Introduction to Biomedical Imaging Systems
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Lecture - 23
CT Back projection

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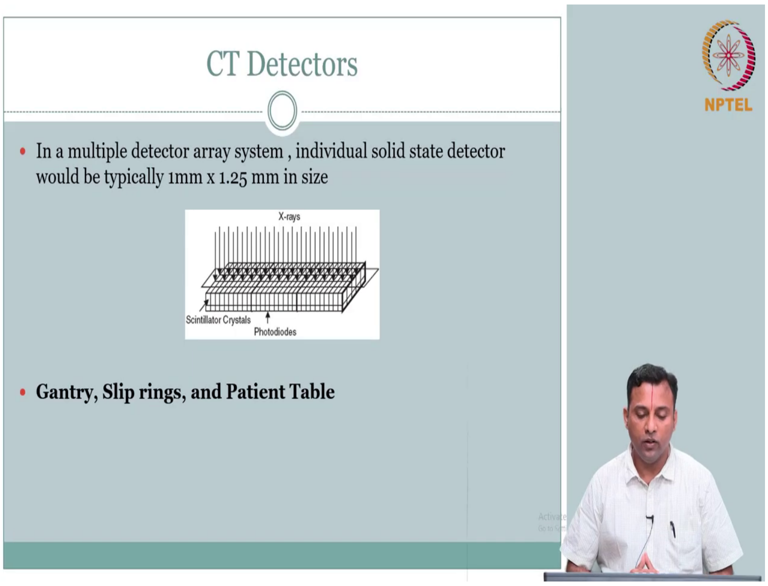

After completing the instrumentation related to CT, now we are all set to move on to the image formation. I would recommend this and probably the next lecture together, this material that we are going to cover could turn out to be one of those you know really really important, yet very interesting right.

In fact, I after going through this, you know did not really have a formal orientation to this course until I really started working in the field right. So, when after your basic engineering, irrespective of whichever program you go through right, this is something that is like so interesting and so powerful, yet we are not really introduced to this topic ok.

We all are introduced to Fourier transform thankfully right, irrespective of whether you are from electrical or mechanical or civil or whoever right. But unfortunately, the material that we are going to cover is so powerful right and so interesting and so much used in so many different fields; but somehow you know it does not get the attention it needs.

So, what the way, I would like to cover this is in a way, where it is like very common sense approach. However, it has a rich you know pedigree and the mathematical complexities are really something that you know is kind of unnoticed ok.

So, with that, I would request you to really go over this part, go through these lectures and go after this go do the homework's right go through the assignments, go self-learning, go read about it and you will find whatever we are going to cover in in this slide, I mean in this module and the next one, so about a couple of hours worth of this material. I think that is a that is a eye opener, at least I felt it as an eye opener I hope you do so ok.

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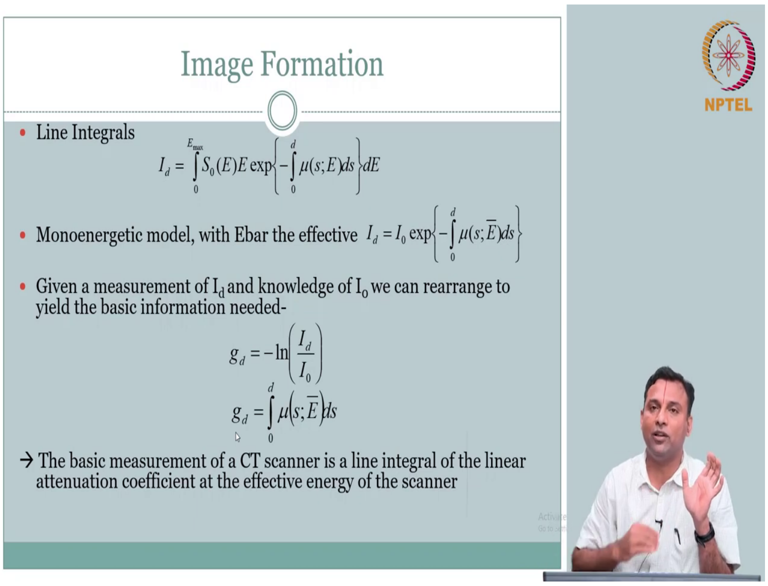

Without much more wait, let us get on to the image formation. So, what do we know already right. Its always going to be what do we know already, what we have done because the physics and the model, we have established already. Because projection radiography, we already talked about x-ray interaction and we talked about what a signal and what is noise right. So, what we know already is here in CT like in projection radiography, you are going to send x rays; but only thing is it is going to be sent in projection radiography, we sent a cone.

So, the whole chest was insonified right, but sorry irradiated; but then, when you take CT, we are going to in the end come up with slice, so through the depth right. So, we are going to now send CT like this over a chosen slice, instead of the area right. So, we are going to do that and we are going to have a corresponding detector that is aligned. In fact, we also talked about the different configuration fan beam, cone beam. So, for simplicity, what we will do just to start on we will start with like how it was first generation, where it was a pencil beam.

So, I had one line go through, I have one detector at the back. So, it was a line. So, we were collecting right.

So, the x-ray was going through here, it was collected behind in the detector. So, it was a line that was passing through. So, we already know what is it that is getting detected at the detector, what is detected is the line integral that is you send something and that integral what is that integral?

e power basic fundamental law right; e power minus mu delta x when we introduce that; mu being the material property, delta x being the thickness of the material. So, mu e power minus mu x that is the fundamental law. Same thing, we will rewrite, we will recognize your what is falling on the detector.

I d is nothing but whatever you sent in spectrum with different energies; mu, the material property is a function of both s, s is your along the line segment ok s. We used I think x or r in our previous variables but right. So, s is just the same through right, the line through just a variable to denote that.

So, your mu is a function of along the line and energy. So, this is the detector. This is what we know from before. Now, what do we need to do? Ok. Given this, we can quickly understand in projection radiography, we did not really bother in image formation. We understood this whatever falls on the detector, we had intensifying screen converted to optical density. We have an image of optical density that fundamentally was related to the underlying mu.

But what you measured was this integral you did not really measure this. This, this I d got converted because of the photochemistry right, the film, all those things and you had an output optical density; whereas, our objective here in in CT is I want to somehow measure this mu along the s right, along the path it came from. I am not in just interested in the net sum.

I want to actually go back and say what is the mu along the path; how does the mu change along the path. Of course, there is another variable; mu is also a function of energy. So, our objective in CT is going to be we are not just happy about this I d. This I d should be simplified mathematically right. This is good.

It has to be this expression has to be simplified so that it becomes mathematically tractable. Why? Because we want to use that mathematical tractable to our advantage so that we can end up getting this mu as a function of s right and E ok. But again, like I said, it is technically a function of E; but in the instrumentation, we also covered where does CT differ from your projection radiography with respect to all the other filtering and stuff.

They deliberately making it harden. Why? One of these reasons. What did we mean by hardening the beam? That is you have a spectra of energy, you are shaving off, you are reducing the bandwidth. So, the idea is if I make it I am trying to get to monoenergetic; I want to reduce the energy distribution. So, I want to get through monoenergetic.

That way, this variable is also a function of E, then it becomes right two integrals. So, what I can think of is instead of using the two variables and two integrals, if I can use only one energy. So, we introduce the concept of equivalent right. It is like your center of mass right of the spectral. So, we introduce this idea of E bar the equivalent or the effective energy.

So, we can consider this as a monoenergetic model. By using E bar instead of having this every energy, so we can reduce this to the first level by saying I d is I naught x, only this variable is there, this summation, this is monoenergetic. This E and d E right, integral over E to E max of this, we have substituted that with only one energy, the average energy or the effective energy or the equivalent energy. It is all similar, it represents the same thing.

So, now, you notice at least one level simplification we did, but we are still not done. We still have one integral here. Our objective is so what you are detecting? This I naught is not a big deal. What is this I naught? Intensity at the center of the detector, when there is no object right that is how we use this I naught concept earlier.

So, essentially, the I naught is you can get from calibration, without even the object. Whatever you are sending into the object, what comes out at the detector, what you send through the detector; sorry what you send at the front end of the body, I d is what comes out of the body. So, the relationship between what you send and what you receive is it is attenuated while it is going through the media with attenuation coefficient mu right. That is what this is.

So, I can get my I naught. I naught is what you are sending. So, without the object if you have my detector, I can pick what you are sending into the body right. So, you can get your I naught. So, essentially that means, I d is something that you are going to measure right in the detector, whatever is going to fall on the detector; I naught is a calibration, so you can do that and you can have that up front. What I do not know is I want this mu, which is the unknown. This d is not a big deal right. I know what is my path, I can I know roughly the object size. So, I can calculate d that is not an issue.

But I need to get out get this mu as a function of s or the length, path length. So, given a measurement of I d and knowledge about I naught, we can rearrange this so that we have I d by I naught minus natural logarithm. So, this is a known quantity. So, known quantity is your g d. So, this is the g d is the measurement at the detector.

But what is this measurement? What is this equal to? Equal to this alone you take, exponential is gone. So, what you have in the right hand side is just the 0 to d mu of this guy. So, what this says is what you are detecting right, what you are measuring at the detector, let me put it that way carefully because what you are remember the CT detector, we covered different from our previous on, where it was converted to optical density. So, this is what is measurement. It is falling on the detector and you are measuring it. So, you have calibration factor.

So, you have you can measure; this is the quantity that you have measured, g d right. This is an estimate you can get g d. What is that capturing? That is an estimate of the line integral right. So, this is what you are measuring in the detector is nothing but it is a line integral of the attenuation values along the line 0 to d. Good, kind of what we knew from before, we have just customized it.

So, what is our goal? Our goal is given g d, given g d right, I have measured this. So, I have g d. How do I say how it came from; what is the mu distribution? I am not interested just in the sum, I need to know at each location, what is the contribution; what is the mu ok. So, the idea is the basic measurement of CT scanner right, the raw measurement, a raw measurement is a line integral.

So, from this raw measurement which is a line integral, we need to construct the image that is why. The image is that you know the object is already there. So, we need to reconstruct the object; object is already there right, it is constructed, but we do not know. So, we have measured the basic measurements in CT.

Using this basic measurement, we need to reconstruct the object, the slice right that we want. That is the goal ok. So, this is the basic measurement. So, now, we are going to see how we are going to get at it. Before we do that, we should appreciate this is a function of mu ok.

But then, if you go look at the values in mu for bio biological tissues, let you have a bone, muscle, fat right; some the difference between the mu is very small. So, just by using projection radiography right, if you got the projection radiography, this is a thing. If I can just can I get to see the different tissues, the contrast out right.

The problem is in in projection radiography, we always calculated the sum right. So, there was enough contrast because the sum was greater than all the p. Now, you want to construct one pixel along the line. So, if the difference is small which is inherently difference between bone and tissue is all small.

So, if you do not do anything and you just reconstruct this, your inherent contrast will be poor; meaning, you will not be able to see much difference between, it will be too noisy. So, what they do instead is that is one thing, the other thing is we talked about mu being material property. You already see here what you measure is also dependent on the system right.

The energy that you are going to use, how it is going to be calibrated right that system what is going in, all of that influences your g d. So, if from g d, if you are getting mu naturally the initial condition of what the g d right, what your I naught is for example is going to have an influence. What this equivalent energy that is going to have an influence.

So, material is same that is ground truth; but the mu will change from machine to machine right that is a problem or even with the same machine, if I change my settings, mu will change or it is the same patient, I go to one scanner in one hospital location, I go to another scanner in another hospital location or over time right.

Any of this will give me different mus or at least there is a possibility to give different mu. So, it is not just that ok, we have g d and how can we get mu. If you get, mu the mu is influenced by all the other factors and therefore, it may still not be as useful ok.

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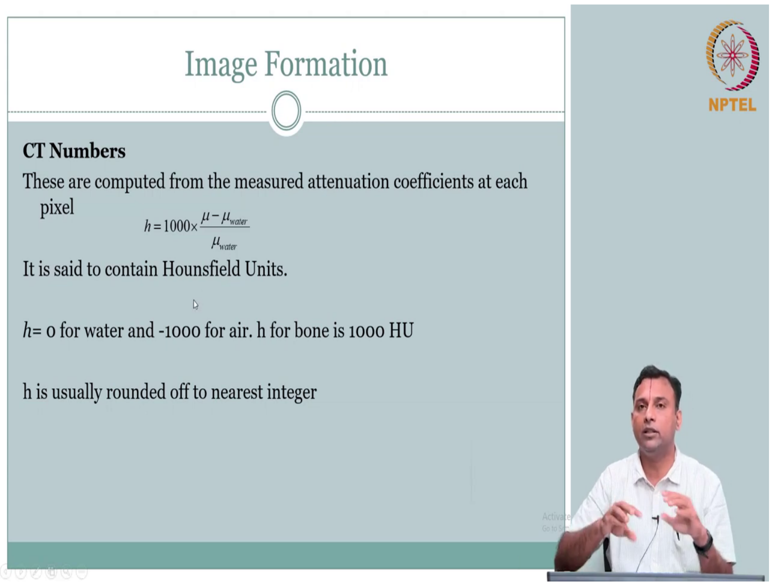

And therefore, what is done is we convert this mu into what we call as CT numbers ok. So, if you look at your CT, if you had a chance to look at it and read the text and all those things, you would have seen the units right. When CT, they call CT number, they also call in Hounsfield's units.

So, what do they do? So, essentially, what are the parameters, what are their tissues of interest to you? Inside the body, you have air, hydrogen, water and then, various combinations like fat tissue, fat muscle, bone some density changes are there. So, the idea is I want to convert that. So, is there something that I can calibrate against, I can have a reference against? Usually, water is taken as a reference ok.

So, these are the CT numbers are computed from attenuation coefficient at each pixel that is you get some mu, but that mu is not sufficient because it can vary from scanner to scanner or settings to settings. So, we let convert that mu with reference to water. So, this you can do.

You can have a test standard for water right, under ideal temperature you know you can have that. So, for that x-ray before you do the human body, you calibrate it with water. So, you can actually get mu of water you know. So, you can convert whatever mu you are getting with respect to water, using this formulation. So, you are making the units 1000 times. So, you look at the you are you are improving three orders of magnitude so that you can separate out the different constituents

Because inherently, the mu between the different biological materials of interest is small and therefore, you kind of do this with reference to water and what you quickly see is if there is water right, if there is water if that location had water along the path, your mu will be mu water minus mu water. So, that is going to be 0.

So, your h right your CT number or the pixel value will be 0 if you have water. So, if you have you know what is the other air for example right. If you have air, you know air does not attenuate or insignificant right compared to new water. So, essentially, you can get this can be ignored. So, you will get minus mu water by mu water. So, you can minus 1000.

So, if it is air, there is attenuation is less than water so, that how much less? So, it is pushed to minus 1000. So, the bone which is naturally you know natural body, bone is supposed to have the higher highest attenuation of all the other material right; fat, muscle, water, air.

So, bone will become 1000 Hounsfield's unit ok. So, h is 0 for water, minus 1000 for air. h for bone is about 1000 Hounsfield's unit. So, now, you see your CT becomes quantitative right. You have an image, CT image, each of the pixel has a unit which is Hounsfield unit. So, what is the value going to range? It is going to range from minus 1000 to 1000.

Usually, within plus or minus 2 Hounsfield's units, you get accuracy. So, it is pretty damn good; pretty damn good. Therefore, you can actually start to do quantitative right. So, I can take the scanner, I can use Hounsfield unit as a parameter and say if this is greater probably you know the material. So, this is a tumor.

So, you can do quantitative. So, you are not worried about which scanner center was taken ok. So, this is very powerful. So, let us appreciate this and let us now go on to the real business. The real business is if I get my mu, I know how to normalize. So, I can get my CT number. So, it is it does not matter on which scanner or what setting you used ok.

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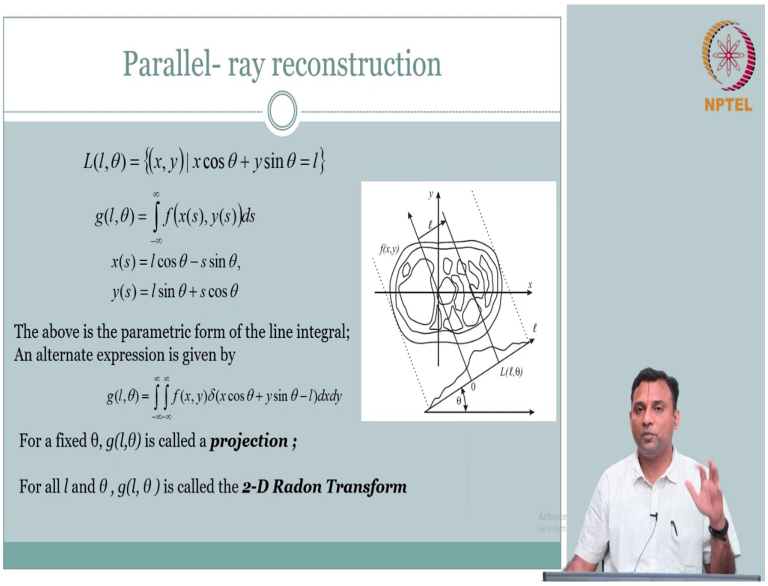

So, let us get into jump into the reconstruction itself ok. So, just to reiterate, we are going to do parallel-ray reconstruction. What is parallel-ray reconstruction? We send one x-ray, I have one detector, one line integral which is what we got. So, we are going to mimic first generation, where then you translate, you send, receive; translate, send, receive.

So, you kind of have parallel lines. Of course, in CT we talked about we are going to move and take it at different angles same thing right. But in essence, if you look at our reconstruction right, we are going to start with simple parallel-ray reconstruction pencil beam kind of word that we used in first generation.

After we do this completely, then we can expand to fan beam and then, expand to cone beam what your slice right third direction cone beam. So, we can do all that, but this is something that is very fundamental and very powerful. If you understand this concept, rest of them is just going to be geometric manipulation on these algorithms ok.

Well, let us start with this first setting the stage. So, here is an object that needs to be imaged or I would say this is the slice, this is the slice right that you need to image. You are standing on one side, so this is the slice. For example, you are going to send on one side, receive on the other side. I want after that reconstruct this object through the slice. So, this is what that is.

So, I am going to have f of x comma y is my spatial reference system. I have this is the object in there. What do we get? We get a projection which is what we saw. What is a projection? If I take a angle right line integral. So, what you are getting here is a projection.

So, how are we going to call? In this case, what you are getting is a 2-D is projected to get 1-D which is a line; correct? In our x-rays projection radiography, the object was 3-D and we used cone. So, 3-D was collapsed to 2-D plane of vapor; whereas, here I am sending only x-ray in a slice. So, even though, I am 3-D, it is only this direction, this 2-D imaging slice is what I want to image right.

So, it is only 2-D, I want to image. So, in this direction right when I collapse, this 2-D becomes 1-D that is what you see. So, what you are collecting? The data, you are collecting is a 1-D data along the length is the extent of this object and the values are; remember, how we talked about if you have one-dimensional variable, you have you can plot it right with the y axis being the amplitude at each location.

So, this is your values at that location, which is nothing but from what we covered so far in our context, this is nothing but line integrals, whatever sum total along the path is this value ok. So, just to get our coordinate system and the representation of this correct mathematically, we will recall we wrote 2-D line right. So, we have to know this is your line which has a length L.

So, that is one axis and it is oriented at some angle that is theta. So, this line is nothing but L of l comma theta. What is this line? This line is nothing but a projection of this object f of x comma y; where this, L right, L is along the line what is that is nothing but this l of this line, along the line of sight right. This is the one that is going to come. So, this is the distance, the perpendicular distance right. So, that is your l and theta is likewise, where is this line making with respect to the this f of x comma y right.

So, l can be defined in terms of where it is located the small l and theta which is dependent on, where this line of projection you are going to take right. So, we are going to write all this out. So, L of l comma theta is nothing but in the x y plane, where this constraint right x cos theta plus y sin theta equal to l, that is what is your l of l comma theta.

So, now, what we want to do? We want to go back to our projection. So, this is projection right that is what we want to do. So, g of l comma theta is nothing but f of x s comma y s d s. So, nothing but your projection is nothing but minus n. So, you are summing summation of f of x y is the object, you are summing along. So, x of s y of s is this line right.

So, your g of l comma theta is nothing but sum along this line that is this value. So, g of l comma theta which is your projection is nothing but sum of along the line segment ok; clear? So, this is our line integral that we covered from this. What we need to do is of course, you can calculate this x of s and y of s is this line.

So, this is fine. This is your parametric form; but this is kind of captures how we can write the line and how we can visualize the you know write down the parallel-ray, how do you get projection in terms of the object and the line integral. But then, there is another way we can write it, which is a more simpler form right. That is you should recognize what is this f of x y is your object here, the 2-D object.

But instead of writing it in this form, we can recognize that f of x comma y into delta along this line; that means, you are picking from f of x comma y, remember we covered point delta and line delta. Why did we cover line delta? We said it will be easier if I want to pick a line from f of x comma y, I can define a delta function, the line delta function and I can move the line delta function to pick values from wherever we want from f of x comma y that is what we are doing here.

So, we can also see this line as nothing but your f of x comma y, you are picking this line from f of x comma y; how am I picking this line? I just say the delta function exists only in this line right. So, my delta function exists when argument is 0. Here when is the argument 0? When l is equal to cos x cos theta plus y sin theta; that means, your g of l comma theta is sum along x and y right of this f of x comma y taken along this line. Clear?

So, this is another way of writing. So, for fixed theta right example here, one theta is what I have shown. This is called as the projection; clear? You knew this from before, but we have just written it out mathematically. We talked about this line integral is nothing but the projection right, you are collapsing the third dimension, when we did projection radiography. Here, you are collapsing the slicer to one line. So, it is a projection. So, g of l comma theta is called as the projection and this projection is at particular theta.

Likewise, what can I do right? Likewise, what can I do? In the instrumentation, I will see it from different views right. So, if I view it from different views, actually I can get a collection of g of l comma theta; clear? So, this is for one theta that we showed here. If you move around and get the projection from different angles, then you have a bunch of g of l comma theta; bunch of projections right.

Each one is registered, you know which direction it is coming from, you know which detector, so you know your l. So, you have what is called as collection of projections. What you I mean it kind of seemed intuitive, but what we did not recognize is the collection is this collection is called as your 2-D radon transform.

So, if I give an object, if I reduce the dimensionality and I have reduced dimensionality or the projections from different views right. So, for all l and theta, your g of l comma theta is called as the root 2-D radon transform ok. So, here at this point, it begs the question; that means, this is not something new.

So, radon transform kind of all this work was published in 1915s, 20s like that right. So, really early 20th century. When did CT come? Post World War II right. They started working on that. So, you see if there is a radon transform, you have projection. You could also imagine that there should be inverse radon transform right.

If you have forward radon transform, you have forward Fourier transform, you had inverse Fourier transform; you have forward 2-D Radom radon transform. So, you can also have you probably can guess there should be some inverse 2-D radon transform. If there is inverse 2-D radon transform, given the projection, you can get given this you can get the object, which is what you want.

So, the thing is mathematically, you know it grew, but our we are very lucky, we can turn back and we have hindsight it is all 2020 right. So, we can turn back and say, this is actually common sense driven way of doing it right. So, given this, I know how to get the image of the f of x comma y given the collection in a very intuitive way which is what we I will attempt to do and then, of course, put it mathematically so that it is correct.

But it is not that easy. I mean the reason I want to do it that way is maybe if you have an appreciation for this, what is not so obvious now, maybe you it you may start to see something that is not obvious and you may push the understanding further ok. So, just for that reason, I am kind of trivializing and saying, this is common sense, but you know nothing is easy.

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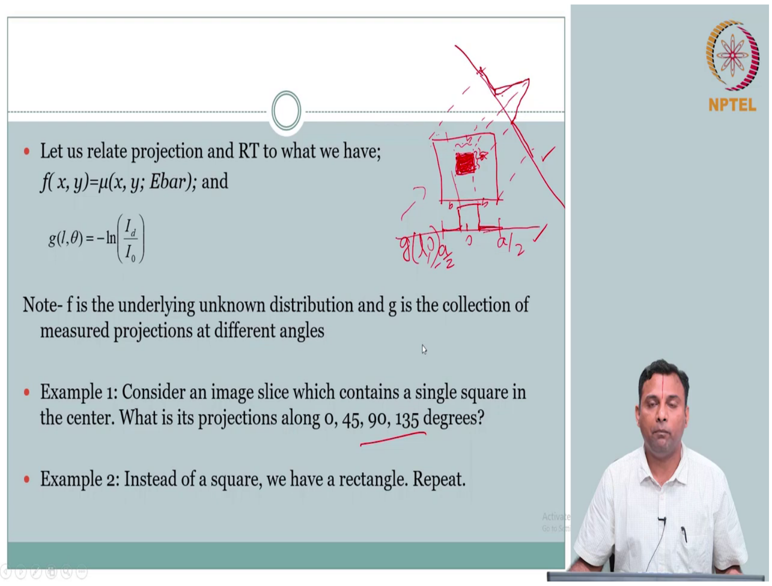

So, what we will do is let us relate right now what we have projection and radon transform right. We just completed that. So, let us just put that because we just said f of x comma y, g of l comma theta, we will just quickly formalize this that was parallel development by radon.

Let us now see why that radon transform that idea is applicable to the values that we do or the CT problem proposition. So, f of x comma y is your mu of x comma y right. You can see that. In our context, we have mu of x comma y which is similar to the f of x comma y that was shown; g of l comma theta, the projection is this measurement right.

So, f is the underlying unknown distribution. In our case, it is going to be mu; g is the collection of projections from different angles. So, now, you see the big picture. Our objective is how do I get mu from g correct. So, let us take a simple example, just to get this comfort.

Consider an image slice which contains a single square in the center. What is the projection? So, what I would like for you to do is when you get time, when you have time to scribble, start to imagine. So, I actually said at the beginning of this module right, CT module I had a Rubik's cube and I told you start to imagine. So, let us so keep doing that; but given that we are here, let us do a simple object at least that will make my life easy right. But you should take complicated objects and try out what we are doing here.

So, all I am saying is consider a image slice which contains a single square at the center right. I am trying to make that colored so that is you can see that. This is the there is a target inside the field of view ok. This is the slice. So, our objective is I do not know this.

This is the ground truth right. I want to image from outside and try to see if I can re come up with something that is close to this right. So, first step towards that is whatever we covered. What are we measuring in CT? It is projection. So, can you draw the projections, let us say for example 0 degrees. So, right so, I going to have my l comma theta; theta is 0 in this case because I am taking a simple.

So, what is this value? What is this going to be? This is going to be my g of l comma 0 for example. What is this going to be? Brute force interpretation. It is line sum of right. So, minus. So, this is my length. So, if I take this to be 0 minus l by 2 to plus l by 2 or whatever length, you want to give this right.

Let us a by 2; minus a by 2 to a by 2; a being the width. You see what is happened? What is my projection? 0 0 0 0 right 0 0 so until line integral, all this is 0. So, I have 0. When I come to this location, now I am going through the square. So, that means, it is going to sum along the square. So, that means, I have to have a value that goes up and that same that remains same because rest of the parallel lines right when we go through the l is same.

And then, how do I get out of this ok and then, it comes down because after you cross. So, I am just trying to sum along the lines right that is what your interpretation is. Again, becomes 0. So, this is some value which is the some integration of this length right. So, if I had one value of one inside the box and 1 per unit length right each value and I have a length of this is a right, square I said. So, this is let me just. So, let I have used a. So, this is b. So, this is b right. So, if it is 1 unit per length is the value, then if I go through b, I will have b right because I get some of that.

So, I have b, so I have 0 and then, it raises to b and then, it is always b inside the box because the length is not changing and all of them are having the same value and then, once you the line comes outside this box, I get 0. This is one simple projection. Clear? So, likewise, you can do say for example, 45 degrees.

These are the careful, you have to be careful. How you want to do it? Again, just for convenience, I am going to draw the amplitude the other side ok. So, here what is going to happen? It is going to be 0. I am going to have some non zero value starting here until here.

So, it is going to be 0. Once I come here, it starts to raise right and diagonal will be my maximum path length. So, I am viewing from here right. So, my lines are going to be parallel like this; sum of lines is going to be 0, start to increase, start to decrease, 0; clear? Likewise, you can go about do.

So, this is the bunch of projections you have. I do not I want to get to this object, but what I have is I have these collection of these projections. I have one projection, I have two projection; likewise, you can do it for the others as well. Clear?

So, I have bunch of projections. So, what we will do is ok given that you have bunch of projections, how do we get back to the object that is what we will see. Again, these are some things that I kind of tell that you do it by yourself. Because I have shown you for square, simple do it for rectangle at your own time ok.

What you will realize is the carefulness with respect to your length and how do you what is your path length and therefore, your shape that you are going to see here, you will get familiar with that is why ok. Because most of the time this is something that you can do by paper and pencil; but a real object, you may not be able to do that, you may want the computer program to do it.

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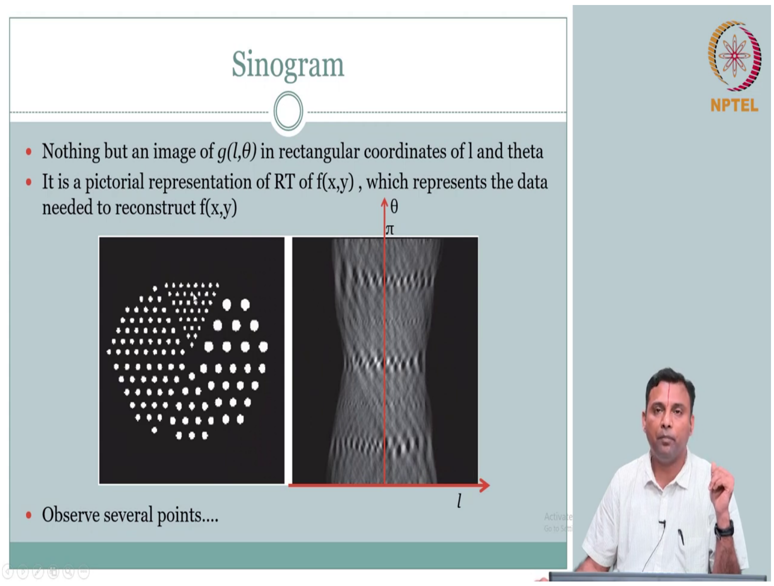

Anyway, so now, first before we go further, I have this a bunch of color you know projections at different angles. What do I do with them? Yes, you have to use them to get to the image that much is clear.

But before that is there a way that I can organize myself, can I organize the data that we have collected in some form so that at least we can see some information from that, before we actually do Brute force reconstruction ok. So, nothing what you have we get what is called as a Sinogram. What is a sinogram? It is a raw data; the data that before any other processing and reconstruction, it is the data that you have collected which is the line integrals.

So, it is nothing but an image of g of l comma theta in rectangular coordinates. So, rectangular coordinates means you have to have two axes; so, one will be your l, the other will be your theta ok. So, it is a pictorial representation. So, this is your pictorial representation of your radon transform of f of x comma y.

So, this is the data that we have collected, this is the raw data. So, if you just plot that, so this is the object f of x comma y. If you were to project this right, you will get one projection like how we did for square just now; if you project this right when you do the sum, you will get some value. So, that will be one projection.

So, when you go to another angle you project, you get another projection. So, if you stack all this, so this will be your l and you stack different thetas. So, your y axis is your theta axis. This representation of the data that you have gathered is called as sonogram. Why is it a sinogram? Well, I will think the quality of the figure is pretty reasonable.

I hope it comes through like that, where you can probably appreciate right. There is some curves that you see. In fact, if you if you tilt this right, rotate it 90 degrees, you will be able to see that the curls are nothing but it looks like a sinusoid right. Here, you have some sinusoids. Now, it is vertical.

So, if you actually rotate it, you will see that it is the typical sinusoid that we are used to plotting and so, the name derives because you see these sinusoids, it is a fundamental. If you have a point, it turns out to be a sinusoidal feature and therefore, this is called as a sinogram ok. So, that is for the name and then, what else do you see? I mean this is important.

What else do you see? I do see that there is a hourglass shape something is there; but if you interpret what is the meaning of each of the projections right, then maybe we can glean one more information, which is if I project from 0 degree for example, this object looks to be the fattest right. It is more wide than tall.

So, if I project from here, my length will be more. So this is 0 degree for example. So, your 0, you have l is more and then, as you move around in small steps, the length decreases and when you get perpendicular right at 90 degrees, this is the projection you are getting. You are getting this width now, which is smaller than this width.

So, gradually, you go you get to 90 degrees, you get the shorter width and then, again when you move start to move away, so you stop just before 180 degrees. Why? Because you then 180 degree you do, you are starting to see the same pattern right; you project from this side and collect on the other side or project from the same, collect on the other side, it is the same line. So, the line integral is going to be the same.

And so, typically this sinogram plot is having l and theta and the theta usually is just less than your pi and you have several of them depending on how is your stepping size, how many different views you are stepping that is going to determine this axis. How many detectors you are going to have or how many parallel in the translate, if you think about its translation or you think about an array detector; how many array elements are there that is going to determine your axis here right, how many points you are going to have along this axis.

So, you see that sinogram is having some information. From this, you can actually measure the size of the object, width of the object and so, we can get that. However, it does not and also you see there are some bright spots, dark spots right, cancellations. Where is that coming from?

If it is bright; that means, it so happens at that angle some of these whites align themselves so that it is high. In when it is dark, maybe it is cancelling right. So, you could actually interpret this image the sinogram as well. There are some information that all the information is hidden though right.

So, even though it has all the details like the underlying information of the object is there, clearly if I tell you that ok, this has all the information about the object. So, you can start to see the object from sinogram is impractical. It does not make I mean for sake of argument, we can say that we can get the length and breadth of the field. So, you know I know the object dimension.

So, we got some information; but this is no substitute to actually visualizing the object ok. So, sinogram is a raw data, it is organized in a form; but essentially, even though it has the information, it contains the information that we may need. It is in a form that is not appreciable from a imaging perspective, as an image perspective ok.

So, what we will do is how do we start with the sinogram and hopefully, get to this image or estimate of this image. So, this is the ground truth. These are the estimates of projections. So, using these measurements how do we construct or reconstruct or get an estimate of this image of this object that is what we want to see ok.

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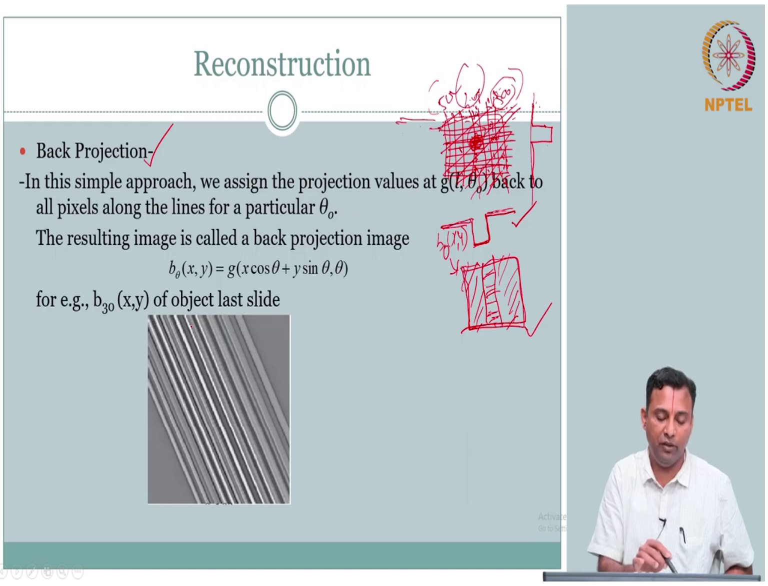

So, here this is what I will do. Quickly, conceptually, I will just tell you what you and I would do right. If we have a problem, so analogous problem, I am saying that sorry I am saying that imagine same square example right. Just for the sake of simplicity, I am just putting some matrix so that you can start to maybe get the big picture of where we are heading to.

So, now, I am saying I am not even worried about x-ray CT. My problem is this, see I have 1000 gold coins right and what I want to do is I have people standing in rows and columns right, have people standing in rows and columns. I have 1000 gold coins.

So, what I do is I tell to I stand here to the first person and I say here bunch of 1000 gold coins, you take 1 or take 2 and pass it along back right so that each one takes and I come to the other side and I take the remaining. So, my problem is I have so many people and initially, I say all of them are good.

I want all of them to take only 2 ok. But I do this experiment. So, I give this 1000, all of them are taking 2 or supposedly instructed to take 2 or whatever and pass the remaining. Next I give the 1000 coins to the next row, next column right, do the same thing. So, I have done that.

Now, the problem proposition is this; I have only this, I know what I sent, I know what I receive. So, I know what is lost along the path. If all of them were honest, then no issues right. But it turns out that few of them somewhere say let us say just for the simplicity of argument, there are about 4 or 5, 6 whatever number of people there, in fact it can be anywhere just for simplicity, I am just putting at the center. I am saying well these people you know they are not they are dishonest, they are taking more coins.

So, now, my problem is how do I find out who these people are, where are they standing. If I know where they are standing, I can pick them up, but I do not have any other information. All I have is I send here, I ask them to take 2 and all of them are same number of people are there; whereas, when I send 1000 here, I got some 1000 minus if there are 500 people and I asked them to take 1 each. I got I sent 1000, I got 500 back ok; whereas, when it comes here, I got only 400. Here, I have 500 or more or less right plus or minus.

So, the problem is at the first guess, what I can do is I could say well, I sent 1000 and roughly there are so many people there and I ask them to take 2 each and so, I would have expected this back. But along this column something came low. But that is not sufficient for me, who see you cannot accuse somebody without knowing who it is right.

I may guess maybe there is something fishy happening; but may not be even here maybe there is someone here, who is taking more and there is someone else, who did not take, he just passed on. So, on an average, you would have got the same number. So, does not really matter. So, the point is how do I suspect? So, without no having more information, I cannot do much with it, all I know is there is a problem. What would you do?

Well, what I what if you have the luxury to do, what you should do is I have a suspicion. So, let me do this, now I will start to send 1000 coins and I will say you start to send not to the person just behind, but at this angle right, then I will go, I will send from here and ask them to pass to their side right.

This is nothing but similar to your projections correct; its very similar to your projections. Now, how will I do? What will I do? My first guess would be how would I start to approach to nail down who could be the guy? My first approach would be look I sent 1000, I got 500; I pretend everybody is nice. So, what I am going to say is from this projection right, I got this value. So, I sent 1000, I got 500.

So, let us say, but when you go here, there was a dip right. This is the data I have. So, I have done this. So, when I do this this side, similarly I get some value dip right. These are my projections from different views. What will I do? I will say look I do not care, I will be honest, I will take I lost 500, I will say I gave everybody, 500 I lost divided by say if there are 500 people, I will say everybody took 1.

So, based on this data, I will say along this column everybody took 1 rupee 1 gold coin, 1 gold coin, 1 gold coin. When I come here, I will say I know it is only 400, there are 500 of you. So, in this column, for lack of much detail, I will say 600 by 500, 1.2.

Each of you are supposed to take 1; in the other cases, they took 1. Here each of you have taken 1.2. I can say that because I do not know I trust equally likely. So, I put 1.2 and I repeat the same process, I will say ok based on that is what I did. When I stand here, I will start to do the same projection right. What is that? From projection, I am giving it back.

So, it is back projection. So, from the collected right from the net sum, I am projecting along the I am back projecting it, projecting it back and saying each of you probably got this much.

So, it turns out that is not a bad idea right. It is a common sense thing. So, if I can collect like that, I will start to project back, what will happen?

Let us formalize one more step and then, we will see how nicely this identifies or makes us help identify the culprits. So, it is a simple approach, we assign the projection values at g of l comma theta naught back to all pixels along the lines right. This is what I did back to all pixel along the lines, I have given that.

The resulting image is called as back projection image. So, we will write it mathematically back projection at theta from one theta value of x comma y is nothing but the same projection that you have g of g right, you are projecting for that theta along that line x cos theta plus y sin theta comma theta. So, you have projected that back along.

So, I had one line right, I had one projection, I have back projected that to this f x comma y space clear. So, we will just take a pause after this slide because we will have to have the energy to do the reconstruction. So, if you take one example back projection from the object that we showed in the previous slide right, it will be like this. Nothing fancy.

Here, I took a square just to explain right. If you project this back, what is going to happen; if I project this back, what is going to happen, just for getting an analogy, if I back project one thing, I am just saying all of them had 1.2 rest of it is; whereas, the other values, if we if you go by our analogy here, the others were 1 right. The This is 1, this is 1.2 in the example; 0.8 coin. This is 1 coin and an average per person. So, I am saying everybody took this.

So, here I am back projecting. So, I am saying along this column, everybody took 1.2; along the rest of the columns, they all took 1. This is my back projection image at 0, at 0 of x comma y. I mean this is back projection of the 0 angle view angle along x comma y. So, 1 dimension becomes 2 dimension right. Likewise, in the complex object that we saw in the previous slide, if you take an example 30 degree sinogram, if you take the one at 30 degree and project it back at that angle in that space of x comma y, this is the object ok.

So, let us stop here. Let us continue further and see the beauty of how from this baby step, how we can actually get the reconstructed image ok.

Thank you.