Introduction to Biomedical Imaging Systems
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Lecture - 18
Imaging Equation_updated

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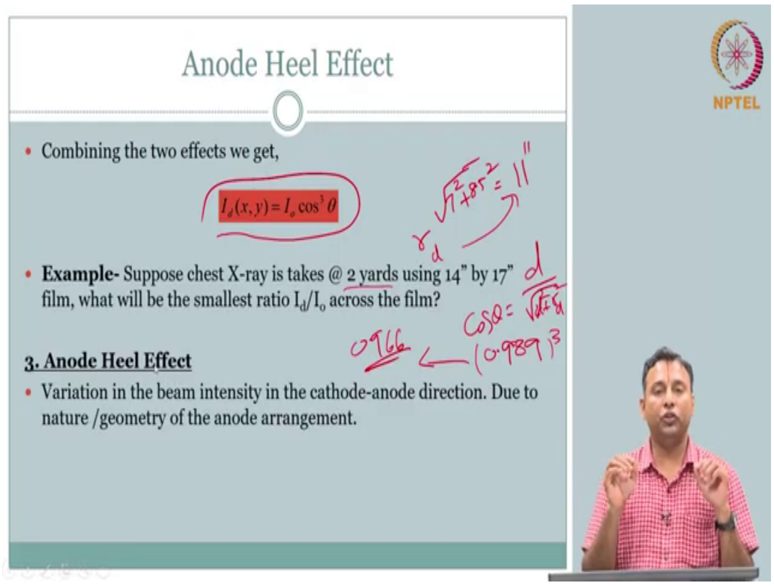

So, one of that effect that is more dominant is actually the Anode Heel Effect. What does this anode heel effect represent? Again, this is also due to geometry. Why is it grouped under geometry? Because, there is something to do with the setup geometry of the set up. But then, unlike the other two where it was the source to distance, source to object distance or the source to detector distance of that geometry.

This one is you have a X-ray tube. In the X-ray tube you have a geometry, right. You had your anode, cathode, and the anode had a thickness. So, it turns out that the electrode, so the electrons that are generated at the cathode come accelerate, hit the anode, and the X-ray comes out.

So, it turns out that the X-rays that are coming out, they are not distributed uniformly in the along the distance of your along your thickness of your anode. So, it turns out that at the anode the thick the edge, right, that is close to your cathode. There are more photons that are coming out, and as you move away to the other side, so when you look at the distribution of the X-rays that are coming out, there is a variation in the beam intensity in the cathode to anode direction.

And that is because of this anode geometry, right. Remember the thick slab of tungsten that was shown, and you had a cathode and anode. So, it is just because of that geometry this happens. So, what does this effect turns out to? Well, if this is the intensity; so, in your source only, if you have this heterogeneity, if you have a variation in the beam intensity, you are going to have that variation in the detector because the source itself is varying.

Then, how are you going to tell whether the variation is from the object or is it inherent variation because of the setup? Right. That is a bigger problem. So, it turns out that this is a more dominant guy. So, how do they affect this? How do they minimize this? They minimize this by remember filtration number of photons you can take.

We did this for example, when we change the path length, right. We talk about the example of a foot where one end of the foot near the fingers you have very little material, when you go towards your back the heel of your foot, there is more tissue coming in. So, in order to compensate for that bath length, we could have some material filtrate, so that it can be inverse of mirror image of this guy of the heel.

So, that the path length is matched. So, go review the physics, where we talked about this filtration to make sure that we do this. Same filtration we can do here also. So, in some sense this is actually a very dominant guy and this will be addressed using the filtration technique. So, going forward we will not explicitly include this effect in the imaging equation. So, we will say that before we write the imaging equation, whatever comes at the source that inhomogeneity, right.

Variation in the beam intensity along this direction is already addressed using some filtration. So, we will so; this is actually a more dominant guy. So, this has to be addressed. This will be addressed using a filter, but we will pretend going forward in the imaging equation the I s is already addressed for this, ok, clear.

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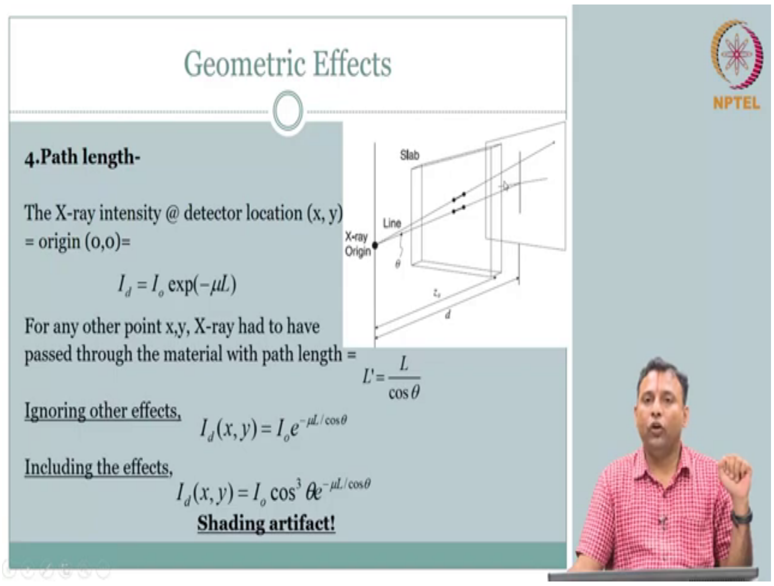

So, now we will move on quickly. So, that is with respect to the source. So, what are the other geometric effects that we can think of? We talked about everything so far without having a objective. So, now, we need to incorporate an object. Even here we are going to incorporate a simple object because this is a slab that we used in our physics also, right, when we covered the path length, change in attenuation, mu delta x.

So, now, you look at it if you have a slab, right, how much the other geometric effects? Because we have a slab structure, what is going to happen? The X-ray that is going along the center is going to go through a path length of object thickness. Same material, but then, what is going to be at the origin here? That is going to be attenuated by the path length that is only the delta x.

Whereas, what is going to happen on the detector point here? That would have come along a line that would have crossed a path in the slab not the shortest distance not the thickness of the slab, but it is going through this path which is more than your thickness because it is at an angle, right.

So, clearly you have a path length effect. So, the material is same, the source is same. So, you have addressed the issue of variation in the beam intensity at the source in the anode heel effect. So, now, you are pretending all of the photons that are hitting, right that it is uniformly illuminating on one side of the object.

But what comes on the other side of the object, not only is determined by the mu, because the slab is supposed to be in this example, supposed to have the same mu. But what you see on the detector is not going to be same. Depending on the detector location it will view photons that have come through a different path length and therefore, different attenuation, correct.

So, how do we incorporate that? We can just say I d is equal to I naught exponential of mu L. This we know. This is only when it passes through the thickness. So, thickness here is cap L, ok. So, material with mu. So, mu L, exponential of minus mu L is your I d.

However, what are what is it going to be at other location? Other location you know from this geometry that L whatever you want to call that length through the material we will call that L dashed that is going to be greater than this L, correct. You have some theta, so you can write that.

So, for any other point x comma y on the, right, you are going to have a path length which we call as L dashed, it will be L by cos theta. So, now, it is straight forward. So, that means, I need to somehow write at each location I am going to have L dashed is the path length. L dash collapses to L if it is 0 degree, otherwise this is how the cos theta is going to influence your path length, and that path length is going to influence your value at the detector using the basic law, clear.

So, we can write, therefore, ignoring other effects, the other effects that we have covered so far without the object, right. If you ignore all that the only variation is going to be due to your path length variation. If that is the case then you can write your I d at every other location x comma y as I naught whatever is sitting at the center and then decaying e power minus mu L by cos theta, so L dashed, mu L dashed, right. So, that is how it is going to vary.

So, now, if I have to incorporate the other effects what is going to happen? Well, we will pretend. The effects that we saw already is irrespective of the object that was there because of the geometry without even considering the object. So, we can think of it as if you have the object and the thickness of the object or the path length is going to affected this way, the other geometric effects are just going to add on, right. It is going to be a multiplicative effector.

So, including the other effects we can, other effects was I naught cos cube theta. So, we are going to have I d, I naught cos cube theta is what we had before, now because you have a material, you are going to have material property, and the path length in that material is going to be incorporated as this guy, mu L by cos theta.

So, now you see the problem. The problem is even if I have the same slab with the same mu, what should be my detector? This is my unknown; this is my f of x comma y. What should be my g of x comma y? That should have the same attenuation like mu value throughout, you should get an estimate of that mu or the attenuation should be same, whatever lost is going to be same throughout the detector phase. That is what is a good estimate of your f of x comma y.

But, what do you see? You are going to have a high value at the center, as you move away from the center the value is going to reduce because your path length is increasing. So, what should be supposedly homogeneously one color, say for example, homogeneously white; white being the highest value. How is the image going to look? It is not going to be homogeneously white. It will be white at the center. It will start to grey out as you go towards the end, right. It will look like you have a shadow, right. So, it is called a shading artifact.

So, this is very important, whether you are going to interpret; see the problem is in reality the object is unknown. Are you going to interpret, if you take an image? I take a perfect sample of a bone and, right, I am going to do this. And I get a shading artifact. Am I going to interpret that actually the material property is varying for that small bone that I took from whichever location.

Are you going to you know interpret whether that bone is actually having this variation in the material property or no the bone is supposed to be homogeneous; with material property is same, it is only the artifact, right. So, this is very tricky, ok. So, this is where the domain expertise comes.

So, the people are involved the doctors who are doing this and the lab operators who are handling this, they are always in communication, the biomedical engineer is also in the loop. So, they know which system, how long they have been using, are they seeing this with the you know application, all these experience that are not caught.

So, if you are going to take; nowadays the trend is I will take this analysis, I have a computer train a model, the model will tell me, right, fine. I mean all that is fine, the computer can tell, it can look at this numbers and it will say that it has varying. But the interpretation is not as simple because so much of domain knowledge goes in to interpret, to avoid the artifact being interpreted as actual variation in the material property, ok.

So, this is a very important effect that needs to be incorporated in the interpretation. So, this is also due to your geometric effect. So, what we will do now is quickly move forward, ok. All this so much for geometry. Is that all? Is there any other problem that we are anticipating? Is there any other aspect of the instrumentation that we are doing this? That is going to come and affect our image. Beyond your actual data, actual object f of x comma y having a variation.

In other words, your g of x comma y, what are the variations that you are going to see that are actually not due to variation in f of x comma y, but due to this system setup, ok. That is what we are going after. So, first is this geometric effect.

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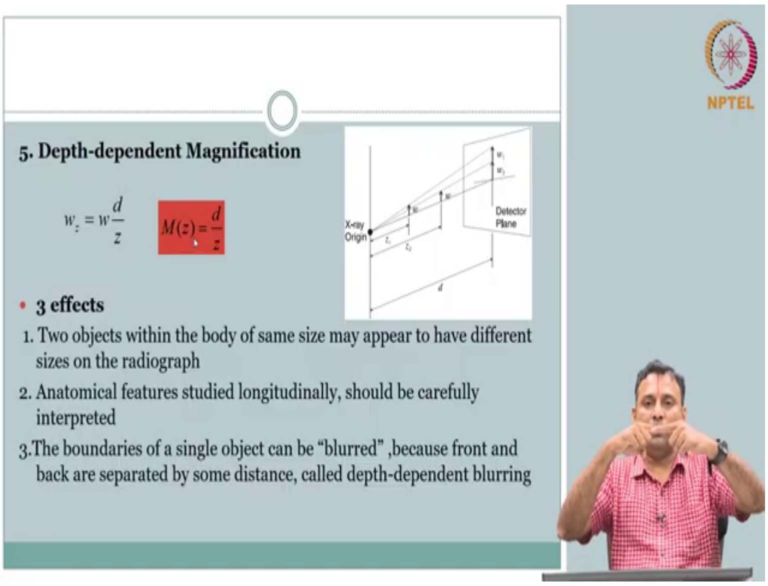

Then, we will say fine we have few other things, Depth-Dependent Magnification. In here, we are this is something that we probably touched upon even when we did as a, when we talked about image quality, and accuracy, remember artifact, and that is that lecture, I said your geometries could be big or small depending on all you know this magnification. So, here is a more formal way to do it.

So, now, you have your X-ray source, you have a detector plane. Where is the object presented? Of course, for convenience we are presenting as stick object, not a not to x comma y, which is what is going to be in reality is going to be x comma y comma d, there is a depth also for the object. But to get the idea here in one-dimension you see if it is going to be a stick, depending on where you are going to place this, right.

If it is of w height depending on where you are placing if it is z 1 or z 2, right. If you remember, this is a example I think I did my; if you want to measure the size of my hand there is a ground truth, right, I am going to measure. But if you are going to use this image where I do not have any scale that you see on the screen, my hand is now big, my hand is now small.

At least the number of pixels that are occupied by my hand is less here, is more here. If I do not have any other calibration factor with of account for this magnification when I move front, then my object size is dependent on where it is presented between the source and the detector, right, very intuitive.

So, here if it is at z 1 you get w 1, if I move to z 2 location I get w 2, move close to the source away from the source, ok. So, you can write this. Your w of z is nothing but w of z is nothing, but w. So, this is a ground truth value, this is your magnification. And the magnification factor is therefore, d by z which is your similar triangles kind of approach, right, so d by z. So, why is it written as M of z? This is depth-dependent, depth in r, x, y is our imaging plane. So, z is your depth direction in this schematic and so M of z is your d by z.

Make sure that you carefully register that this is capital M of z which is a depth-dependent magnification. We will quickly have one another magnification that we will talk about. Do not get confused. So, here we are talking about object is a stick object, right. If it is close or further away the magnification is altered by d by z, ok.

So, depth-dependent magnification is an important factor, right. You do not want to make a measurement on the image and advice, ill advice that the tumor has grown big or whatever. So, this is a very important factor. Why? I mean. So, the crudely that measurement is one thing. Is that all? Is there any other effects because of this problem is there any other problems that you see in the image because of this effect?

There are 3 other effects that come because of this problem, because of this depth-dependent magnification. One is we talked about. So, two objects within the body, right depending on a same size, but they can appear at different size depending on where they are. If you recall our introduction slides in image quality, we had a circle transform to ellipse or two ellipse showing one as circle, the other. Remember we talked about two effects depending on where they are front of the body or back of the body, right depth.

So, depth-dependent magnification two objects within the body of same size may appear different on the radiograph, just because of this. Same tumor, one I have in the front of my chest, the other I have at the back of my chest, they both will appear of different size because of this effect. That is one effect. The other is again, this is a more challenging one. Anatomical features studies longitudinally that is I go here, I take a you know chest X-ray for example, or abdomen X-ray, whatever, I take when I am say 150 kgs.

And then I go after do some treatment, dieting, all those things I slim down tone down to say 75 kgs, 80 kgs. I am the same person technically my id and all is same, but then next time when I go for the imaging after 6 months or a year, probably because the weight is shed the location of certain organs would have moved front and back, right.

And so, if I have to go now do a X-ray and I do a measurement, I cannot say this this is grown big or this has grown small in relation to my time 0 when I went first time, when I was 150 kgs. You cannot use that and say, now it has come down. Maybe that is not physiologically, it has not come, it just that geometrically I have toned down, right and so the object distance or the tumor distance within the object have changed.

So, we will have to be really careful when you interpret. So, that is what I mean is this depth depend, you would really do not know its inside we want to see what is there inside non-invasively from outside that is our big picture.

So, if I know the ground truth no issues. The problem is I have to interpret from what I see what could be the ground truth, ok. So, if I have something like that that is where the domain knowledge, domain expertise, other medical reports, all these are very important. So, it can go to cause misinterpretation if it does not carefully done.

Then, is this big picture. This is a bigger problem. Boundaries of single object can be blurred. How is that possible? Because you have a front. So, you have a 3D object you have a front and you have a back, right. There is a thickness. You talk about the slab, right.

Now, what this is saying is because the front face of the slab is at one distance and the back face of the slab is another distance, intuitively based on this magnification that you have seen, if it is the front end is more close to the source, the back end is away from the source. So, from this magnification, what do you what are you going to see? If you are moving towards the source, it is increasing, right. If you are moving away from the source, the on the detector, the size is reducing.

So, if I have a object which has a front end and a back end separated by a distance, the front edge is going to be grow, the back edge is not going to grow, it is going to grow less, right. There is going to be some magnification unless this is on the detector. Again there is going, but there is this differential magnification because where it; so, that means, the two edges probably can come close, right or what was same and if you project it should have been a line.

But these two are back one behind the other, but because of the depth-dependent magnification one is going to move up on the screen, on the detector. So, that means, one edge is now blurred, right. You going to see a blurred image of the edge, because front edge and the back edge of the slab are magnified differently or projected on this height differently because they are at different depths, right.

Very, I mean I think you should do this yourself to interpret it. Of course, I will spoil that imagination in the next slide showing how it will be. But, I think you should do it to convince yourself that it is pretty straight forward actually. So, only thing is you have to account for it which we will do. But the effect itself to appreciate it is not difficult to imagine what it is.

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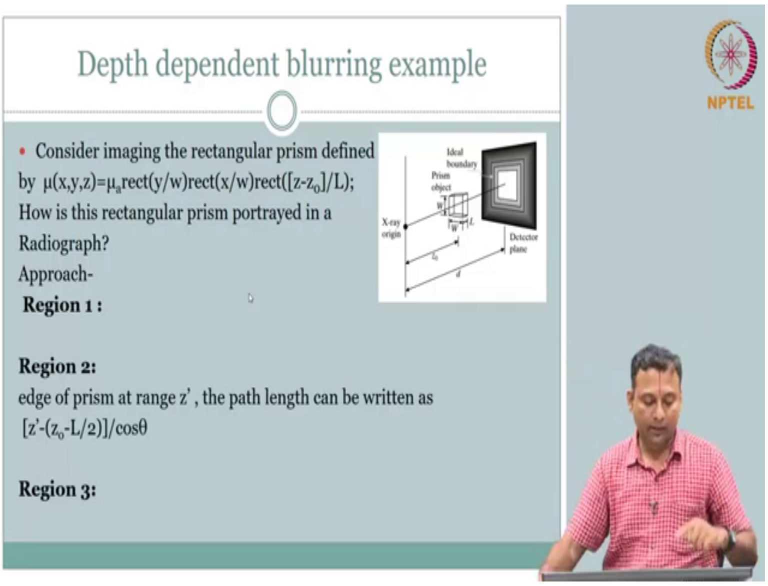

So, what we will do is; I will spoil your imagination. So, here is what that slab is going to be. It has a width, height, right, thickness. So, supposedly, it is the object is going to have same mu, so ideally what should you what you should get is when you project this rectangle, only that phase should have the same value. So, you should have only this, right. That should be our ideal boundary.

But what is going to happen, yes, because of this depth-dependent, you have your front face, you have your back face, they both are separated by some distance L which is your thickness. So, each one is go. So, therefore, because of that what is happening? The front face is at different location with from the source, the back face is going to have a different distance.

And therefore, you are going to have different magnifications. We will write out what we wrote. And so this is how you see the, so you are going to not see a sharp edge, you are going to see a fading away, a blurred edge because of this reason. So, the way we can look at it is, a image consider imaging a rectangular prism defined by, so essentially I am saying this is my object.

Remember silently what I have incorporated here. When I talk about object thus underlying f of x comma y, I am not talking about, I am talking specifically about the object being a distribution of your attenuation coefficient to X-ray, right. Because you are dealing with X-ray imaging, we are talking about the material property of the f of x comma y, the meaning of f of x comma y is a variation or a distribution of mu which is a function of x comma y comma z. So, it is a spatial distribution of mu that is what I am, ok. That is what this prism is.

We are interested in the attenuation property which is distributed in this prismic format. So, it is mu a times defined by this boundary condition, width, height and depth which is in L, ok. So, how is this rectangular prism portrayed in a radiograph? So, it is going to be portrayed like this.

So, visually you can see what it is and you probably can appreciate the effect, but we need to write this out in terms of the imaging equations that we already have, ok. So, what are the 3 regions that you are going to have? Straight forward, no problem in the region when you have this go through the entire prism. We already wrote it. It is going to be that exponential with different path length, right, e power minus mu L by cos theta.

So, when it is passing through, when the X-ray is passing through entirely through the prism, the front and the back, then I know already how to write my imaging equation which we wrote already. What is new here is, I have front face I have a back face, so the diverging X-rays are coming. So, it could happen that, so this is my back face, this is my front face, X-rays are coming diverging, right, X-rays are diverging.

So, it could happen that and here is my detector plane, I have my detector plane. So, it could happen that the X-rays are coming, front, it is entering the front face, but it is not going till the path length complete, it is exiting in between. And then, go hit the detector. You are still going to have a value there because it is causing a smaller path length, right that could be one region.

So, there is a partial region where it enters, but not completely goes through till the back end. It is exiting before that. That is one region. So, first region is no issues. Everything is going in coming out through the other face, inlet, right, front face, back face. Everything goes in the front face, comes out to the back face, you will get an image which we know the equation already.

But then in region 2, which is complicated is it goes in, but exits, because of the angle, right, it exits before it reaches the end of face. But that is also going to have a observe with the detector plane, partially. The third phase is not, third region is not a problem. It is diverging that it is not even seeing the object. It is just going and hitting the detector screen. So, its other field of you without the object is not there. That is also straight forward. Without the object we already wrote what will be our variation, right.

So, and this one we already wrote, I naught cos cube theta e power minus mu L by cos theta, no issues. The problem is this one edge of the prism in is in this direction, right. The path length can be written as z dashed minus; so, essentially I am going to write my path length in terms of take for example, the center of this L by 2, right. This is L is the thickness, I can have L by 2 is a center of the thickness. So, I have the front face is at z naught, so I can write any location z dashed along that L, right, along that length.

I can write the path length to be z dashed minus of this guy by cos theta, ok. That is your path length. And therefore, that is the path length, and your I d is going to be everything else is same, only the path length is going to be changed here for the formulation. So, the third region no issues. You do not have any object. So, it is only your I naught cos cube theta.

So, this is an important you will find in kind of problems home works where you have to identify given this x comma y plane, right. Where are these? Region 1, how do you describe in their coordinate? What is the range over x? What is the range over y? When this region is applicable range over x and y when region 2 is applicable, right. Here you do not you just see that x and y, this is the variation, and you have the equations.

But, what is the range when it transitions from r 1 to r 2 or r 3 in terms of whatever geometry that you see, those things you should be able to work out, ok. But intuitively this is what is happening. And all of that is one step geometry only, all your triangle, similar triangles, sin theta, cos theta, tan theta, right. Those things you will have to use to describe the range, ok. So, much for depth-dependent blurring. It is a very important one, right. So, here you have the object which is 3D.

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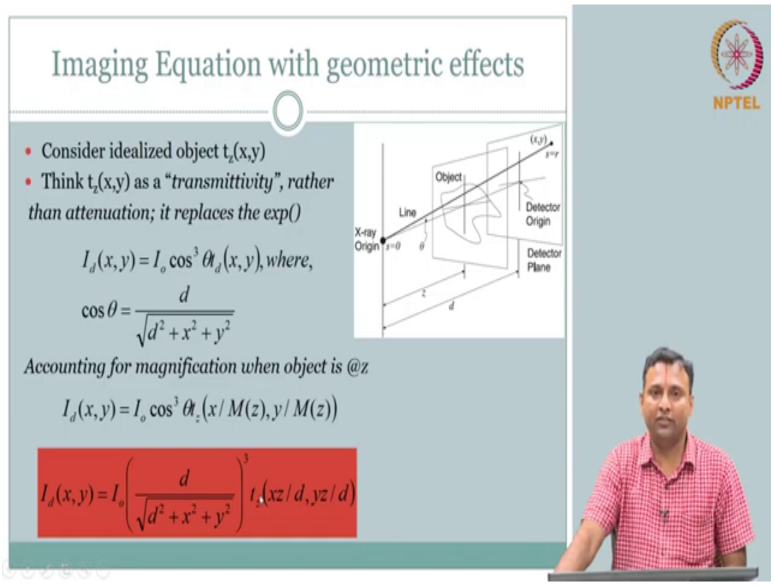

So, now what we need to do is, ok all this is fine, right. Now, we need to we had a imaging equation, individually we have incorporate. So, how do we incorporate all the effects we have seen so far and also instead of having a see, right now when we talked about object we simplified it as one mu with one thickness. Generalize it, what do we want?

We want an object which has different distribution of mu, right. Mu is changing in x, y, z, ok. Not only that, in other words that is changing, what we are interested is change in mu and the change in thickness, both of them contribute to whatever is coming out attenuation. So, how do I make my life simple? So, instead of talking about three-dimensions, we can conveniently collapse the object to a two-dimension and say instead of writing it as e power minus mu delta x, right.

I can make that whole e power minus mu delta x as a factor. So, I can have a ideal thin object where the distribution is mu delta x, right. Mu into delta x if you have third dimension we had mu into path length over it. Instead of dealing with that, I am collapsing that I am saying I have a thin object, infinitesimally thin object where the object is described by the total loss, not just attenuation coefficient, right, which we will call as so consider a idealized object.

Why is it idealized? Because it is infinitesimally thin, it does not have a thickness. And the mu, instead of just varying the mu we are saying that object is actually mu into delta x. So, that is the value that you see as the variation, ok. So, think t z as transmittivity rather than the attenuation, ok.

So, this is very straight forward. In some sense, you can think take a transparent sheet, just analogously, take a transparent sheet and if I, right this is optical not X-ray. If I write using the optical marker, what is going to happen? The marker wherever I am writing is going to attenuate the light that is coming, and that is what you see in the projected screen, you will see what I am writing as a shadow, right. So, it is similar.

So, in here maybe you can think about this as a infinitesimally thin sheet where I have something like a attenuator that is placed the whole thing as some points. So, object pattern is there in terms of mu, and the depth that collapse into it. So, in some sense, it is like inside it comes I have this transmittivity function, so depending on the transmittivity function what comes out is determined, ok.

So, in in our thing, instead of exponential minus mu delta x, we can replace that, right. This is same. I naught cos cube theta, your exponential of minus mu delta x, right that part is replaced and convenience, we are calling it as t d of x comma y, where your cos theta is like before I cos theta is nothing but your d by d square and on the x y plane. The d is your distance from the source to detector, clear.

Why is this maneuvering done? Because it becomes easy to write. What write? Account for one more magnification [Laughter]. Remember, we talked about position magnification remember. We talked about depth-dependent magnification. Now, we are talking about you know front is magnified, back is magnified, remember that example of thickness that we gave we are going to incorporate that, how do we incorporate that. So, it becomes easier to use this denotion.

So, account for magnification when the object is at z, we can upgrade the equation to I d x comma y, I naught cos cube theta. Instead of t d x comma y, we are writing as t z, what you see at the detector is whatever is there at the z location, the object t z, right, object is t z that is here magnification. So, what you see at the detector is a magnified version of what is there at location z, clear. So, very that is why we wanted to collapse it into ideal object, so that it is easy to write this out.

So, now, you see one more incentive that you know if the object is on the detector plane, right then you cannot worry about your magnification, otherwise you are going to have that effect. So, one of the incentives is when you go for a chest X-ray, they would like for you to go back and be as close as possible to the detector. This is one of the reasons. They want to reduce the blurring, right.

One of the reason for magnification is what? Depth-dependent magnification is blurring. So, you want to reduce that blurring. How can you avoid that? So, one more incentive to be as close as possible to the, there, of course, there is all trade off, but these are certain things that are exploited. So, you can write your I d of x comma theta, right as I naught. Your cos theta is written here, so d by square root of these.

And your M of z, right if you recall we wrote it as z by d, right. So, you can write. So, this is your full imaging equation incorporating the depth-dependent magnification as well, ok. Are we done? Are we done? Is the imaging equation; now, it seems like we have addressed several aspects, right without the object, with object, with object, with the depth-dependent magnification.

Is there anything else that we need to worry? Well, I think so much so far geometric effect is good, but it is still not completely done. Because look at the image that we have here. We talked about these distances, we talked about the object, what we have not really talked about now is this a good approximation. You have only a point source that is sending out diverging beam.

Is this really a point source? If it is not a point source will we have any effect? So, if instead of this being a 0 point geometry, if it has if it is not infinitesimally small, right, if it has some finiteness does this geometry of this finite aperture or source, does it have an effect on the image? Right.

Last but not the least, we will also have to address, we have talked about imaging plane imaging detector plane, but we know what is happening in the detector plane. You have a film screen, intensifying screen, and then film. So, is there any effect because of that instrumentation on the image? These two we will have to still see.

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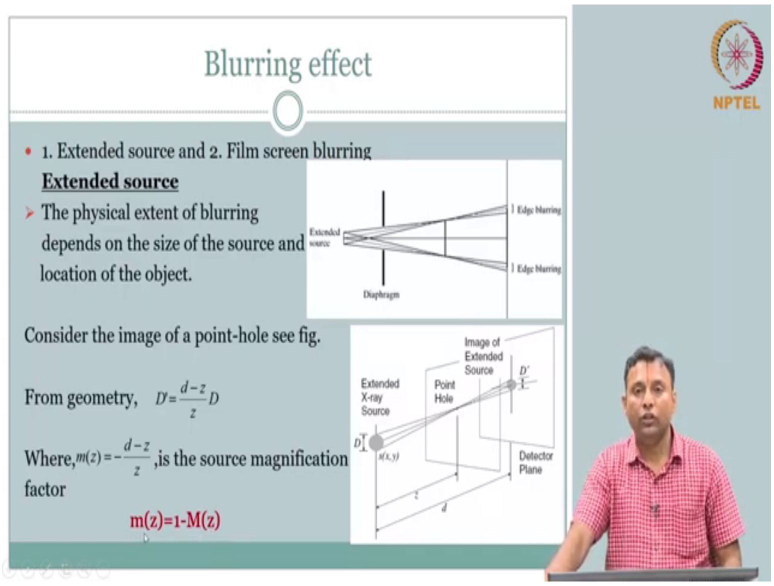

So, first is extended source. In extended source, as the name suggests it is not point source, right. You have a source that is extended. How do we tell that? Well, if you have I mean in some sense you can look at it. We talked about diverging wave, right diverging so far. So, there is a point and the source is a point, source and it goes at different angles, mostly in the cone shaped is what we are talking about.

So, now, we need to account for what if that is not a point, if that is I mean just think about it, if it is not a point how what else can it be? It can be a line, right, it can be a area. So, instead of a point it can have some geometry, ok. The physical extent of blurring depends on the size of the source and location of the location of the object. So, this is the size of the source.

Now, the question is magnification we already talked about some sense location of the object is there, and there is a blur that is happening because of where the locations. So, in the image that you see, if you see blur, right, it could be because of the object dependent magnification or and or because of the source not being a point source or source being extended source.

So, let us look at what do we mean by that. Extended source mean, so in this example, right just because we are projecting it, instead of a point I have a line, right. So, I am interested in the case where I have an object, this is my object. So, instead of object being hit by a point source that is diverging, when you have a extended source, extended the denotion means the source is not a point it extending in one direction.

The dimension is extending in one direction, right here. So, it become a line. So, here what happens vividly? Vividly, you see that you could go from here, you the same edge is caught here or it is caught, right caught from here. So, one edge is caught by the extended source. Instead of ray emanating from a point and that is being the only ray that is capturing the edge of this object, now you have the whole extended source all of these points go capture. So, what you see is the net sum, right.

So, in some sense that is what instead of one edge you get a blur. So, it is uncertain over this region. This region is, you can see here is dependent on the extent of this source. So, if this was a point source you would have had a point here. Now, that the source is extended your object edge is spread over some length, right. So, your edge is called as edge blurring.

So, clearly there are two things whether this edge, so this instead of this one line that you have we saw that if the object is that the rectangular prism, we already saw depending on where it is located you could have a blurring of the edge in the image. Now, if the source is not a point source also you could have blurring. That is what this means. So, how do we. So, we can see the effect. So, this is a problem, right. So, how do we account for this? Right.

So, we will write this first. Consider the image, so instead of doing a edge now, let us characterize this, right. So, we are going to say let us have a point hole, meaning my I have a object which is a point, ok. So, you have a hole through which you can see through. So, now, my source is not a point, my source is a extended source, but my object is a point object, ok.

So, what happens is this is my source. So, it is a circular disk of area instead of a point source, now it has a finite extent, and we have capital D is the diameter of that for example, of the source. But my object, there is no object. My object became point meaning I have a point hole only through which this has to go, right and then I have my detector.

So, now, what do you notice is you are going to have this source is finite dimension, capital D. Going through this point, we talked about, right, this guy going from the bottom and the top, so, this is going go through the hole come here, this is going to go through the hole come here. So, what are you going to get? You are going to get a source, right.

In the detector, you are going to get a version of the source, a copy of the source, of course, it is going to be scaled, right. It is not going to be of the same size. It is going to be scaled. And one more thing is yeah, it is going to be scaled D dashed is; then one more thing is to do with your see you are going to interpret your what you see on the image on the detector as essentially your object, right f of x comma y.

But what is this object? Object is supposed to be point. So, what are you going to see here? This is not point, this is a spread version of the point, right. Start to think about it, right. But that is not the case. So, I have a point here, my source, I have a copy of my (Refer Time: 43:55) scaled version of my source, but my object is just a point. So, what I am recording is a some version of the source, even though I would interpret it as a version of the object, ok.

So, from the geometry that is given here you could actually write out what is going to be D dashed. D dashed dash is going to be this d minus z by z times your capital D. So, what is this d minus z by z? Right. You can you can write that. You can write that as your magnification, source magnification factor. Remember, when I said capital M of z what was that? Depth-dependent magnification. I told you will get confused that we will use another m, but this is a small m of z. This is also you know dependent on z, but this m of z is your source magnification factor.

The magnification that is occurring, right what is; so, the object is supposed to be a point, but it is magnified and that magnification is because of the source, right. So, that is why it is called as source magnification. So, m of z; so, your D dashed is this guy d minus z by z of this capital D. So, your source magnification factor is minus of this guy.

So, what does that physically mean? Well, you look at this. You have your D, you have your D dashed, both of them look like a disk. So, here it is difficult to appreciate. But then, what does this negative physically mean? If I draw this the top, right this line, where it did go? It went through the it went to the bottom. Where did this go? This line is going to the top.

So, that means, on the on the detector this is a inverted image, the top gone to the bottom, the bottom is gone to the top. So, it is a inverted image and therefore, this negative essentially says the m of z is magnification due to the source, source magnification factor. But this negative essentially implies the magnification happens, but it is a inverted image that you are recording, ok. So, that is your effect due to your extended source.

So, now, we will write depth relationship between depth-dependent magnification and your source magnification factor. So, we can have this relationship because it is all in terms of d and z. So, you can manipulate one in terms of the other. So, our objective is extended source has a effect which leads to this magnification. This can be written in terms of your depth-dependent magnification. So, all of the equation in your imaging, we have already written, right. So, we can just go quickly upgrade those equations.

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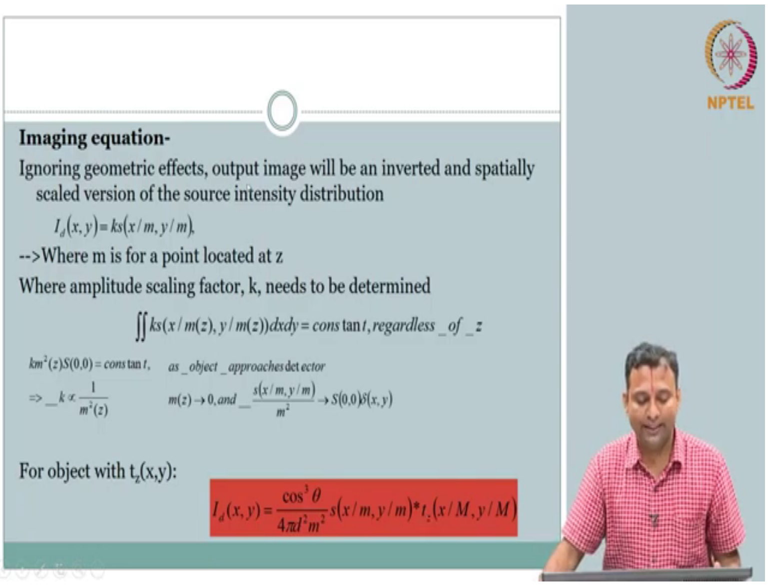

So, we can first write your ignoring other geometric effects. We already see, right. It is a inverted spatially scaled version of your source intensity that is what we saw. So, you can write your I d as scaled amplitude scaling, k of your source, which has this magnification is incorporated x by m and y by m, nothing fancy we have not incorporated the other of that can be incorporated. Just because of the source, extended source idea, this is how your detector is incorporating that. So, where m is the point located along z, wherever, ok. So, this is fine.

But what do we need here? We need to find what this k is. What could be this k? What do we know about this? We saw the disk. What is clear is, you are sending the same number of photons, right, I is same. In the point depending on where it is in the plane, the size on the detector is increasing or decreasing, right.

But then if it is increasing the intensity will go down, right inverse square law, but the sum of all the intensities in this detector area should be same as sum of intensities that came in, right. What went in came out; only thing is it is spread out depending on the magnification factor.

But if you take the net sum over the area that has to be same, assuming there is no other loss and other things, right. So, where the amplitude scaling factor needs to be determined, right. This is fine. This has to be determined. How will be determined? We will make use of the fact that if you sum what is falling on the detector, right your intensity, double integral because dx dy, so ks x by m of z and y.

So, if you integrate this that integration is a constant irrespective of; so, if you are coming close or going further away from the detector, right. That does not matter. Magnification is going to change. But based on this my intensity is going to be reduced or increased. So, as long as I sum all of them what is falling on the detector that is going to be same, ok.

The sum is going to be same. However, this itself can be different your k, your amplitude scaling factor is also dependent on depth, but this double integral which is you serve whatever falls of the detector, if you sum that that that should be same irrespective of how much of the detector is eliminated depending on a magnification, right.

So, where amplitude scaling factor needs to be determined; before we determine k, we can use this fact and say that this integral is a constant irrespective of your z. So, if that is in spatial domain, right, Parseval's, you can take the Fourier domain you can have this is to be a constant, right, k m square z of your s of z should be a constant. So, if that is a constant you can kind of see that you can have your k is proportional to like, inversely proportional to your square of your magnification, ok.

Of course, best cases, if your object approaches the detector. What is going to happen? If the object is approaching the detector my magnification is going; if I am on the detector I do not have any magnification, right. So, if the I am not going to have any magnification. So, if approaches the detector, we can write this m of z tends to 0; that means, it essentially you are saying that you have a delta at 0 comma 0 whatever is there, right. So, s of x by m y by m square which you can write it as everything is packed in the delta function.

So, in this sense, you can essentially take the effect of your magnification factor and depth-dependent magnification factor, right. So, m of z captures one in terms of the other we can write. So, we can get a k, if you get a k we can; so, we will, what we will do is we will just incorporate because we had our object, we really did not have a object, we had a point. I want to incorporate this extended source effect and other things for a object. What is a object? Easier object that we talked about is idealized t z comma x comma y

So, now, again the beauty of our simplification of the systems, what did we say? We said linear systems. What is the advantage of that? If you really look at it I have a point response, right my object in this case is one point.

So, one point I have a response. Now, my t of z of x comma y is nothing, but a collection of points, object is a collection of points. So, if one point I have the response, what will I have when I have a collection of points which is t z of x comma y? Convolution, right. We will convolve right. We have a point response then input convolve with system gives your output.

So, we will have I d of x comma theta, other things we have incorporated, cos cube theta by 4 pi d square. So, we have incorporate or also your inverse square law, right; m square s of x by m scaling magnification convolved with, so this we got for point therefore, we are convolving it with your t z of x by capital M y by capital M. So, for one point if you do that we can treat this as convolution, clear.

So, t z, you see the advantage of using t z. We have just written the object. So, its convolved with the object. All these effects are incorporated, ok. Is there anything else that we need to cover? Yes, there is one more aspect. This is only the source that we have covered, right. This is only the source. Is there anything else we need to cover? Yes. In the setup that we have, we are covered about the geometry, the object, the source. What about the detector? What falls on I d we have, right; we have come this far.

But this is not what falls on the detector is not what you see. What you see is after it falls on the detector there is a film screen that happens and then you get a printout like your developed film that is what you see. So, is there an effect due to the geometry of the detector or the not the effect of your detector; how do I incorporate that in the image that is actually viewed, right.

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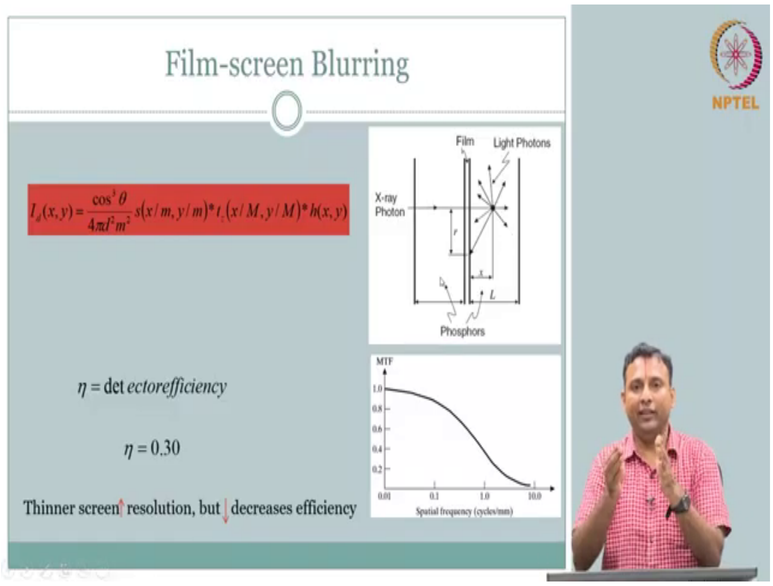

So, here, the only aspect that is there on the detector is your film screen, otherwise there is no big deal. So, we covered film screen. So, in the film screen what we talked about is you add a X-ray film and on both sides you add phosphor. Remember the cascade packing. So, you get the X-ray photon, X-ray photon comes, it generates light photon. So, here you have a one

X-ray photon is interacting, creating several light photons, and these light photons go and hit the film, clear.

So, in some sense just by looking at this proposition, what should from a systems point of view, what you should start to think is film screen blurring, blurring, blurring, blurring. What did we (Refer Time: 55:41)? We talked about point spread function, right. Is there a point spread function due to your film screen. You look at it, I have a point. Technically, this one X-ray photon is interacting at one point and that creates, so that is the same location where you want the X-ray photon is coming. The X-ray photon is coming through some x comma y of the object that is what I want to recall.

But then, the X-ray photon is coming through that location in that object, but when it comes and hits the detector, right your intensifying screen, that one location now creates photon light photon that goes in every direction, clear. So, I want to know where that one location is. But now what happens? That point is now spread on the film. One point is not going to be the point alone; it is going to be spread out. So, one point is spread out. So, you have a point spread function, right.

So, if you have a point spread function of this sub unit. If this screen, film screen subsystem, how can I incorporate that into my imaging equation? Cascade it, convolution, ok, convolution. So, if I have my screen, film screen subsystem, if I can characterize the point spread of this, then I can update my imaging equation. So, clearly you can see the effect of this geometry, right. The thickness of the film screen has a important role to play, correct.

So, you can update your imaging equation with just this convolution of h of x comma y, h of x comma y is your screens response, your film screen, right your intensifying screen that you had this combination that is what it is, ok. So, I can update this h of x comma y.

And if you have this system, we already use this idea of transfer function, modulation transfer function. So, you can characterize this film screen that is what I said, right. You can characterize means you can capture the system performance h of x comma y. And we already saw that this kind of plot which we call as modulation transfer function can be used to capture the system h of x comma y, right. So, this is how it typically looks for the film screen.

So, what happens is here, the detector efficiency is about 30 percent. So, if it was directly X-ray it was about 2 percent, right. Original, when without any film screen, when we talked about this in you know instrumentation of film screen, the idea of intensifying screens, eta is equal to 0.3, 30 percent only efficiency, even this.

So, how do I; what is my trade off? If I have thin screen, what is going to be good? If it is if the screen is going to be thin, vividly you can see if the screen is thin, this point will become closer to the film; that means, it can spread less, right. It can spread less.

So, my point spread function will reduce if this is coming closer. If it comes closer, that means, thin. So, thin, if I reduce the thickness, thinner the film I will increase my resolution because point spread function is related, right is measure of your resolution. However, what is going to happen? If it is going to be thin, that means, I am going to have less material.

If I have less material, the number of X-ray photon that has to be absorbed and converted to more light photons that efficiency is going to get reduced because I have less material, right. So, it is always a tradeoff; what do you want to play, am I more bother about resolution or am I more bothered about signal or your contrast noise, right. So, this is always a trade-off, ok.

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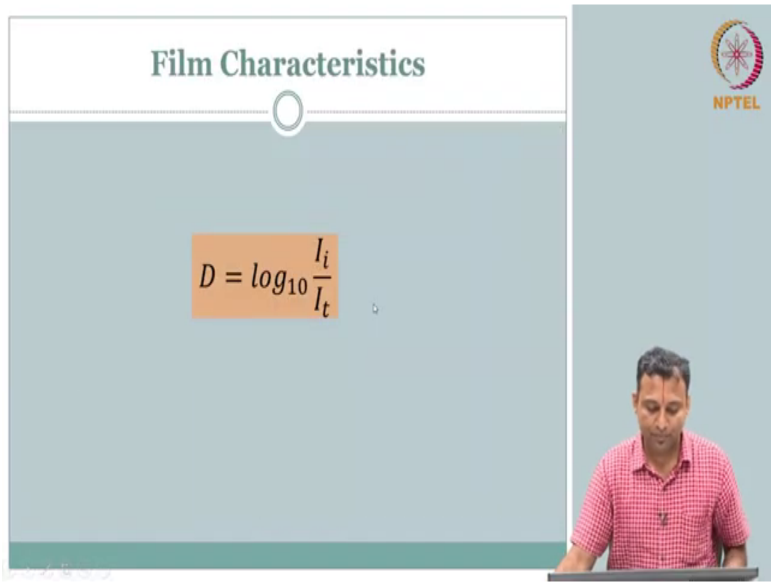

In the next lecture, we will kind of complete the effect of your detector and how do we interpret the image. In the end; see, what we have covered so far is what is falling on the detector and what effect it has the other aspects.

We still need to complete the imaging with this the interpretation that is after it forms the detector, and after you have some transformation that you have done here what happens. The image that the doctor sees, right to interpret, what happens from this point to that point because in the end image quality and how he is going to perform the doctor is going to perform is a is the real you know impact factor.

All the other things that we have covered is mathematically fine, physics wise fine, but that part is a key part, how do you interpret the image based on after it goes through all this, right, is there a you know is there something that we need to understand, from this point till what actually the doctor is seeing that we need to account for, ok. So, that we will do it in the next lecture.

Thank you.