Introduction to Biomedical Imaging Systems
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Lecture - 24
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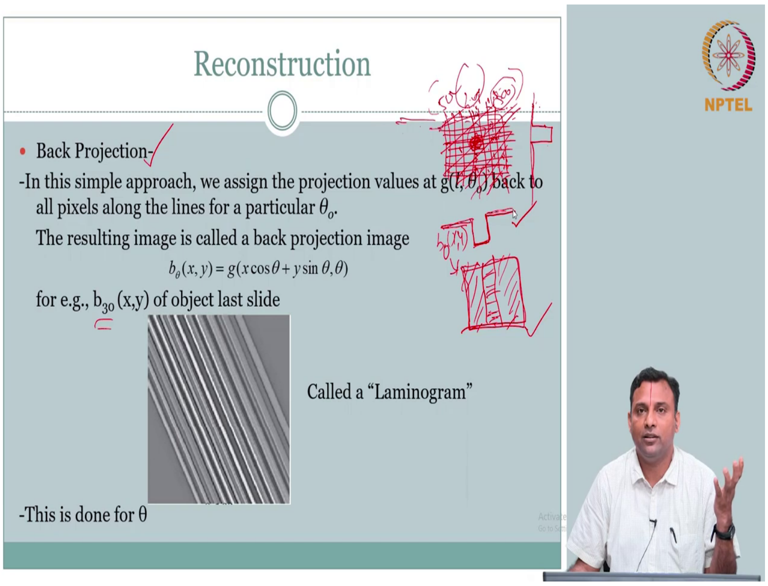

So this is for one angle, this image is called as a Laminogram ok; because you can see laminates line feature, so this is Laminogram. So, what can you do? This is just one example, we can do this for all the theta, right. So, you can in fact reconstruct this b right at all the different thetas as well, which will be very similar to this. So, you get several different Laminograms.

So, we can do for all theta, you can do this; then what will happen? Well, important step. So, in our analogy, each projection right, each time you did this experiment, you get one guess, which is you are saying average they all gave so much, that is one guess right, that is one estimate.

Because based on this, you did not apply anything else; you said on an average this is what everybody contributed. So, that is one estimate, but then you did that experiment several times different theta, each time you got guess. So, if I have to make use of all the guess what will I do, if I have to make a judgment right; if I have to come up with identifying who where that person is who is stealing more than what I asked them to do. Well, I did several experiments, I will essentially average out right; I will take the average of all my experiments correct or I will sum. So, sum divided by the number is your average, right. So, in some sense I did measurements theta different times. So, I will just make a judgment based on the average of that. So, what happens, I sum each of that what will I get.

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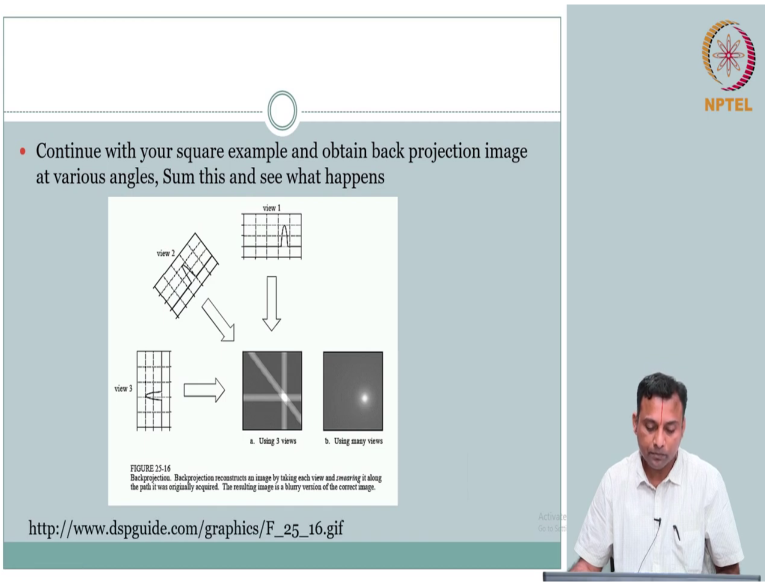

So, continue with your square example, you can get your back projection and see what happens, see what happens. What I will show here is you know I have done that, there is a textbook example of a circle ok; you have a circle instead of square that I did, you have a circle and you get it from different views and you back project that exactly the steps we did, only thing is we will back project and sum. So, you do it on your own for the square example that we did, see what you get.

So, here what you see is, you had a field of view where there was supposed to be a circular disk; we put a square at the center, this was having a circle. And same logic view 1; when you project it this way what happens you have zero, zero, zero, zero zero and when you entered the circular disk, it started increasing and then the path length decreases. So, this is one view. Since they took a circle, it is any theta you look at it the profile should be same, right. So, different views, you had the same profile, but register a different location.

So, then what did we do? From the projection we did back projection. What is back projection? I sent it along, so here the same value I sent it along, same value I sent it along, same value I sent it along. And then when you sum this right what happens; wow in different views, this person was behaving or here these pixels were behaving differently and they got caught in different views, so partially in each of the views. So, when you sum it, this one stands out, rest of them does not stand out, clear.

So, stretching back to our analogy; that means from the guess that we did for each of the time right, projected back and say average you got one rupee, one gold coin. In this column you got 1.2 gold coin, if I did that from different views and started projecting right; people would get one gold coin, if he comes back in the next one, he will still get one more gold coin, whereas this person will get only 1.2.

So, if this person is coming back in different views right, he is going to get less. So, the others are going to have more and this person is going to start to get more and more. So, if I sum all of that, I could clearly spot that there is a few person or people at the center who are you know the culprits, who are taking away more than they should. So, here you can see this, you would not have imagined from the projection, it would be very systematic straightforward, very common sensish approach to do the back projection you get something like this.

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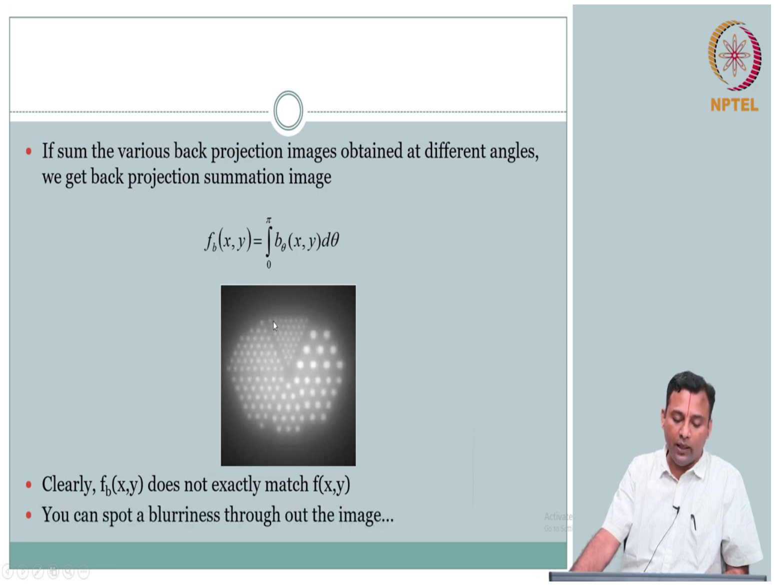

In fact, you do the same thing, you do the same thing in the complex image that I showed right; you had several circles and you had this width and height, so a back sinogram that we showed, right. So, we will see how that comes, but just to formally rated what we have done is, we have our back projected image which is a sum of back projection sum, right. So, this is a sum 0 to pi of this is b of theta x comma y this is what we have done; we showed one example of b 30, so you do it for all different thetas.

So, that 2 D image you sum over 0 to pi, that gives you a back projected image right, sum of the back projected image. So, this is just for consistency we call this as back projection sum image, this is your back projection image, back projection summation, this is f of f b of x comma y, ok.

If you do that in the sinogram that we started with example; wow this is pretty damn good, I mean there are two ways to look at it, one is wow without knowing much right, I just collected projections from different angle and the projections were weird.

In the sinogram we looked at the sinogram of this data, you could not really tell; it was not even anything close to the object, whereas a simple educated guess, that is I back project average value and then sum from the different views, I get this estimate f b of x comma y, which is really it is a you know pretty damn good, right. I mean you see the object, of course they are not really happy all the time right; because we said this is fantastic, without too much fancy just common sense you know going logically we got this.

But then with that for that reason this is good, but actually you look at it, it looks like hazy is blurry right; I am greedy I want the object, look at the contrast that we had in this original object it was black and white right, white wherever that circle there, whereas here you see there is so much grayish, you have a whitewash. So, it is good intuitive, but then not good enough; why because of the blurriness.

So, it does not really match exactly your f of x comma y, ok. So, now, what do we knew? Given that we are unhappy right, given that we are unhappy; what we did is actually, we did not really do anything sophisticated mathematically, we just applied common sense and our intuition of how to go about it.

So, it turns out that this is a very good start, back projection is a very good start, very common sensish; but analytically, so there is something that is missing which makes this problem of blurriness. What is that missing?

So, now, we will wear our theoretical hat, we will go through some theorems and try to you know go from the analytical expressions that we know for several different formulaes, like for different concepts that we have covered so far and see if we actually get to the same problem when we do systematically and see if we can identify what is the missing link, why are we getting this, this is almost there, but something is missing.

So, when I when we do our common sense based approach, we all almost got there; but we are missing a critical aspect, critical step which could perhaps address this problem of blurriness, ok.

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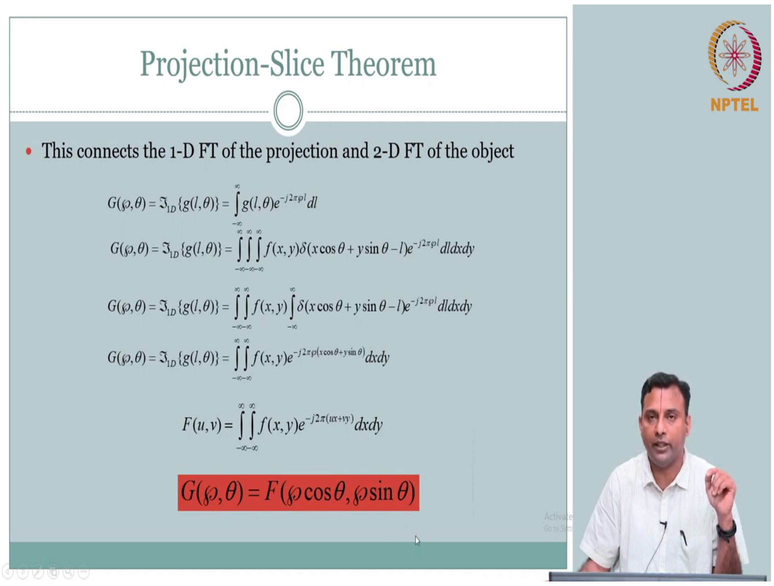

So, that is what we will do. So, we will take a look at it and we will talk about what is called as a projection slice theorem. What is projection slice theorem? That is what you have is a projection right, what you collect is a projection. What is a slice? Slice is something that you are getting this projection from the slice. What do you want to do? I want to use the projection to reconstruct the slice, right. So, is there a relationship between the projection and the slice?

From what we have seen so far, yes this projection is nothing but line integral right; from the slice you take a line integral and that along the dimension is projected onto this point. So, we know that in the spatial f of x comma y coordinates, we know the relationship between projection and slice; but that is not what we are interested, what we want to know is this relationship between the projection and the slice in a different domain.

What could be that domain? We are always we spatial domain or frequency domain right, time domain frequency domain; so spatial domain or spatial frequency domain. So, there is another way we can look at things. So, if I have my projection, I know radon transform that is the relationship between slice and projection.

Is there a relationship between the Fourier transform of the projection; because that is 1 D signal right, your projection is a 1 D signal we saw that. So, is there a relationship between Fourier transform, the 1 D Fourier transform of the projection and 2 D Fourier transform of the object, right.

So, your projection slice theorem essentially connects this relationship of how does the Fourier transform of your projection relate to the Fourier transform of your object; object is 2 D, projection is 1 D. So, the 2 D Fourier transform of the object you have, you can get 1 D Fourier transform of the projection. Is there a relationship between these two?

In the spatial domain, we know the relationship is radon transform between the slice and the projection. In the frequency domain, is there a relationship between these two, ok? So, we will try to we know all the material in pieces, separate pieces; but we will we will we will have to connect them in the context that we have. So, we can write down for any signal, we can write the Fourier transform, right.

So, we are just going to use g of l comma theta is any signal for me right; I am going to take the 1 D Fourier transform of that, which I will refer as capital G of this rho comma theta. What is Fourier transform 1 D Fourier transform? G of l comma theta e power minus j pi this is arbitrary variable denoting the frequency, ok.

So, this is your Fourier transform; we know that I have just used a variable, so that it is consistent with the ongoing context. So, I have written g, small g as your signal which is your projection signal; but nothing more than a regular 1 D Fourier transform formulae, ok. So, now, what I need to do is, I need to find out if how does this relate to 2 D Fourier transform of the object. What is my object? F of x comma y right or mu, but ok f of x comma y.

So, we have to try to relate what will be your Fourier transform of f of x comma y, how do you relate this to that is our object. So, what we will do is, we will start to write out; what we know is the relationship in spatial domain between the projection and the f of x comma y.

So, we will go here and say I know my projection, what is my projection? Projection I actually got it from my f of x comma y. How did I get it? I got it because of remember it is a sum along the line, so we use the line delta; so essentially I will express g of l comma theta or expand it, it as a line integral, right. So, x and y, so you have two integrals for the two x and y; I have written this in this form where I am picking from f of x comma y using a line delta, straightforward right this is how we represented the start, what is your line integral, ok fantastic.

So, now, you look at it, what do you do? I have f of x comma y, I have three integrals; can I squint my eyes and look at it and say look I have a delta function right, what does this, can I regroup them, can I see if I can simplify it any further.

I have a delta function right, I have a delta function exists when this guy gets to l, right. When this guy is l, that is when this exists. So, instead of grouping it with this guy, can I group into this guy, you can do that, right. So, what happens if I group it to this guy? Delta is going to pick only when l is equal to x cos theta y sin theta delta is going to exist; so that means when l is equal to x cos theta y sin theta, only then this is existing and therefore this will right, I can kind of conveniently look at it that way.

So, I will split it, I will group right f of x comma y; I will group this delta function not with f of x comma y, I will group it with my exponential here minus infinity to infinity delta function of this. So, quickly I can reduce this right; because I know when delta function exists. So, I can quickly reduce that.

So, it will be f of x comma y exponential of when l is equal to x cos theta plus y sin theta; fantastic not done anything fancy here, we are just putting what we know, but conveniently looking the grouping from a mathematical perspective, right. But when you do that from a physical perspective what is happening, how do you interpret this now? I have seen this before, what is this; f of x comma y I have two integrals d x d y exponential of negative of something.

Immediately you should recall that, this form I have seen; if and also now that I know this variable has to do something with frequency, this is nothing but my 2 D Fourier transform of the object f of x comma y. In fact, you recall the formula, we wrote f of u comma v your 2 D Fourier transform of f of x comma y in terms of frequency, u frequency and v frequency; only that here it is arbitrary, it is some other variable is used,

But this is a frequency, right. So, in fact these two become identical right; you can really recognize this as a 2 D Fourier transform, if you make this u right, here rho x cos theta plus rho y sin theta. So, you can recognize that when your u is rho cos theta and v is rho sin theta; then this is exactly your Fourier transform, 2 D Fourier transform of your f of x comma y.

This is fantastic, what this is saying is; if you organize your data correctly and you get from one domain to the other, you actually see there is a relationship between the 1 D Fourier transform of the projection and 2 D Fourier transform of the object. What is that relationship?

This 1 D Fourier transform of the projection is nothing but in the 2 D Fourier transform of the object, you evaluate along a line right which makes which passes through the origin, which is at a theta value that is what it says.

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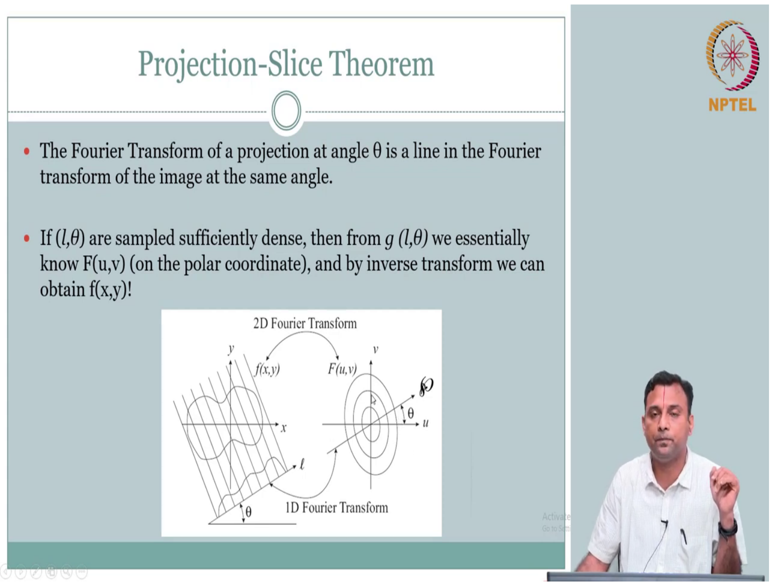

So, the Fourier transform of a projection at an angle theta, Fourier transform of a projection at an angle theta is a line in the 2 D Fourier transform of the object at this along the along the origin at the same angle. So, if I did a projection at 0 degree, then in the Fourier domain that 1 D Fourier transform of the projection will sit along the 0 degree line or same angle of your u comma v space.

So, if I make my projection at 45 degree view angle, the Fourier transform of that projection at 45 degree will be in the 2 D space will be aligned in the 45 degree orientation, clear. So, this is very powerful. So, if l and theta are sufficiently sampled right, if that is the case; what have you done? By using this theorem, we are essentially saying I get the Fourier transform of the object from the collected projections right, just by reorganizing the data.

So, I got my projections, I take the Fourier transform; this all I can do, so 1 D signal, I take 1 D Fourier transform. But now I am organizing it in the 2 D space; whichever theta I got, I aligned them exactly in that theta in the Fourier space, the 2 D Fourier space. When I do that what do I get? I end up getting the Fourrier, 2 D Fourier transform of the object.

What is my goal? My goal is to get the object; I have bunch of projections. What I am getting by organizing and doing a Fourier transform? I am getting the 2 D Fourier transform of the object. So, how do I get the object? Done inverse Fourier transform, right. So, intuitively you see this powerful relationship because of the projection slice theorem, ok.

So, you have an object, you project it. So, you get a projection at a particular angle; when you have that projection at a particular angle, it turns out that the Fourier transform of that is along a line, also oriented at the same theta in the 2 D Fourier space, 2 D Fourier transform of the object, right.

So, only thing is you have to recognize that; you are writing instead of f of u, it is on the polar coordinate, because of the way right l comma theta. So, you are getting the projection, you are taking the Fourier transform and you are placing it at an angle. So, you are going to get you, you can imagine that this f of u comma v space, you are filling it in a polar format, ok.

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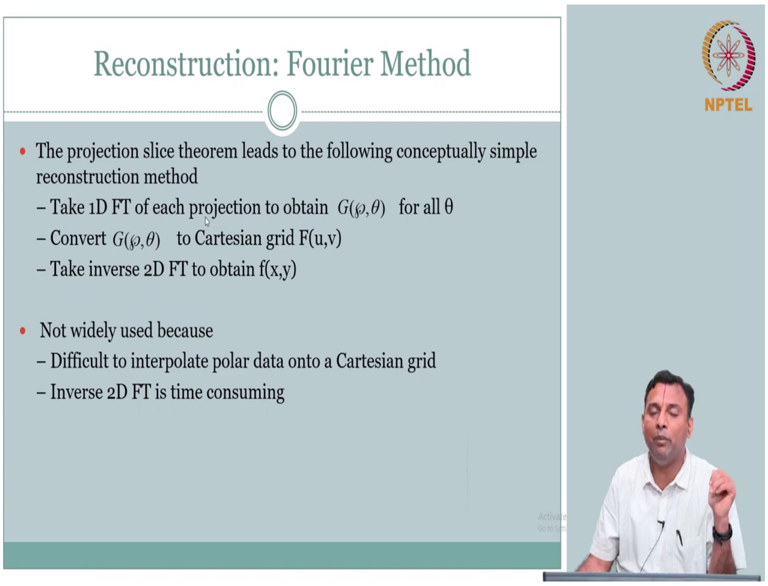

So, now get back to all the reconstruction, from now on it is a breeze; we are going to cover beautiful algorithms right, but you know a big picture, that is all it is straightforward, ok. So, first is Fourier method, what do we mean by Fourier method? Directly from the projection slice theorem, we know what to do; what we have got is, we got one projection, take the 1 D Fourier transform of it, right.

So, you will get 1 D Fourier transform at a particular theta. So, you have a bunch of data at different thetas; take 1 D Fourier transform of all that and align them, right. So, convert. So, you can get capital G of rho comma theta right; you are collected the 1 D Fourier transform, organize them because of projection slice theorem in the Fourier space with the corresponding theta.

Once you do that, you get your G of rho comma theta; but this is in polar coordinates, so convert that to a Cartesian grid. Once you convert that Cartesian grid, you get f of u comma v. If I have f of u comma v, what do I want f of x comma y. What will I do? Take inverse Fourier transform simple, right.

So, take inverse 2 D Fourier transform, you get the object f of x comma y, clear. So, this is one simple reconstruction method; however it is not that preferred, why because you have to do this conversion. So, you have to do some interpolation, convert the polar coordinates to rectangular coordinates and then do a 2 D inverse Fourier transform.

So, intuitively it is straightforward; but we do not really do it like this, because of computational reasons it is not convenient. So, what do we do? Well, concept is correct maybe we can look at it how to regroup and do some maneuvering, so that we still are able to implement in spirit the steps; but maybe it is convenient from a computational point of view, right.

So, what we need to do is ok, we got to the Fourier transform, we understand this idea; can we now go back and look and see if there a better way to implement this, ok?

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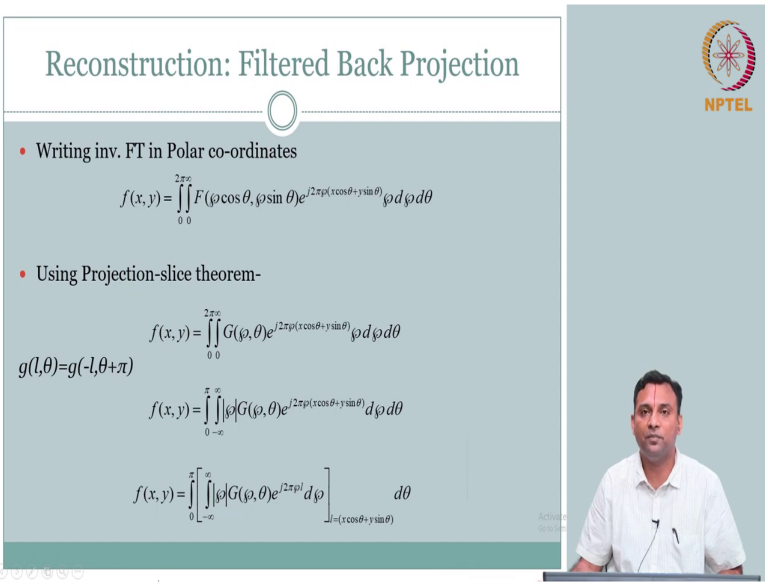

So, then what we call as filtered back projection. We already saw back projection ok, we already saw back projection. What is the problem in back projection? We did not have any fourrier slice theorem that time; it was just common sensish, we just did what we would do. But the problem was there was a blurriness; but now we come to an algorithm called a filtered back projection ok. So, we will see what is happening; maybe that is a missing link that we did not incorporate when we were just thinking through the problem, ok.

So, what we do now is ok, I have we have this bunch we have this Fourier transform; I can collect the Fourier transforms of the projection and organize them. What happens if I write my Fourier transform, inverse Fourier transform in the polar coordinates, right? If I do instead of converting to rectangular coordinate, how does the Fourier transform inverse Fourier transform look in the polar coordinates, right?

So, f of x comma y, if you write it in the polar coordinates; so your r d r d theta right r rho d rho d theta. So, you can write in Fourier transform in polar, inverse Fourier transform in polar coordinates. But then the point is I want to look at this and see whether I can simplify right; can I see it differently, how do I simplify this that is the problem.

Well, you quickly put the context I have 0 to 2 pi, 0 to infinity. What is the meaning of theta here? The different projection angle. What do we know? We know after 180 degree, you are doing the same thing right; you are doing it will be reverse of whatever. So, remember we always collected only till 180, because whether you are going from here to there or there to here, it is along the line you are integrating. So, we said that is not going to change much.

So, we did not collected the data, in fact we recognized the 0 value is 180 and so on and so forth what you would get; only thing is that depending on the view you could have negative sign right, you could have. So, what we quickly recognize is the 0 to 2 pi does not, it is not a big deal; we are going to collect the theta only from 0 to pi, right.

And also, so we will essentially from what we know using projection slice theorem right; from using projection slice theorem, I will just substitute this guy f of rho cos theta, rho sin theta is your capital G of rho comma theta. This is from your projection slice theorem and I recognize that 0 to 2 pi is not going to happen, 0 to infinity.

So, now, G I know what it is, my G is the projection and so I know g of l comma theta is same as g of minus l comma theta plus pi, this I know right; 180 degree view after 180 degrees. So, this I know, for the projection, I know this behavior, right. So, what does that give me? That gives me the liberty to just do some rearrangement.

So, what I can do is, f of x comma y I can write it as 0 to pi minus infinity to infinity, this can become modulus right and then G rho whatever. So, how does this look? How does this look? Again if you look at it, something should be very familiar; I have a frequency domain, I have plus e power those frequencies, integrate that my output is in spatial domain, right.

So, what do you see? Essentially I am multiplying this frequency response of my projection, right. So, this is frequency domain, I am just multiplying it with some frequency function. So, this can be seen as, I am filtering right, this can be seen as a filtering operation, right.

So, now what is happening, I see this as a filtering operation and then the 0 to pi; if you read it loud, it is summation right integration, summation of the different angles. So, something rings a bell, I have some filtering; so I can see this as filtering, I can see this as summation, is there something that I am leaving it deliberately, what is the other integral doing, right.

So, write this as 0 to pi minus l to l sorry minus infinity to infinity all this, of course I can recognize this x cos theta plus y sin theta as l; anyway that does not vary right, your running variables has nothing to do with your l. So, it is evaluated at a particular l. How does this look? What is this? Can you relate it to the title of the algorithm, ok?

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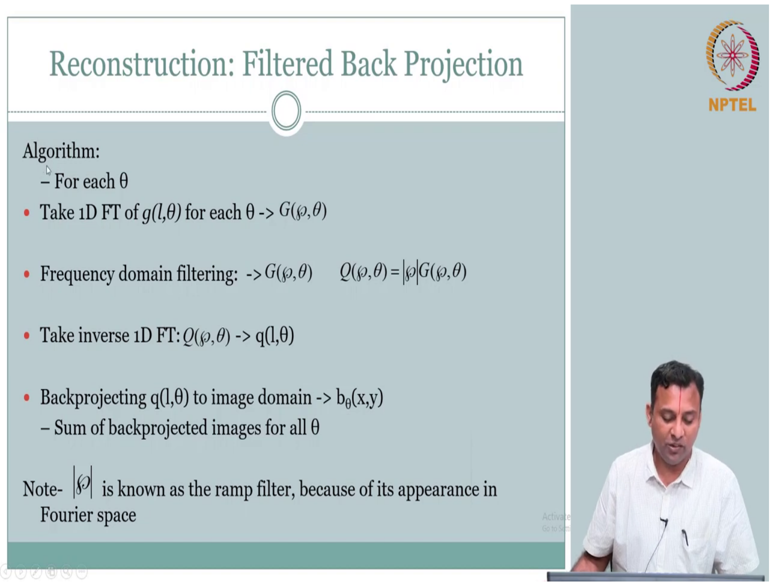

So, what you have done just to recap? For each theta you have taken the Fourier transform of for the projection, right. So, you have g of l comma theta is measured at different. So, the projections are measured for different theta. So, for each theta you take the 1 D Fourier transform, you get G of capital G right your frequency domain representation of your capital G.

So, now, this is a Fourier 2 D space. So, now, frequency domain filtering. So, I can see this filter that you had right, I can see this as a filtering operation. So, I have a filtering operation, I have a filtered value. Now, what do you have? If I take 1 D Fourier transform, inverse Fourier transform I get q of l comma theta; so your q of rho comma theta right, so now, it is a filtered version.

If you take inverse Fourier transform, which was the operation that you saw in the previous slide; inverse Fourier transform you get q of l comma theta. What is this? This is similar like your back projection right; back projecting q of l comma theta to image domain. So, now, I got my filtered projection that is what it is, I got my filtered projection.

So, when I have a filtered projection, I can do the same back projection algorithm, which is I can construct my b of theta comma x comma y right; sum of the back projected for all theta, which is what the formulation in the previous slide. So, it essentially talked about filtered back projection. So, instead of back projecting the, back projecting the projections that we got; what we are now back projecting is, the filtered version of that. Where is that filtering happening?

That filtering happened in the Fourier domain with this operation. So, perhaps when we do it systematically right, when we write it in polar coordinates; that r the rho right length in the frequency domain that, that ramp that was acting as the filter, that was kind of missed when we did only in the spatial domain, we did not incorporate that, ok.

So, this is known as the ramp filter; why, because you this is in the frequency domain, right. So, with respect to frequency if you plot this, this is going to be your mod function. So, it is going to be a ramp, it is going to go up like a v, right. So, that is why this is called as a ramp it is ramping up. So, clear filtered back projection. So, we back projection made sense already that, missing piece was we did not think about that filtering.

But when you actually write it out as a formulation of inverse from starting from projection slice theorem and incorporating the polar coordinates; then you recognize there was a filter that is happening analytically and that is the missing piece, ok.

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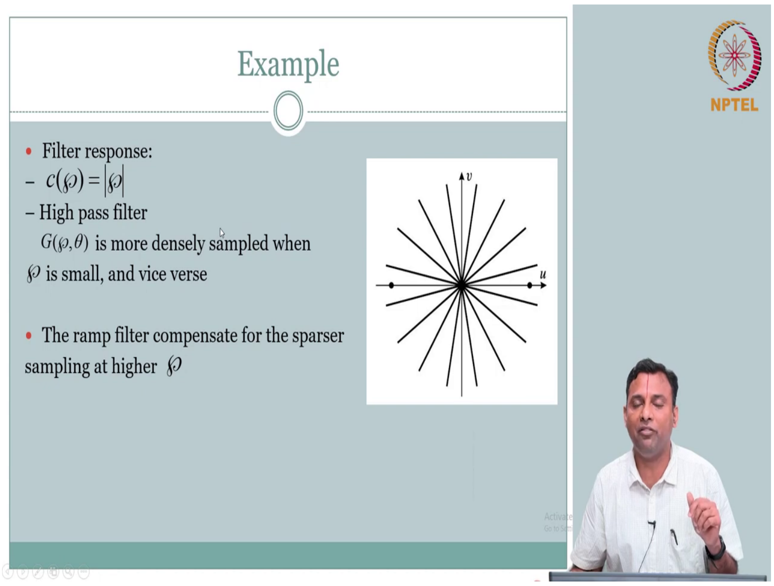

So, let us just quickly take an example to appreciate what we have done so far. So, I say that you have sampled right, you have a Fourier space and you got the data; you got projections at, so eight different angles, eight different views and I have aligned that, this is my eight different views. So, I got my g of l comma theta, I have taken the Fourier transform and I am organized that in the space. So, now, if I tell you the object from where you got the projections is nothing but a cosine x function.

So, essentially remember there is a when, we talked about this, you had frequency only in x direction, spatial frequency in u right, spatial frequency is u we talked about. So, when you have a cosine oscillation along the x direction with the frequency u right, what will be; so if that is the object. So, remember the objects that we started when we did 2 D signals and how the sinusoids look vertical line, horizontal line.

So, like that we have a cosine function right, some oscillations at a particular frequency; it has a u frequency alone. How will that object look? If I take the projections at eight angles, this is the data that I get. How will the object look in my reconstruction? Well, just now we said no problem; reconstruction is what this is the Fourier transform of the object, inverse Fourier transform of this is going to be the object.

But what is going to happen? My object I said this going to have a cosine function only in u directions right, there is a frequency only in x direction; so that means I would expect a frequency to be as a delta function on the u axis. Where is my data I have sampled? I have no one, two, three, four, five, six, seven, eight; I have five thetas, but when I did that, I do not have the axis is not sampled.

So, I do not have. So, the frequency is present only in this axis in the object. So, I will not, I will not get to see the object, the variation, the cosine is lost; why? Because I did not sample enough, right. So, the problem is, you have your filter response, right. So, this is the point, this is where the data is; that is where the frequency of the object is.

But when I sampled, I did not really sampled that frequency; I got different projections, different views. In one of my views, where supposedly the signal is present, the frequency or oscillation is supposed to be in this direction, I did not take that view. So, I do not have it in my raw data. So, I cannot I have lost it completely.

So, it becomes important to appreciate that, dense sampling of the space is important. When I talk about dense sampling you notice; by because of this polar coordinate and the way we have done it, there is always going to be dense sampling in the low frequency, as you move away the spacing is going to spread out. So, it is going to become sparse as you move up, close to the center you have more information.

So, if I do not. So, what is my filter doing? My filter this is a ramp filter or this is a high pass filter; what is it doing? So, it is passing very little around the center; as you go up, go out right higher frequency, it is giving more weightage. Why is that important? Because I am going to have fewer signals; so at the zero which is a dc, there is no oscillation, if you just without filtering if you do, all the contributions will come from here, because there are many samples here.

So, what does that mean physically? Zero frequency, it is going to all add up more, it is going to increase the bias; whereas the high frequency because there are fewer samples, they are not going to be seen, because you have many of these.

Now, you recognize without doing any filtering, when we back projected, you had a whitewash right; that was the basically a whitewash, meaning there is no oscillations, there was a systemic bias and that kind of spread out the image, throughout the image. So, your contrast went down; whereas here by applying this ramp filter, we are accounting for this discrepancy, which is as you go higher in the frequency, you have to give more weight, because you may have less number of samples.

At the center, you may have more dense denser sampling and therefore, you want to give less weighting to that; that is what this high pass filter is doing, is more densely sampled when rho is small and vice versa. So, this is what it is trying to compensate, ok. The ramp filter compensate for the sparser sampling at higher frequency. So, just to put everything in context; so we got filtered back projection, Fourier method, filtered back projection, of course in the filtering, now it says just maneuvering, ok.

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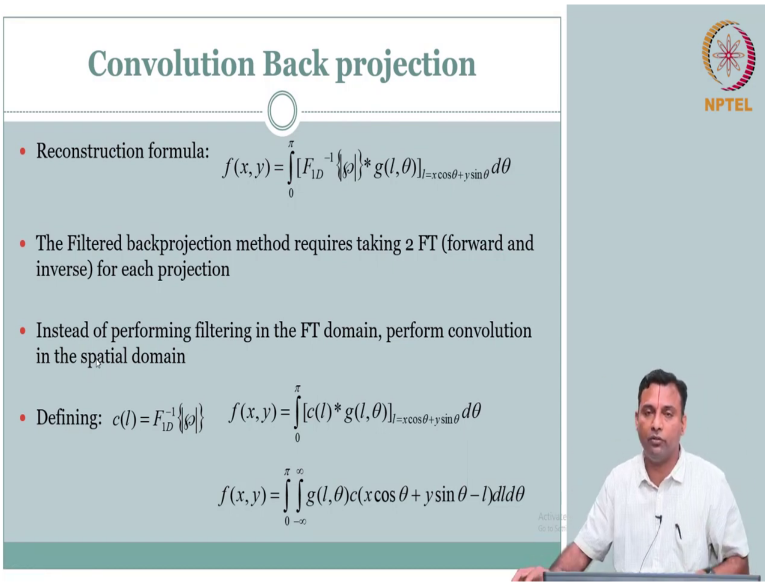

I like filtered back projection, but you know what why should I do the filtering in frequency domain; why cannot I do in time or space domain? You can quickly recognize ok what is frequency domain, you multiplied; if you have to apply the filter in spatial domain, it has to have convolution, convolution back projection. So, it is I mean small way it is a it is a variant and the reason for that is, it is convenient to operate, ok.

So, reconstruction f of x comma y is you take the inverse Fourier transform of the ramp function the filter that. So, this is your filter coefficient, convolve that with your projection and sum over the different back projection sum. So, nothing but instead of doing the Fourier transform and then multiplying with this frequency response, this ramp filter; you can bring this ramp filter to time domain, get that coefficient and convolve with your g of l comma theta, that is called as your convolution back projection method, ok.

So, the filtered back projection method. So, in filtered back projection method, you have to do two Fourier transforms right; you have to first take Fourier transform, go and then come back taking inverse Fourier transform for each projection.

So, you know there is lot of computations instead of doing that, we could just compute this ramp filters coefficient and keep it in time domain. And so, it becomes very implementationally efficient right and therefore, instead of doing filtered back projection; I would rather do a convolution back projection, because all I need to do is this can be precomputed, this does not depend on the data.

So, I can have this filter coefficients ready, the moment you are recording this g of l comma theta, just convolve with them and integrate for different theta; instead of performing filtering in Fourier domain, we perform convolution in spatial domain. So, this is very convenient and so, therefore it is very popular; some variation of this algorithm is what is routinely used in commercial scanners, ok.

So, I can compute this and keep my coefficients, filter coefficients are nothing but inverse Fourier transform of the ramp filter and therefore, you can readily implement this, ok. So, nothing fancy about this; the only thing is the ramp filter is not convenient right, it has hard boundary, hard boundary, it goes and falls of.

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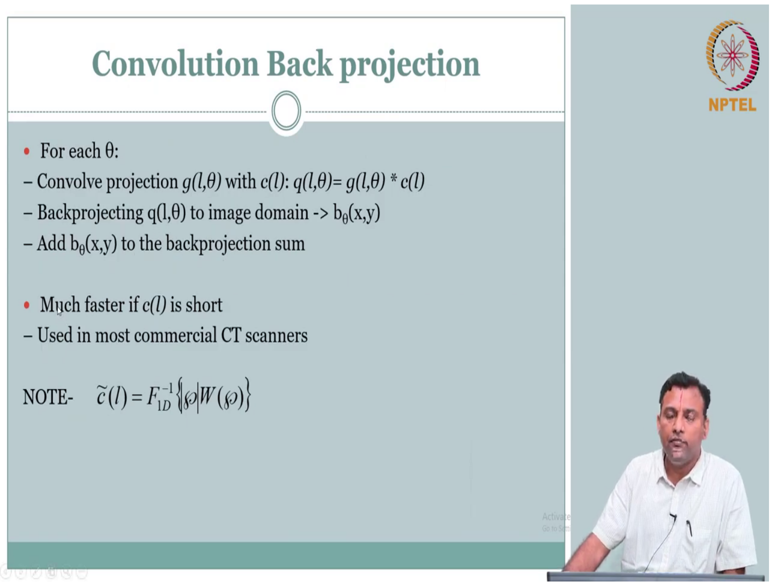

Therefore, instead of that, we may want to use some windowing functions, ok. So, for each theta, you do this convolution g of l comma theta is convolved with your filter coefficients; then you do the same back projection, you get this b theta of x comma y and then you do the sum same back projection algorithm, only thing is that projections are filtered and it is filtered in spatial domain using convolution of with the filter.

So, this is very efficient if this filter is short filter right, because you have less number of computation. So, most commercial scanners use this; of course you cannot get perfect c, you have to get an estimate of that. And usually what they do is, instead of perfect ramp; you have some windowing function, ok. So, that it is mathematically tractable and therefore, you get an estimate of your filter coefficient.

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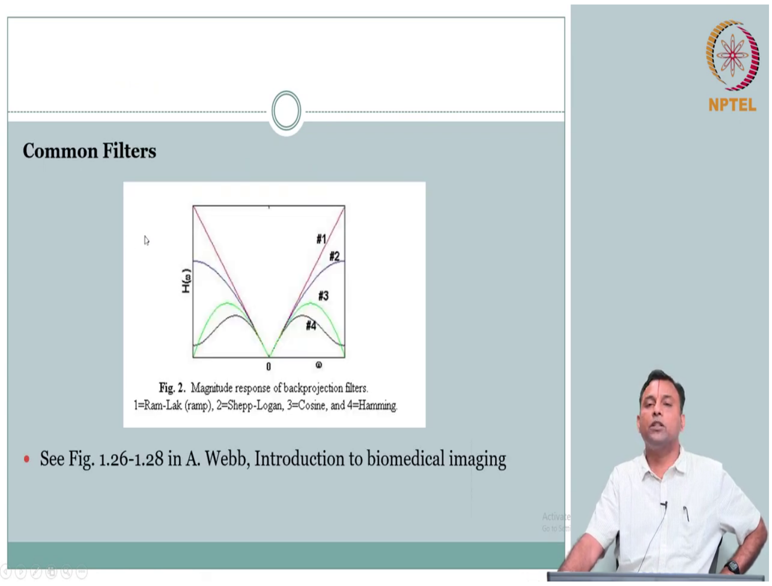

So, we will just quickly some common filters, this is your ramp filter right; but then you can typically operate that with a window, some of windows are given here, handing window, hamming window, hand is not given n. Hamming, cosine right; so you have several windows very similar to your windowing functions that you do in your signal processing for example, right.

So, you can make it smooth. So, still the same concept of going to 0, you 0 frequency you have less response; when you move away from that, you have a better response. So, this is windowed function and so this is straightforward right; this coefficient can stored done on the fly clear. So, this is for your convolution back projection.

So, with this I think it is a good time to stop; because we have covered the reconstruction mathematics from very you know rigorous way in some sense with respect to using the projection slice theorem, we showed how we get at these formulation. But more importantly, we also understood back projection as a very straightforward intuitive concept, ok.

So, without doing too much sophistication, from simply know using what we knew from before, putting them together in the context of our problem; we actually covered from real powerful algorithms. Of course, starting from here, we will have to do few more things, which is this parallel ray projection. So, if you change your detector to fan beam, we will have to you know change the formulation to account for the geometry, then your cone beam. So, that is there.

So, it is simple geometry of parallel ray we have developed, but the concept is powerful; after this it is going to be very careful in mathematical description of how you want to organize the data, right. Because the geometry is changing, that will be the key; but otherwise concept I think this is very powerful, this can used like example that I told right, it has nothing to do medical imaging, right.

But the mathematics, the understanding this formulation can be used in verity of field; the physical meaning of the you know the values that you have, the physical meaning of that depends on the context. In our case we are going to call that as mu, which is the attenuation coefficient; whereas the example that I showed it is like the person's ability to steal, right. So, it is you know very powerful, what we have covered is very powerful.

So, we will stop here; what we will do in the subsequent lectures is, kind of go through some examples of all the steps, review the steps, go through some examples and then see how we can spread out to fan beam and cone beam. I think that will cover the reconstruction part that we want to, ok. For now we will take a break here, please go try out different objects, see if you can get the projections; because after you get the projections, you can do back projection, you can do sum of back projection, you will see the image.

So, take simple geometries square, circle at the center or something like that; play with this, play with this and see if your estimate is close to the object that you had.

Thank you and see you next time.