this slide here, we have our Taper table. So, it shows height above the ground, diameter outside the bark and diameter inside the bark put different heights.

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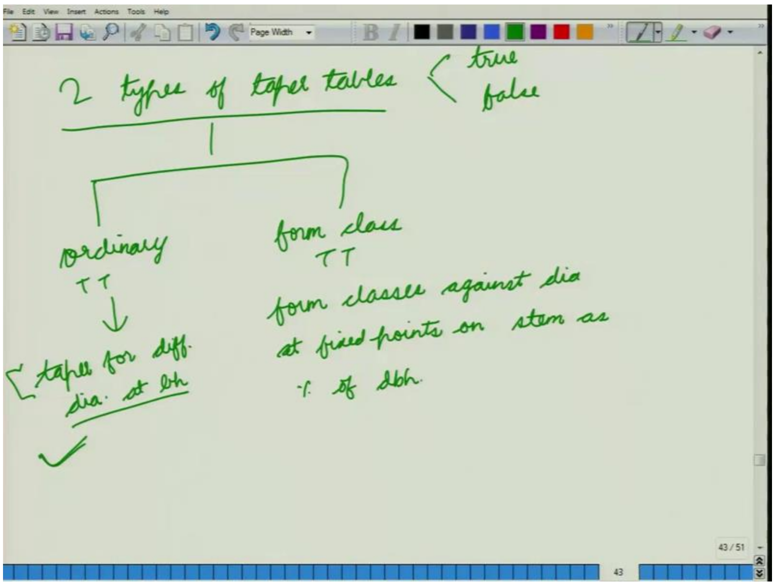

Now, there are two types of Taper tables. So, one classification was true or false Taper table and the other classification is that of ordinary Taper table or form class Taper table or negative Taper table form class Taper table. Now, an ordinary Taper table gives Taper for different diameters at breast height. So, what this Taper table or this ordinary Taper table does is that for any dbh, you can get a Taper equation using the values that are given there in the case of form class Taper tables. They show different form classes against diameter at fixed points on stem as percentage of dbh.

Now, in India we only have ordinary Taper tables. We do not have formed Taper tables for most of the species. So, this is the only Taper table that you are going to witness.

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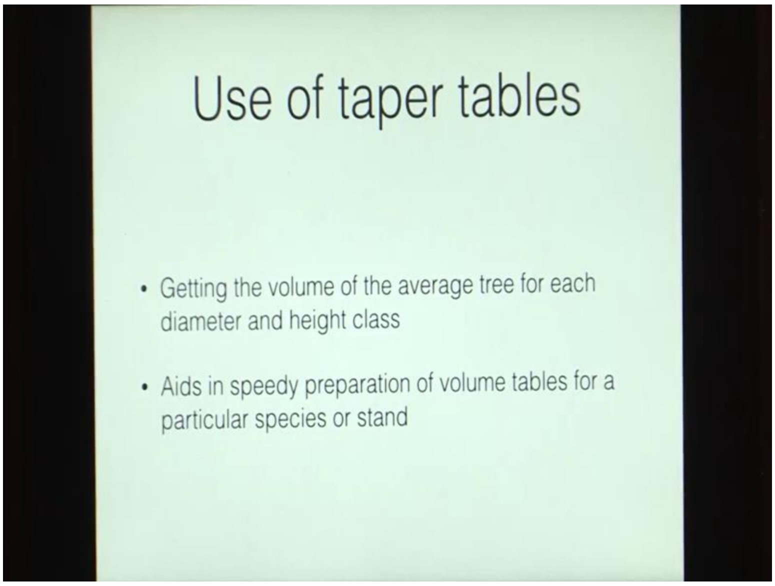

Now, what are the uses of Taper tables? We can get the volume of the average tree for each diameter and height classes.

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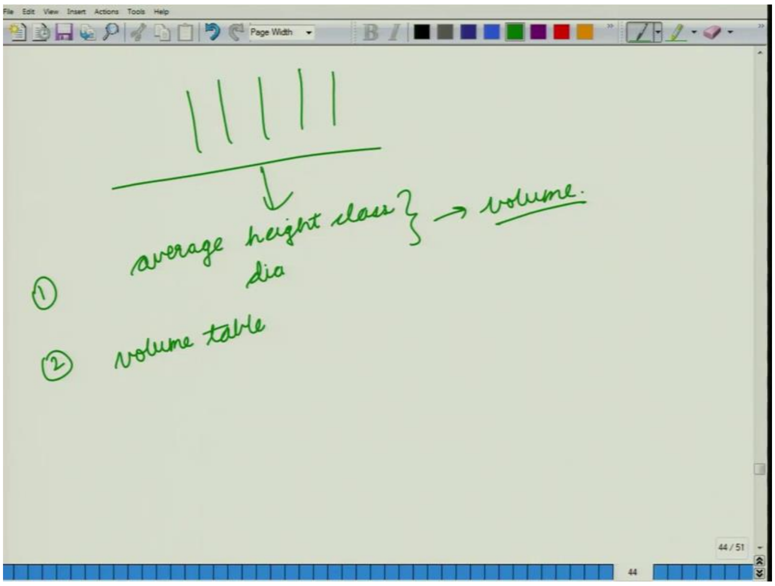

So, if you have a stand and for that you can figure out the average height class and the average diameter, you would be able to predict the volume. So, that is the first advantage getting the volume of an average tree. Now, we can further use these volumes in the construction of volume tables. Now, what does the volume table do similar to the Taper tables? It gives us volume for trees of different heights classes and trees of different diameters. So, if we represent it in the form of a table, we get a volume table.

Thank you for your attention. [FL]