


Introduction to Biomedical Imaging Systems
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Lecture - 13
Attenuation Models

(Refer Slide Time: 00:16)



Summary observations

- Compton scattering is equally likely in various materials and invariant of incident energy
- Photoelectric effect is more likely in high Z material and less likely with high incident energy
- Overall, Compton scattering is more dominant with higher incident energy in the same material
- But the percent of energy deposited due to photoelectric event is larger because all incident energy is absorbed.
- Table 4.5



So, let us move on. Now, that we have set the stage let us get into the, you know some more specific terminologies and usage because we completed the interactions right.

So, we understood the different types of interaction different types of ionizing radiations, what is of interest and you know what is of interest to a X-ray tube from a generation point of view, what is of interest from a imaging point of view right, what physics what


phenomenon's are you know involved what types of interactions are involved all that has been completed.

So, if we have to progress further because our objective is imaging system, understanding the physics and then engineering it so that we can capture an image of the unknown distribution right that is our big picture. So that means, we need to you know march forward with the understanding of the physics can how do we capture how do we engineer that right.

So, before we can engineer I think you know you need to have some equation. So, there is this phenomenon's there is this physics there is this intuition somehow we need to have a common language. So, that we can put them in a mathematical form and then exploit it to get the imaging system model and the image right.

So, having said that let us move on. Now, we want to talk about the interactions that we just talked about some interactions right, electromagnetic radiation interacting with material. So, now, let us introduce some terminologies to how to describe them more quantitatively how to model them more quantitatively ok.

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Attenuation of EM Radiation

- Measures of X-ray Beam Strength

Simplest → # of photons, N, in a burst, area of spread is also important


Photon fluence - $\Phi = \frac{N}{A}$

Photon fluence rate - $\phi = \frac{N}{A\Delta t}$

Energy fluence - $\Psi = \frac{N\hbar\nu}{A}$

Energy fluence rate - $\psi = \frac{N\hbar\nu}{A\Delta t}$

Also known as Intensity-
 $I = E\phi$



One of the important interactions that we want to capture right, we did not use this term so far we probably use it in introduction lightly, attenuation of electromagnetic radiation. In some sense if you recall I said you shoot an you know you shoot X-ray energy something comes out. So, through transmission is what we will discuss.

So, when you do that the electromagnetic radiation is interacting with body and we talked about important interactions of this electromagnetic radiation. One is photoelectric, the other is Compton. What happens in photoelectric is absorbed we said; that means, there is loss of energy.

So, more correct terminology to describe it is the attenuation. So, you send some energy, energy is reduced. You send some photons number of photons reduce, it depend. We will have to be little more clear on that, but the idea is you send something in something to do

with X-ray in and that get reduced when it comes out because it is interacting with the it is losing out to the tissue or the material or the medium that it is interacting. So, we now about talk about attenuation of electromagnetic radiation.

So; that means, we need to first and foremost ok, I am sending X-rays it is interacting and something is getting lost and that is what it is getting lost and it is getting reduced or attenuated because of the interaction with the material. So, the material is able to you know make something happen here ok; that means, we need to talk about how do we describe that.

So, we can talk in terms of X-ray. What am I sending in? You know loosely we said X-ray, but what more specifically of X-ray. So, we need to talk about how do we characterize how do we quantify how do we describe objectively what do we mean by beam strength ok.

If we can send a beam strength right that is getting reduced or because of attenuation due to the material specific property right that is what we want to and then it comes out. Based on what comes out we know what we sent in we know what comes out we need to see the attenuation is the property right that is the unknown of the body that we are trying to estimate, how much is attenuation where it is right distribution X-ray beam.

So, easiest way to do it is what? We talked about X-ray energy as photons packets of energy photons right. So, easiest thing is I send so many photons in right of X-ray energy in that energy range. I send so many photons outcome so many photons. So, it is easier to talk about beam strength in terms of number of photons that are going through right.

So, that is the simplest measure. Number of photons that is good, but then we should contextual it little bit further because you are sending I am interested in imaging this body for example. So, there is a target medium right. So, I am sending it. So, I am not interested whether this X-ray is going and reaching the side of the room. I am interested in the X-ray that is going to interact with the body, right.

So, X-rays are going and then I have a volume let us take a surface right that is perpendicular to the propagation. So, my interest is how much of the photons are crossing per area or

something to do with the cross section right. If I can normalize that that will be the even though number of photons itself is good enough. Number of photons per cross section is probably lot more meaningful right.

So, N is a burst area of spread is also important right. Whether I am sending how many over the surface area right, it has go through and come through the other side. So, the cross section perpendicular to the direction of the N photons that are going alright that is important. So, we can normalize that. We call that as photon fluence.

So, there will be several definitions right that we will incorporate describe here. If you just read at it try to byheart it will be confusing ok. So, just go with the flow that I try to organize here. Talk through loud explain the explain the concept loudly. So, that it gets you understand you will not get confused. You understand the flow of how these are built up; fluence, photon fluence photons fluence flow right.

So, number of photons travelling across a cross section normalized right. So, N over A is your photon fluence. Carefully look at I mean these symbols all those things will complicate life right. I mean usually that is the confusing part. So, do not worry about all that concept. Photon flow, photon fluence is photons per area perpendicular the cross section perpendicular to the flow of the photons ok.

That is good, but then we should also be interested if I send some photon outcomes the photon I have to detect it. I cannot wait indefinitely, right. I have to send the next burst of photon next detect that. So, in some sense yeah photon per cross section is good, but then the rate at which it is coming that is also important.

So, that is photon fluence rate. What is flow rate is a temporal you know it talks about the normalizing with the time. So, this is the fluence per unit time, right. So, N by A per Δt that is the fluence rate right ok, good. So, fluence is fine fluence rate is fine. So, this is good enough, but then one may argue look we talked about in X-rays we were talking about energies right.

In fact, if you really paid attention to probably one or two sentences I would have made in the introduction is in PET we end up counting the number of you know the radioactive number of photons whereas in X-ray based modality of radiography or X-ray CT we are more interested in the energy part right.

So, you might just wonder oh he talked about N number of photons, beam strength in terms of number of photons, but reality even just before this we were worried about the energy levels. So, would that be a better measure of describing this X-ray beam strength right.

So, yes you can do that. You will end up doing that at least for the x part you are not really interested in number of photons per se. You are interested in the energy that is flowing X-ray energy that is flowing through. For simplicity we will see if there is only one energy that is coming out, so, the N photons that are coming out are all coming out with the same energy. What is that energy?

E is equal to $h \mu$ right. So that means, I could write instead of N by A , if each photon has $h \mu$ energy then I can write the energy fluence right. So, the energy fluence is N times each one has $h \mu$; N times each photon has $h \mu$ right. So, this is your energy fluence naturally extending this analogous here. If you have fluence flow and photon fluence rate you can have energy fluence rate which is going to be per time right.

So, $N h \mu$ by $N A \Delta t$ $N h \mu$ by $A \Delta t$. Wow, probably you would not have heard right energy fluence rate. What is it saying? Its talking about the energy that is flowing across a cross section A right unit cross section across time Δt X-ray N . So, but you would not have probably used this sophisticated term.

Most commonly you would have heard are you would be more familiar with the usage called intensity right intensity of X-ray that is what you would have probably if you had a chance to brush through or you know this is what you will find intensity. Intensity is nothing, but energy fluence rate. So, do not get confused right. So, that is why I said a lot of terms, but then it is

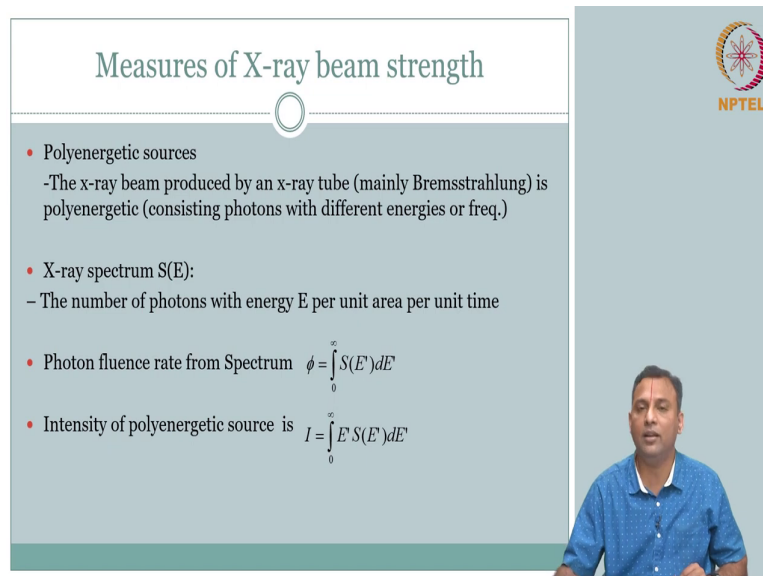
very intuitive very straight forward very common sense extension of each of the concept. So, intensity is this guy.

So, we will write it intensity is I is equal to energy times; of course, this variable did not come out correctly in my fonts here. This is supposed to be right $N h \mu$ is the right $N h \mu$, $h \mu$ is the energy per photon. N by $A \Delta t$ is this guy fluence rate. So, intensity is nothing but energy of the photon and the flow fluence of that photon right.

So, number of photons at that energy level that are crossing area A right at temporal Δt some rate ok. So, this is what you will ok. So, for so good So, we know about intensity, but do not you think we already made some simplification? What could be that simplification? If you are observant you will say oh this is fine life can be little more complicated. Why did not we talk about energy of X-ray coming out from X-ray tube that is what is going to be given to the body right.

What is the nature of X-ray energies right? What is the interaction that produces X-ray in X-ray tube? Is there something that we need to clarify further right?

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The slide is titled "Measures of X-ray beam strength" and features the NPTEL logo in the top right corner. A presenter is visible in the bottom right corner of the slide area. The content of the slide is as follows:

- Polyenergetic sources
 - The x-ray beam produced by an x-ray tube (mainly Bremsstrahlung) is polyenergetic (consisting photons with different energies or freq.)
- X-ray spectrum S(E):
 - The number of photons with energy E per unit area per unit time
- Photon fluence rate from Spectrum $\phi = \int_0^{\infty} S(E) dE$
- Intensity of polyenergetic source is $I = \int_0^{\infty} E S(E) dE$

So, here polyenergetic sources; what do we mean by polyenergetic sources? Poly is many, energetic means so, different energy. So, sources that are sending different energy we talked about this. Just in the previous lecture we talked about X-ray tube generating the X-ray this X-ray right Bremsstrahlung remember.

So, there were discrete characteristic X-rays, but then predominantly the X-rays generated in the X-ray tube is due to the Bremsstrahlung interaction where you saw the continuous. So, there are energy levels different energy levels that were there right. It was going up and the hill shape right, it is going up and then coming down yeah.

So, what we covered just now is for photon with one energy level, but what I am sending is going to be having different energies right because different photons are coming there is a distribution. It is all statistical right that is it is going to be distribution. So, how do I kind of

describe that how do I capture that right when you have; when you know the polyenergetic is the source?

So, N photon with each having the same energy is fine. How do I write about or how what do I how do I capture polyenergetic sources? Straightforward so, now, we need to just recall what we meant by that right Bremsstrahlung. So, literally you have to look at this definition and recall that plot that you saw and you know internalize the concept there as to how do you interpret that plot.

What was that plot? That plot said there was a energy axis number of photons right that was the relative intensity that was coming out and the energy that it was operated right. That was the hill shape. So, Bremsstrahlung means, it is going to give out each burst is going to have different energies at each energy you are going to have some number of photons right.

So, that so, you have a distribution of spectral lines that is you have energy in one axis number of photons in another axis then you are going to have a distribution several energy. At each energy level there is some N number of photons that are coming out. See that is how to view this right. So, you can talk about this as spectral lines remember.

So, we are talking about X-ray spectrum S of E if it is only one energy no problem. If you have multiple energies then I have what is called as a X-ray spectrum; that means, I have S of spectrum of energy. At each energy level I may have some N number of photons. So, at the highest level remember just to you know complete the recollection with what we did already are the highest in the Bremsstrahlung when the thing is annihilated right.

You have complete loss away. So, the highest energy does not come out. The I mean at least relatively speaking we said that if it was 120 kilo volts applied voltage you will get 120 kilo electron volt energy and if you look at that plot it was almost 0. You did not really have any energy photons coming with 120 kilo electron volt because we said that it is a rare interaction complete annihilation.

More number of photons come you know at a range that is less than you know the peak of the hill is much less than your highest voltage or the highest energy that you are sending. Just recall that. So, you could expect that means, X-ray spectrum is different energy levels you have different N number of photons that is going to be a distribution of photon energies.

So, number of photons with energy E per unit area per unit time. So, we can define it this way right. So, we are interested in number of photons with energy E per unit area per unit time correct. So, we talked about number of photons in the previous slide we talked about number of photons per unit area per unit time each having the same energy.

So, here we are just telling that oh this has a energy E , but the burst may have several energies. At each energy level you may have different N ok. So, in some sense photon fluence rate if it is a spectrum that is going in right. If it is a spectrum that we are describing, it has to be integration of all spectrum of $S E$ dashed $d E$ dash. So, this is your dummy variable.

So that means, what is this saying? Fluence rate, number of photons per unit area per unit time, I am given the energy spectrum. What does that say? Number of photons at each energy. So, total number of photons that are going in is nothing but sum of all the photons at different energy levels that is what the sum is what is your integral.

So, your ϕ is the total number of photons with so, each energy level whatever is that spectrum number N number of photons at that level you sum the total photons that are going in right. So, that is what your fluence rate. So, what is the other thing of interest? Intensity, ok; this is the number of photons, what will be the energy?

Well, when we had only one energy we equated $N E$ is equal to $h \mu$ and so, we talked about total $N h \mu$. Here you have N_1 with energy 1, N_2 photons at energy 2 right, N_3 photons at energy 3. So, you have a distribution.

So, what will be the energy? Guess? No, without knowing much we could already guess that why do not you multiply the corresponding number of photons with corresponding? So, if I

have N_1 photons with energy E_1 then the energy is $N_1 E_1$ plus $N_2 E_2$ plus $N_3 E_3$ right. I could stretch your imagination its straight forward extension right.

So, intensity of polyenergetic source is therefore, number of photons at each energy level into that energy level sum of all of them sum of all of them which is your integral here. Each energy source right E dashed into S of E dash. S of E dash has number of photons. S of E dash at each energy level this spectra has S of E is N number of photons at that particular energy level.

So, E dashed is the energy of energy level, number of number of photons at that energy level E dashed S of E dash is number of photons having E dashed energy. So, E dashed into number of photons having E dashed, you do that for all energies right. So, since we talked about that as a continuous space right when you saw the Bremsstrahlung it was a continuous.

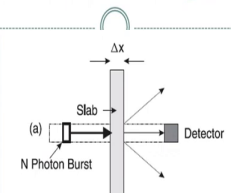
So, dE is that small increment that you are doing. So, your intensity is this guy, very good. So, now, what we have done is we wanted to cover attenuation right. So, now, what we have done is before going to attenuation defining attenuation or something is lost we are actually quantified. What is going in we can actually talk.

We can talk in terms of X-ray beam strength either write it in terms of photon or you write it in terms of energy you write it in terms of monoenergetic or you write it in terms of polyenergetic we know how to describe how much of what you are sending in. Next what is the logic?


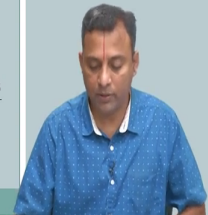
Next that means, we need to be able to define how much is coming out and relate what is coming out to what is going in terms of the loss that is happening or in terms of the material properties that is contributing to this loss or what we call as attenuation, right.

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Narrow Beam, Monoenergetic Photons



$\gg N' < N$
 $n \rightarrow$ photons lost = $\mu N \Delta x$; where, $\mu \rightarrow$ linear attenuation constant $\mu = \frac{n/N}{\Delta x}$
 $\rightarrow \Delta N = N' - N = -n = -\mu N \Delta x$; $\frac{dN}{N} = -\mu dx$ $N_0 @ x=0$;
 Fundamental Photon Attenuation law $N = N_0 e^{-\mu \Delta x}$
 For Monoenergetic case, $I = I_0 e^{-\mu \Delta x}$ $\frac{N}{N_0} = \frac{1}{2} = e^{-\mu HVL} \Rightarrow HVL = \frac{\ln 2}{\mu} = \frac{0.693}{\mu}$
 HVL- thickness that will attenuate 1/2 the incident photons

So, narrow beam monoenergetic photons. So, let us take simple cases at a time. So, what I mean is now we need to tie down what you sent in what comes out in terms of the medium right. So, we will take a simple case first before going into complicated human body with 3D volume right, which is what we will do. Make its make life simple.

We will just take a slab right have a particular thickness delta x, small thickness delta x on one side I am sending N photon. So, I am sending N photon. I am taking a small slab outcomes I have a detector to capture what is coming through, clear. So, what do you think is going to happen? You see some interaction. So, the photon electromagnetic radiation is going to interact with this material right.

That is what we talked about two effects. One is photoelectric effect. So, there is going to be absorption and then characteristic stimulation all that, energy is reduced and then it comes out

of the slab right. The other thing is it is interacting scattered across. So, what you will capture, the detector is anything that is coming through and hitting the detector that you will capture.

So, what you are going to capture is going to be less than what you sent. In terms of photons if you see number of photons that you sent if it is N , you are not going to capture all N because of this interaction. There is a loss because of this interaction, because of the material. Of course, there is also loss because what is exiting through the slab is not hitting the detector the scattering part.

Whatever said and done we will tease out that part later, but at least we are clear what you detect is going to be less than what you sent in and that is due to this loss due to the medium interaction right ok. So, that is fine. What do we mean by monoenergetic photons is straightforward right. Simple case is just deal with photon having only $h \mu$ right that all the photons that are coming have the same energy $h \mu$. So, that is monoenergetic. What do you mean by narrow beam?

So, it has to have a context right. What is narrow? Narrow with respect to what? Why is it narrow? We are sending here x right, the photons are coming the cross section area that we talked about right. So, it is coming. The detector is going to capture only this cross section right. Only this cross section the detector is going to capture. So, narrow means; the beam that is coming in relation to the size of the detector ok.

So, here the detector is small. So, I can only capture this one. So, it is narrow because of this condition because of the size of the detector ok. So, narrow beam means, the width of the beam is of the size of your detector, ok. So, now, what we need to do having said all what we have said it is just now question of putting all the concept together.

So, I know how to describe what is going in, I know how to describe the interaction or at least I know what the interactions are I have to model and then I know something is going to come out I know that it is going to be less than what I sent in. So, what do we do ok? So, let us do a simple thought experiment.

So, attenuation is just loss ok. Just keep that level of simple simplistic interpretation. Attenuation is the loss reduction ok. So, let us do this start experiment. What happens if I do not have this slab right? And assume and assume the medium that is there right, separated between these two is vacuum. I mean even if it is now there is there is not much to interact. So, the medium is absent. What is going to happen? What you sent is going to come out as no loss right.

So, if I put the medium I know there is a loss. So, instead of I send I put in N I have Δx is the thickness through which it is travelling and there is a reduction. What would happen right if I double this Δx ? If I have $2 \Delta x$, what do you think is going to happen? If that is the case I will get even less right.

In fact, if I double this I will you know I will probably expected to be halved right because if there is a Δx there is a small loss. If I double that $2 \Delta x$ where as the loss will also double right or I have the same Δx , but instead of sending N photons I sent $2 N$ photons, what do you think is going to happen?

Well, the number of photons that are lost, now you will have 2 times; because percentage is same right the medium is same. That means, 2 times number of photons would be lost. So, in other words you can sense that the loss the number of photons that you are losing right is proportional to directly proportional to this Δx or what you start with right. So, what you know is N dashed is less than N . N is what you are sending in, N dash is what you are receiving, you know N dash is less than N .

So, if you do this start experiment then you can write n is your number of photons that are lost right, n is number of photons that are lost. You can number of photons that are lost is proportional to Δx proportional to capital N , right. So, this proportionality constant μ is your linear attenuation constant. So, number of photons lost is directly proportional to N and Δx . So, you use this proportionality constant linear attenuation coefficient right. This is to make it equal. So, photons lost is equal to $\mu N \Delta x$.

So, this has to do with the material property right. So, you can write that as μ is equal to n by N which is your number of photons lost divided by total number of photons that was sent in over the length Δx , clear. So, this is a material property ok. So, in this case we have only one material with certain value μ . So, we will just finish this part and then add to make it little more realistic ok.

So, now, we consider monoenergetic photons, narrow beam; there was only one material, Δx thickness. So, you can relate from the input output based on the loss based on the attenuation right. We attenuation was not there is a loss right N dash less than N we can write our μ ok. So, let us try to put that together. So, ΔN this N dashed minus N . What is N dashed minus N ?

N dashed is the output number of photons that are coming hitting the detector, N is the number of photons that went in. So, N delta N is number of photons that are lost is N dashed minus n which is output is less than input right. Therefore, it has to be minus n and we know that minus n . We know this N , N is $\mu N \Delta x$. So, ΔN is the number of photons that are lost is equal to $\mu N \Delta x$. So, we can quickly look at this and say there are these Δ quantities right.

How can I you know when what happens when this material Δx goes to 0. We never said, we just said Δx . What happens if this Δx approaches 0, right clear? How what do you get? You get a you can get d differential right. So, you can write d by $d N$ by N equal to $\mu d x$ or you can solve this use the boundary condition and say at x is equal to 0, which is basically where there is no material.

Whatever you send you get some N naught. N naught is your without any obstruction whatever N you send the estimate of that is your N naught right. So, boundary condition if you will right. So, N is equal to if you solve this using this boundary condition you can write N is equal to N naught e power minus $\mu \Delta x$ very powerful.

So, what you are saying is your N_0 whatever you are measuring the photon N after it has travelled a Δx through a medium with μ there is an exponential decay exponential loss right, $e^{-\mu \Delta x}$. So, N_0 ; when N_0 is sent in through a material Δx having a μ coefficient right, the attenuation constant. If that is the case what comes out is equal to $N_0 e^{-\mu \Delta x}$. So, it is decayed exponentially ok, fantastic. This is the fundamental photon attenuation law, very powerful.

This is what we will exploit time and again in not only the X-ray part. It is after all photon; photon is decaying with the material interaction. So, right so, same idea can be used for your PET also. So, projection radiography, CT, positron emission tomography, SPECT all of those we will use this concept ok. So, the idea is you get the big picture view.

Big picture view is if I can know how much I send in and if I can detect what is coming out based on that input and output my job is to get the material property μ right, clear. So, of course, there is some Δx that also we need to account for, but the idea is this is the inherent property that we are trying to capture ok. So, how we do that with specially volume all that will become interesting, but all of that captures starts from here ok.

So, simplest cases like we said N is fine, for monoenergetic case very straightforward. I can write it as intensity is equal to I_0 N is equal to N times remember each N has $h \mu$ energy. So, we can directly monoenergetic case is straight forward. I is equal to $I_0 e^{-\mu \Delta x}$. I send some intensity through this material having a property μ and length of Δx or which it is coming out.

So, then the intensity that comes out will be reduced or exponentially decayed of what went in ok that is how you read it, good. So, that is your monoenergetic case. Once important concept or important metric that usually you capture from here is half value layer. Why is that important? See here you talk about attenuation. You send something, something comes out.

This half is always important for us right, full width at half maximum minus 3 dB half power point; half amplitude point. This half is always something, right. Something is reducing.

We would like to know for whatever reason 50 percent is some salient number right . So, the idea is ok, what is the for a given material what is the thickness Δx when it will be 50 percent attenuated that is something that people would like to know ok. So, that is called as half value layer which is a thickness.

So, if I give you material right Δx ; I give you a material of lead, I give you a material of the soft issue, I give a material of slab right of bone, I give a slab of titanium, I give a slab of whatever; aluminum each of the material has its own μ . If I say their job if I send energy N right, N photons outcomes N by 2 photons, if that is the requirement do you think the thickness of each of the material is going to be same?

Intuitively not right because each one is some are more attenuating some are less attenuating. If it is less attenuating it will be probably thick because you have to go through if μ is less Δx has to be more. So, that so, has to keep I by I naught a constant at 50 percent in this case right. So, if it is more attenuating right μ is high then Δx has to be small right. The thinness of thickness varies as the material property and therefore, this half value layer thickness is a important concept.

So, it is straight forward right. So that means, you apply this definition, your output is 50 percent of your input. I have some μ ; I will call that thickness Δx as my half value layer thickness right. So, N by N naught is half which is equal to e power minus $\mu \Delta x$, but that Δx which reduces to 50 percent right, the input. The thickness I am calling it as not Δx I am calling it as HVL, half value layer thickness.

So, if you just do some manipulation right you have e , you have half, take logarithm natural logarithm right, \ln by 2. You can have this factor 0.693 by μ , clear. This is a very important concept. This you have to do quick calculation that you will see very handy. For example, without we will get to the details, but so, I want to have only my you know small portion in my chest. I need to do X-ray right.

I do not want the X radiations to go to different parts. What I can do? oh I can put a you know lead shield or whatever that will attenuate, wave more than your tissues. So, whatever N I am

sending when it has to go through certain thickness of your led it will probably attenuate and whatever after the attenuation comes in going into the body will be very minimal.

So, you can design this thickness depending on the half value layer thickness characteristic of a material. You can say if you do not have led maybe you can use some other material, but the thickness of that should be more right. So, all this can be very handy in designing how much attenuation you want to control ok good.

So, that is an important concept that you need to be very clear about. And I would not say memorize this, but if you recall this definition we should know from the word half value layer thickness you should be able to quickly relate this and in that context it does not hurt to remember it is 0.693, but just do not remember in the formulae, remember how you arrived at that is more important ok.

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Narrow Beam, Monoenergetic Photons


• Non-Homogeneous slab $\frac{dN}{N} = -\mu(x)dx$

$$N(x) = N_0 \exp\left\{-\int_0^x \mu(x')dx'\right\}$$
$$I(x) = I_0 \exp\left\{-\int_0^x \mu(x')dx'\right\}$$

>> Can be thought of as the **integral form** of the fundamental X-ray attenuation law

>> Most important physical model for projection radiography and CT. Also valid for PET


NPTEL



So, that is for your monoenergetic case. What we need to still do is kind of complete and say narrow beam monoenergetic photons that is fine. So, I am not changing the dimensions of my you know beam or the detector right, the narrow beam or the energy is still only photons with one energy that is going in. One another variable that is there is my thickness slab. So, that I varied I got HVL.

So, only other thing is why should I have only one slab, cannot I have two slabs next to each other, cannot I have more slabs next to each other? Each slab right has different material. So, I can have one lead, I can have aluminum. So, why should the path of the photons when it goes through the slab be only one μ or only one material?

Can I update my equation, right? Can I update it? When you have a non homogeneous slab, so, when I mean by non homogeneous slab the slab is one slab you think about it, but then you have multiple slabs that are next to each other. So, the path of the photon it has to go through material 1, material 2, material 3 and so on and so forth and then it exits. What we know what we have derived so far is when it goes through one material we know how to how much is the reduction.

Now, we need to when you have cascade when you have several slab next to each or what is the output? A straight forward extension would be what comes out on the first slab goes into the second slab, what comes out of second slab goes into the third slab and so on and so forth.

I know the attenuation basic fundamental attenuation law which is whatever goes in exponential of minus that material property μ , right times the thickness of that material that is the input to the next. So, that is the output, but that output is the input to the next slab. The output of that slab is a input to the subsequent slab and so on and so. So I can basically cascade this right, I mean to think about it ok.

So, that is a case we will now do, non homogeneous. So, we have d by d N is equal to μ of x . What is this μ of x ? That means, my attenuation is a function of where the length scale. So, if I have multiple slabs, each slab has its own μ ; that means, along the x μ is changing

right. So, that is how it is denoted. μ of x means which slab the x is corresponding to different slabs. As the x changes; you can have different slab right, different material property. So, you are writing μ is a function of x .

So, I can have number of photons that are lost right dN by N is dependent on μ at each location and therefore, you can write at each x that is whatever is coming at any given location what is the number of photons that is less than whatever number of photons you sent in. It is lost due to every length that is before that right.

So, whatever you sent in say let us say N_0 is what is there without any material. So, that is your estimate of your input, what you are sending out from the X-ray tube that is your N naught. At any location x , number of photons that are there is going to be whatever you sent in times exponential loss right, when you arrived at that location.

So, 0 to until that x whatever material that the photons have travelled through, it is a loss each μx dash dx dash meaning each slab has its own attenuation. So, x dash is one material at that location right. So, your μ is changing. So, sum of all the losses until you arrive at x . So, whatever you went in the input right, it is getting lost depending on the material property and the length over which 0 to x of the material property right that is your N of x , straight forward.

So, you can write that as intensity also which is I of x is nothing but I naught whatever was sent in and loss until before that. You can have multiple layers multiple slabs right. So, you have μx dashed dx dashed, this is just a dummy variable along the length ok. So, why is this important?

Is it is an integral form of the fundamental attenuation law what we saw already right, same thing is captured here, but then instead of just having one slab writing one fundamental equation $e^{-\mu x}$ or $\mu \Delta x$, if you have non homogeneous case, which is all interesting right. Interesting is; because in eventually we are not interested in only one slab with one material we are interested in biomedical imaging.

So, you are going to have different tissue materials; bone, soft tissue, fat right. You are going to have several things, contrast agent you can have several material. So, what you are sending in on one side of the body what you are receiving on the other side has to go through multiple different materials that is why this is important. and therefore, integral form right.

Integral meaning it is this form that you are talking about where you are account for different materials, non homogeneous slab, it is a very important part. In fact, whatever we have covered so far this model right of what you send in and what you receive and the model this input output relationship using this exponential law right.

This is very handy even for PET. This is how we will do. We have to do our see this is physics right. So, we need to do our imaging. We need to talk about image as a model of model capturing the physics, ok. So, this form will be very handy. We will start to write this more often not only here you know in projection radiography, but also in CT and also for PET ok.



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Linear attenuation coefficient

The linear attenuation coefficient depends on the medium property as well the energy of incident photon (E)

Photon energy (keV)	Bone (cm⁻¹)	Muscle (cm⁻¹)	Fat (cm⁻¹)	Soft tissue (cm⁻¹)
10	~30	~10	~5	~4
20	~15	~5	~3	~2.5
50	~7	~2.5	~1.5	~1.2
100	~4	~1.5	~0.8	~0.6
200	~2.5	~0.8	~0.5	~0.4

Figure 4.8
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So, let us quickly do one more right. Linear attenuation coefficient depends on the medium property as well as incident photon. So, this is the part that I kind of mentioned even when we talked about the different interactions right of your electromagnetic photon with your material interaction. What did we talk about? There is a energy range at least for the signal part. Compton scattering; we did not worry. We said you know for all the practical biomedical tissues, it is not energy dependent.

But for signal we did talk about ok. So, we did talk about that and it turns out that this is how the plot is for your attenuation coefficient or your material property that we just talked about as a function of what energy you are dealing with. These are different material; bone, muscle, fat right these are all. This is the plot.

In fact, we will stop here. Is a good point to stop here because we need to observe few things here and then proceed towards generalizing this a little bit more. What we covered is monoenergetic that is fine, but then narrow beam ok. We expanded it for non homogeneous slab, but we need to talk about perhaps; if it is not narrow beam there could be wide beam not one energy multiple energy, not one slab multiple slabs all this we need to quickly generalize it further.

So, we will stop here. In the next lecture we will actually it is a very nice material. It is all gradually building. Very nice material. This is a heart of the physics that we need to understand before we jump into each morality and say I know my signal model, I know my noise model. So, you know let me write it out and this is the image is an estimate of your unknown underlying. So, we can actually get there with a very interesting fashion ok. So, we will stop here for the moment and see you in the subsequent concepts in the next lecture.