Introduction to Biomedical Imaging Systems
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Lecture - 25
Fan beam_IQ

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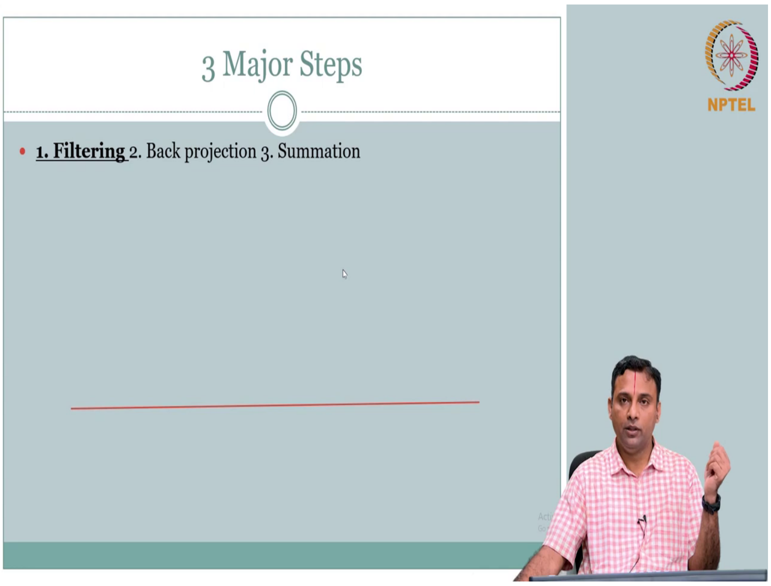

So, before me we move on just to recap, we have three major steps, all of this reconstruction that we saw so far has three major steps, first is your filtering right. So, you get your data acquisition as projections that projection data has to be filtered. So, we had filtered, the filtered can be filtering can be accomplished in frequency domain or in time domain.

So, that came as filtered back projection or convolution back projection, but filtering is the first step after you collect the raw data and then, you have to do back projection finally, you have to submit. So, these are the three major steps. So, what is a raw data? It is a collection of; it is a collection of projections right. So, we talked about how to view them. How did we view them? we could plot them as a sinogram, image of the projections as sinogram.

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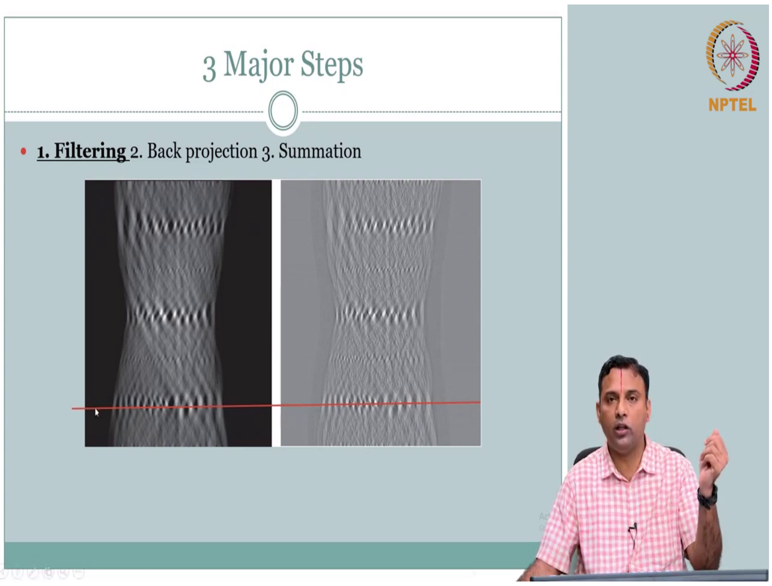

So, the raw data what you get is the sinogram, this is the one that we saw before and then, when we did the reconstruction formally, when we built that material, we realized when you project this back by itself, you have a hazy picture and that is not appropriate. So, what we need to do is take a projection, apply filtering. So, after you do that line by line right for every view angle whatever projection you get, you do the filtering, you get a sinogram like this.

Notice that it appears in the sinogram, it appears of poor contrast compared to the sinogram on the left, but this is not of our interest, this is not an image of interest for us, we do not want the blurring in the final image. So, we are not really interested in making the sinogram look good. So, this is a filtered version.

The second step is so, for example, when we did this earlier, we said take a line and then, we have to back project. So, at every view angle, you have a projection, now that projection is filtered, you have to back project in the field of view at the orientation, along the orientation in which it was acquired right.

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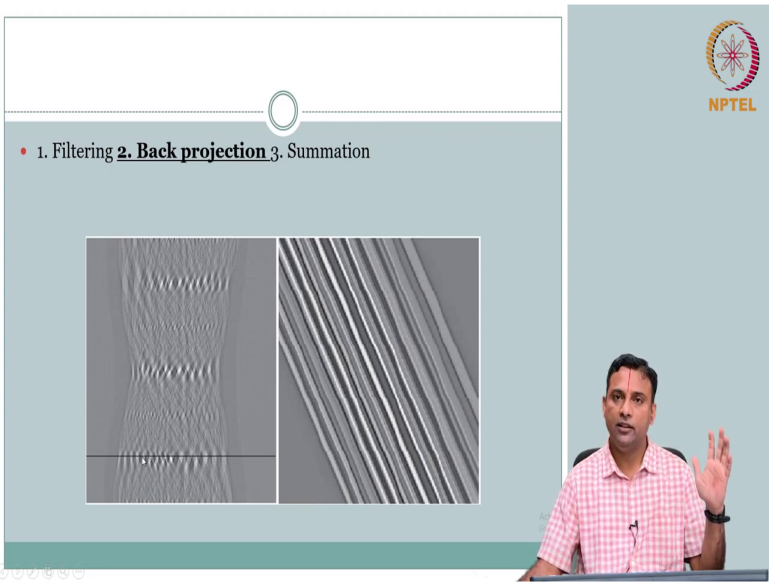

So, we took a particular angle, remember b30 is something that we showed as example before. So, you take the projection along this line right and that one you project it back along the field of view at the angle so, this angle is the angle that you see here ok. So, this is your back projection, but the only same as back projection, we saw before now we start with the filtered projections that you back project and then finally, summation.

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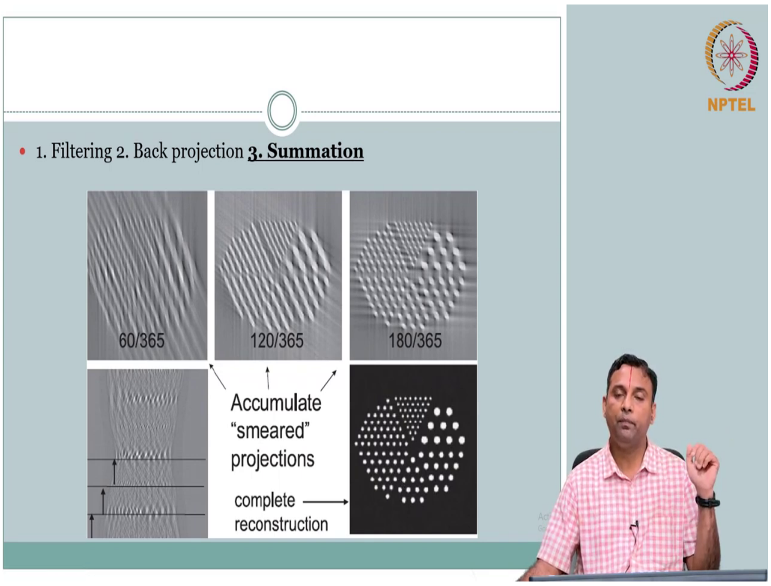

So, you notice each of the back projection image itself is lousy, but when you start to add them right, more the number you start to add, you get the final image, reconstructed image like this. Now, you compare this image right, you can go back, look at the notes when we started, we gave this template image, we said ok what happens to this image and look at the contrast when we got only when we did back projection not filtering right. This one is pretty damn close, this is like an object right, this is fantastic..

So, how did this get? The more each one of the back projection itself contributed little bit, but then, when you start to add all of that, you get a complete reconstruction which is pretty damn good. So, this is all it is this is what we covered ok.

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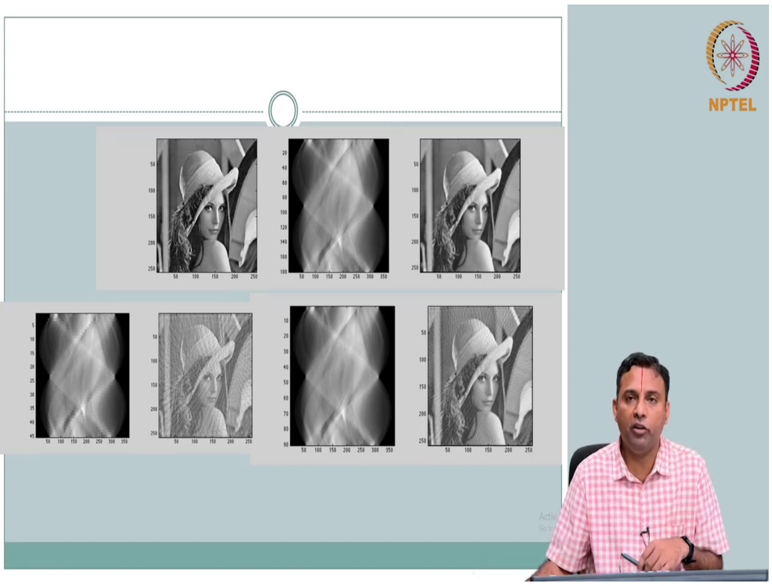

So, moving forward, these material is important that you actually try it, there is no I mean this is not going to be paper, we can do some exercises, but then, these are algorithms and these algorithms you learn, understand more, visualize more when you implement it.

So, please take an effort to whatever you know way you have, simple back projection of square, cube right which we didn't, you can try different geometries which are easy to get a feel for it, but if you have a real object to project that, you cannot use paper and pencil and calculate right it will be tedious.

So, you cannot say no, I do not have X-ray raw data and therefore, I cannot work on it, that could not be an excuse here. You know very famous image from image processing textbooks examples that you will find Lenna dot JPEG right, this is a very commonly used example for image processing in classic textbooks. So, what you can do? You can actually take this is a matrix right.

So, what you can do is take this raw data and then, this is your image, you can create projections right, you can write program in whichever language you have, you can write program to basically collect the projection some along the line in each of the different orientations.

So, you can create your own sinogram for this and you can play with this, how many projections can I use? What happens if I reduce the number of projections? What happens if I reduce the sampling along the l right? If this is l, you are going to project from top, down like we did for the square example, this 250 is your l number of elements, this is 0 degree.

So, if I have line projection, you add all this and you get one value here, add all this, you get another value so, you get 250 array with 250 elements and each value is sum along this 250 right. So, you can do that, you can write program to create whichever projection angle you want.

So, you can create this sinogram and then, see what happens if I b sparse, if I do not use certain view angles? What happens if I reduce the number of view angles? What happens if I reduce the number of projections, number of samples in the projections? All this you can try it on your own right. So, this is something that I encourage each of you to try on, it is actually very interesting ok. So, you can take any image and do it ok.

So, we need to move on to say fan beam so, we will not do exact derivation like we did, we will not go step by step like we did for parallel ray projection. For fan beam, we exactly understand the instrumentation right, we talked about the physics so, we know what, why, when a fan beam is employed, and we now know the physics of ok the reconstruction is along the line you take and you back project. So, here, you back project and the back projections are all parallel.

So, what happens if it is a fan beam? It is not going to be parallel. So, the only major difference is going to be how do we align our data right. If we get that, rest of the equations you substitute back in the way we did the back projection algorithm for parallel ray, you have to modify that ok. So, we will not go too detailed in this because it will be an advanced topic if we really go into the detail, but just for completeness, we will highlight how you have to modify, where you have to modify and if you modify, what happens.

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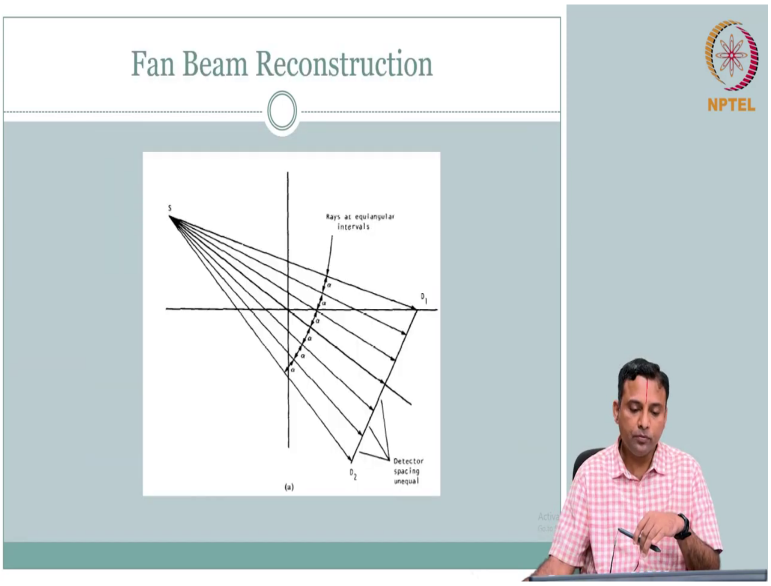

So, fan beam reconstruction. So, we can start out with first geometry right because it is not parallel ray, first thing is this is fan. So, when you say when it is fan beam, you have to know how to describe the geometry. So, if you have a so, this is like your 3rd generation system where your source and detector can move around right so, this is a fan beam.

Notice different configurations that are possible. You could have the spacing between the lines to be angle could be same ok though that will if that is the case, if this is line, then you may have unequal spacing here, then you might say no no, I do not care about this, I want my l direction, I want evenly spaced detector. If that is happening, then maybe your angles will be different ok.

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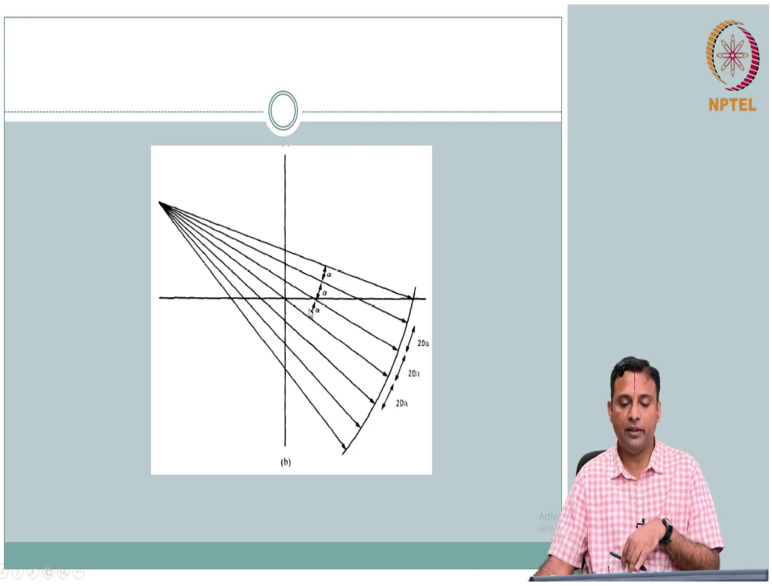

Or if this is not a line, if it is going to be a curve, then we have a better situation where I could have a constant angle and a constant spacing. So, what we will do is we will take a simple case where it is ok, we have a constant angle right in this configuration like this. So, we will start with this and the reason where you can expect a change is this is the line.

So, we started out our reconstruction pretending this is the detector and it is summing everything along this lines and then, we had to back project and the geometry was parallel whereas, here, you have to back project along lines which probably can be described more in the angle or length depending on how you define the system ok.

So, we will take this; this configuration. So, if we take this configuration, is this sufficient for us to describe the geometry? Well, no. What do you have? You have a source, you have a object, you have a detector. So, let us we have to have a reference frame so that you have a field of view where the object is placed right in relation to the data acquisition, one side is your source, the other side is your detector. So, we will take iso center in this configuration that is where prop the patient is there so, that is your object near the iso center. On one side, you have your source, the other side, you have your detector.

So, now, how do I? I need to have some way to tell which is my view angle right, this is going to rotate around the patient. So, I need to know the view angle comes from where my source, this source which angle is it right. So, and then, how far the distance is, how far is the source from this iso center, how far the detector is from this iso center, how far these lines are from the iso center?

So, if we can define everything with respect to one coordinate system, then we may be able to quickly; quickly integrate that or you know look at our reconstruction algorithm for parallel ray and make appropriate changes in the geometry.

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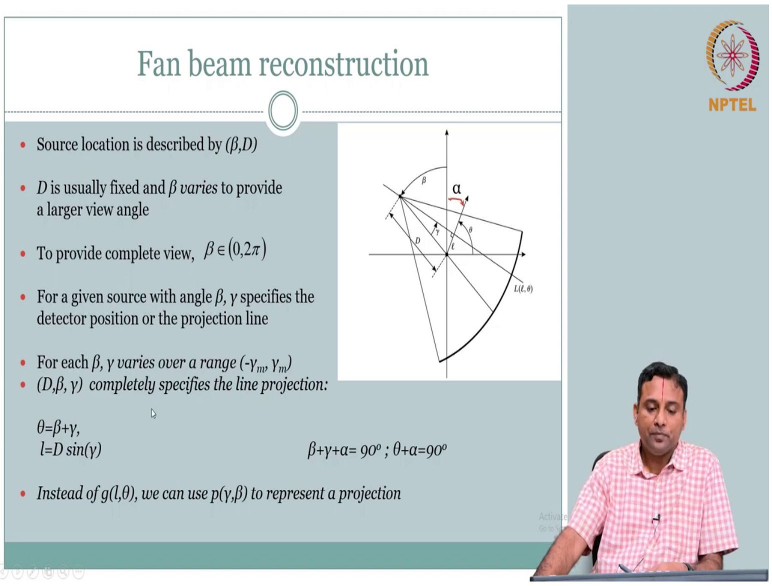

So, what we will do is the fan beam reconstruction, we are going to call so, this is the source so, the source is going to move around the patient so, it is going to be at different angles right so, that angle we will call at beta, but each beta, you have a fan beam, the source is sending out fan beam.

So, the source can be at an angle beta, but within that fan beam, you can refer to the angles as gamma right, the lines that are going out in this fan, they can be at an angle gamma. So, we can use this to describe the angle of the rays that are coming out.

So, likewise, we could also use right this line can be referred to by your l, this is your theta right. So, we can refer to this line in l comma theta. Why do I want to do that? Because, we derived all our material using g of l comma theta ok. So, here you notice g of l comma theta is fine, but everything was parallel so, only the for a given view, only the l's were changing whereas, here, for a given view, l is changing with respect to where you are on this line ok. So we will come to that.

So, then, you have to place your sources, how much distance is it from the center? So, that is capital D ok. So, we can define this L and this is an angle, we will talk about this alpha, alpha is nothing, but this angle right, what this perpendicular is making with respect to the vertical ok.

So, what we have is a source described by the angle and how far it is located from the iso center. Usually, this is fixed right because the gantry is there and the patient is sitting so, usually the D is fixed, but you can change your beta, you can coverage can be increased or decreased depending on the beta.

So, to provide a complete view, what do you need? I need to be able to rotate around the patient so, I will have complete view I can get by beta taking 0 to 2 pi. So, essentially, what we are saying here is if you are given a beta right, if the source location is decided, the view angle is determined, then within that view angle each of the lines are determined by your gamma.

So, gamma specifies the detector position or the projection line. So, this is defining that. So every time you take a data, your beta right your view angle could be different so, if that is changing, then the gamma is changing over some minus gamma m to plus gamma m depending on the angles that you are taking ok.

So, in some sense your D beta which defines your source and for a given source, the fan angle of the line right gamma, they completely specify the line projection. So, they if you to tell me these three, we know exactly which line we are talking about ok. So, in from the geometry, you can see that theta is nothing, but beta plus gamma and your l is D sin; your l is D sin of gamma; your l is D sin gamma. Also notice that this is 90 degree and alpha plus theta is also 90 degree.

So, essentially using this geometry, if you go back, we can recognize that we had g of l comma theta representing the projections. Instead of g of l comma theta, now we can call to avoid confusion, we can call it as projections at gamma comma beta, beta is your what was theta before right? View angle so, here we are referring that to as beta. Your gamma which was aligned there, we did not care about that now, your gamma is specifying the line which detector location. So, we can use p of gamma comma beta to represent projection.

So, now, you see you have to maneuver this and then, the same filtered back projection that you are going to use. Only thing is you your you now have a geometry where you have defined where you got the data so, you have to project it back along this geometry that is the key ok. So, before we so we can do that and we you can do that for both filtered back projection and convolution back projection, both you can derive.

Start from the equation, substitute these changes, before we run on to do that, there is one more effect that we can see I mean one more coordinates that we can introduce why because you want to project it back, you will realize that it may be convenient because of the geometry, it may be convenient to represent a point in r comma theta rather than a line right, line was good when it was a parallel.

So, now, maybe in this scheme of things in the reference coordinates, a point can be better represented, or better tracked down if we can write it in terms of r comma phi some polar coordinates ok. So, to do that, we kind of start to use different variable, variation from here because you have to transform that. So, we will now call this right D dashed and then, you have some phi angle so, we want to relate everything in terms of D dashed and phi. We want to refer to this point in terms of D dashed and phi instead of l and theta ok.

Just for operational convenience ok so, you do this coordinate, you do this geometric transformation of the variables in your back projection. The some of the steps are actually given in the textbook, you might find if the reference textbooks also have these derivations ok essentially, substituting back into your formulation is there.

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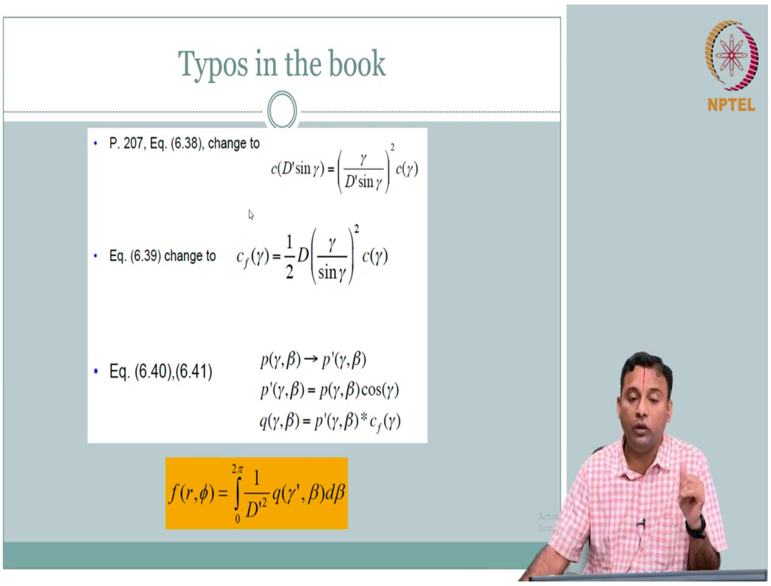

So, while you so, we are not going to derive that is little beyond the scope of what we want to cover at a introduction level, but if you were to go, look at the textbook, at least the prints and links that I have picked from here, these are some of the typos that you may want to change ok, there is a dash missing or a square missing or the variable is wrong. So, this is a corrected version of it, but you have similar derivations in the other textbooks as well ok.

So, you start with your known back projection formally, put this geometry so that the D, l, cos theta, all the terms were there right, all that you have to change, theta should be changed, we have the relationships so, you substitute back, you will get up you know get a very similar looking back projection. So, if you do this, for convolution for example, you will get the image in r comma phi coordinates. So, this is same 0 to 2 pi sum of that means, you are summing from all the views right, all the views that was theta, here it is beta so, no surprise there.

What is this q gamma dashed beta? This is just a filter. So, here if you notice somewhere, you can see this is here you have your projection right, projection convolved with your filter filtered that is what we did even filtered back projection convolution back process, this is convolution only thing is this is instead of we called it g of l comma theta now, we are calling it as projection at gamma comma beta because we changed the variables.

And so, this is your q of so, this is your projection which is filtered ok. Only new thing is you have this term. What is this D dashed? D dashed we saw which was the distance of the point from the source ok in the previous slide, we saw this. So, what this says is the convolution back projection for fan beam slight difference has happened. Here, you can think about it as without this term it is filtered back projection, convolution filtered back projection right, convoluted filtering is implemented in convolution so, convolution back projection is what this is same as what we did for parallel with geometry change.

But in addition, if it is fan beam, this is a distance; D dash is the distance of the point from the source. So, essentially, you have a inverse square. So, there is a waiting based on the distance where the point is in the from the source. So, you can think about this as weighted convolution back projection.

So, for fan beam, you can think that it is a back projection formula that we convolution back projection of the parallel ray accounted for the geometry after that you get this term. So, you can think about this as a weighted convolution back projection ok. So, I recommend you can actually read this material in from any of the textbooks, only thing is the variables might be slightly different, meaning of the variables will be analogous to how it is done here, but otherwise, it is a very standard material. Like I said again you will probably end up learning this better if you have access to data and you try to do the reconstruction by yourself ok good.

So, let us move on. So, this is our major topic of recon which is beautiful material I mean I hope you appreciate when we started, it was like we had this projection how are we going to get the image right without much without flooding with theories or complex mathematics, intuition and common sense we believed got us to the back projection as a option. After that the mathematical correctness gave us that, there is a filtering term that should come.

So, it is actually when they talk about recon, the basic recon algorithms are here, we have we have covered it. Starting here, you should be able to again expand it to cone beam, a popular FDK algorithm if you search, you will look at it, again those are self-driven, we are not it is a very introduction class where we cover several different modalities so, we will not go through that but with the material that you have covered if you understand this, you should be able to quickly get on top of things so iterative reconstructions what is that right. So, you can go star if you understand what we have done so far, you should be able to quickly get on to the current state of the art ok, but within the scope of the you know syllabus here and the timing, we will not go that is going to be self-driven ok.

So, image recon is done physics was done for X-ray modality first, instrumentation for CT is done, image reconstruction is done, what is the pending part for CT? Image quality. So, now, we need to talk about image quality.

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So, when we talked about image quality, these are the things that we will cover no surprises here. We will talk with resolution, noise. When noise is not always talked in separation, it is a signal to noise or contrast to noise and then, some of the artifacts ok.

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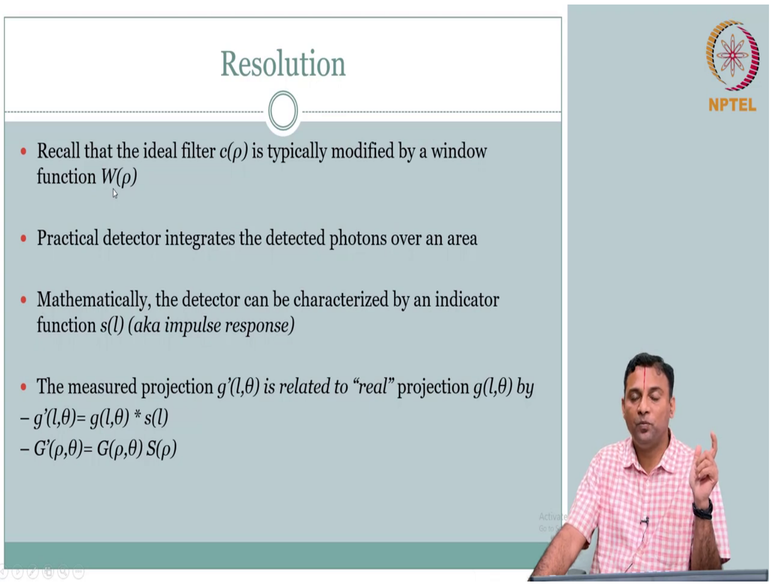

So, we will quickly go to resolution. So, what do we think is a problem in resolution? We are making an estimate right, your object your underlying ground truth is f of x comma y, but what you got is using this recon algorithm and other things, you got an estimate of that f hat of x comma y is what you have got.

So, that is going to be a poor cousin of the ideal. So, what happens to the resolution? Is it going to be same as the ground truth? No. What you are going to get is going to be as close as possible or you want it to be as close as possible to the ground truth f of x comma y but having said that you still are only having estimate.

So, what are the aspects that are coming in? This is not ideal, we covered the reconstruction algorithm, but we kind of covered it from a theoretical aspect, but you have to implement, you notice that there is a filter, a filtered back projection so, there was a filter, you have to implement that filter. So, we kind of alerted you that you cannot implement the ramp filter even though we just put you know filter as a ramp, you cannot implement directly that.

And therefore, you will kind of use a windowed function. So, that means, when you have a windowed function that could introduce some spreading and then, we talked about all of the line integrals going to a point right, point detectors where detector is not a point detector, it has an area.

So, what you are measuring at one, what you are saying that you got it at particular point is actually sum of all of the things that are falling on the detector surface right, the area of the detector. So, practical detector is not going to be infinitesimally small point, it is going to have some area and so, what you are getting is going to be good only over that area, there is a size effect. So, what we can do is we have done this before, all we are going to see there is a ideal and then, because of these f x, we can use these as incorporate these as a convolution ok.

So, detector, it is not ideal detector so, it has a characterized by some indicator function s of l right remember, impulse response of that detector. So, we can have impulse response of that detector and then, you can have this is your filter impulse response right, your filter function. So, essentially, you can think about your projection, the estimate that you have.

So, what we said is what you are detecting right, what you are measuring here is the projections. So, you have a ideal projection, but because you are measuring this using a finite detector and doing a filtering, essentially you are you can think about your projections are an estimate of the ground truth projections. So, you are estimating the projections.

And the relationship between the real projection what would be in ideal scenario which we use and a practical scenario would be your ideal 1 will be g of l comma theta, you have to convolve that with your impulse response of the detector right. So, this is not sufficient so, this you are going to apply another W ok.

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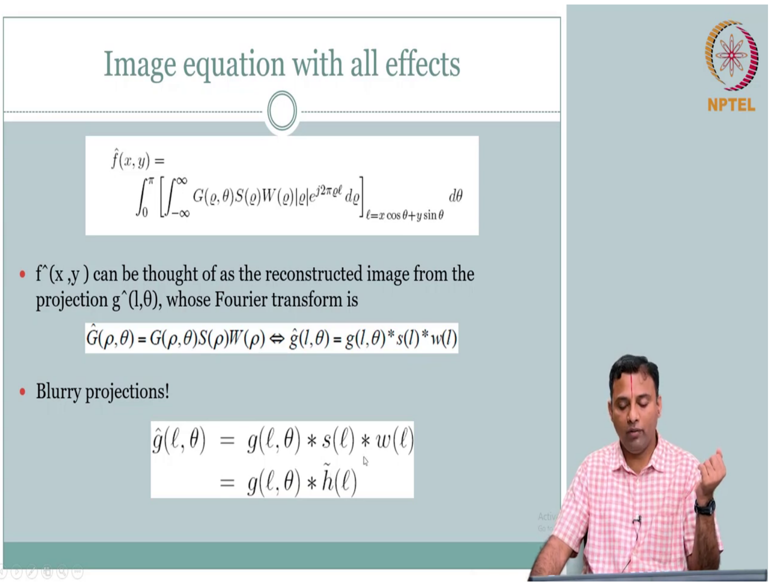

So, recall we had this estimate of the image, this is nothing, but your filtered back projection ok, this is your filtered back projection, this is your estimated image. So, what we have done is we have incorporated the real effects. We did not ignore, we ignored this before, now we have introduced, there is a response of the detector. So, detectors response function of course, this we said earlier, this ramp cannot be implemented so, you will have window functions. So, these two are incorporated in the equation.

So, now, you look at this, you can start to look at it, you have an estimate, this is not ground so, f hat should be as close as possible to f of x comma y. Now, we are going to think, the f of; f hat is not f of x comma y because not because of the back projection algorithm, not because of back projection summation, it is because of this filtering, the detector and filtering so, your projection is an estimate, and that estimated projection is what is causing the blurring ok. So, f hat can be thought as a reconstructed image from a estimated projection, that is the key.

So, the back projection, the projection is not we have been talk talking about back projection, back projection right, but now, we are saying, it is not back projection, that projection that of the data that you got that projection itself is an estimate. So, g of l comma theta was ideal we said when we developed the formulation.

Now, we say, g of l comma theta itself we actually have only g hat of l comma theta, we have only a measurement of that, we do not have the ground truth and that is the one that we are using to back project and get the image and therefore, the estimate of image is affected by the starting point which is your estimate of your projection.

So, now, what is this, how is this affected? So, let us see what is estimate Fourier transform of your right this is in time domain or spatial domain g of l comma theta, what is the Fourier transform of this? We saw that, its a fourier transform is the ideal G of Fourier transform of the g of l comma theta frequency domain so, multiplied with your detector response function and your window function or in time domain, spatial domain, you can do as a convolution. So, this is the problem.

So, this estimate, this is an estimate because of including these two effects. So, these two have an effect on the blurring because that is the point spread functions that we have. So, because we start with the blurry projections, you have a blurry image. So, what determines the amount of blurriness? It better be something to do with the s and l right. So, your g of l comma theta gets blurry because of this guy, this we can call as h tilde of l, this is the in some sense, the blur caused by these two.

So, if you really look at it, what is your g is the projection so, your h is operating on g, g is sorry g is one-dimension. So, this is also applying in one-dimension, but what is g? g is Radon transform, g is Radon transform right that is what we say projection you sum you get g. So, it is actually a Radon transform of the so, it is a sum integral right. So, in some sense, that means, we will have to look at this as also we are talking about this is a Radon transform so, you collapse the two-dimension to one-dimension.

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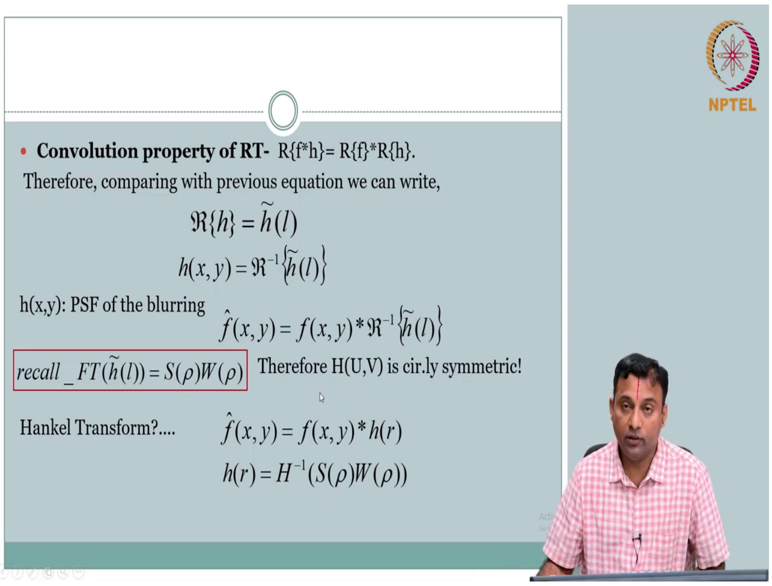

So, what about this guy? So, you have a convolution here so; let us just recall convolution property of Radon transform, very similar to your Fourier. So, Radon transform of a convolution of two function is Radon transform of each of the functions convolved, it turns out that this is the case and therefore, looking at this, in the previous equation that we just put, we can tease out that by comparing, we can tease out your h tilde of l that you saw is nothing, but a Radon transform of h.

So, that means, see g is similar, what is g? You had a g of x comma y right. So, that is projected you get g of l comma theta right, you collapse along the line so, two-dimension became one-dimension similar thing here that means, your h tilde l is actually arrived from a two-dimension h of x comma y that is h of x comma y is the inverse Radon of this guy ok so fine. All we need is h of x comma y is your blur function in some sense right. So, if you are looking at h of x, y, it is the point spread function.

So, this is how the ideal image f of x comma y is blurred by this function h of x comma y to get the estimate f hat of x comma y ok. So, f hat is the estimate, f of x comma y is the image, this is the image, this is the object right ideal, it is d rated or is blurred by this blurring function, what is this blurring function? This is nothing, but h of x comma y which is inverse Radon of this h of l ok.

How do we compute this? We actually know something more about this h of l, what is that h of l? I know that this h of l the free Fourier transform of h of l is what we wrote already right. Fourier transform of that h of l is your detector function multiplied with window function; this is what you know about this. What is this?, this is for applying the filter for every line of your projection right for every theta, your g of l comma theta, it is a collection so, each of the 1D detector data you are applying this the detector as an effect, and you are doing filtering right that is how you did it.

And so, you did do this for every theta same thing and therefore, this is circularly symmetric, it turns out that this is circularly symmetric, the Fourier transform of this right. If you arrange it is circularly symmetric. So, when you have one-dimensional Fourier transform, one-dimensional relation, two-dimensional Fourier transform, we did talk about the relationship and there was a transform that we said is useful, what was that? Especially in the circularly symmetric case, we said you could reduce the two-dimension to one-dimension right, the frequency now we can also talk about only the radius of the frequency because theta is it is circularly symmetric.

The spatial domain likewise you can only talk about the r, the x and y square root of x square plus y square is equal to r. So, we could just talk about radius because in the spatial domain also, it is circularly symmetric, anything ring a bell yes, please go back, we talked about

Hankel transform. So, in order to calculate this guy, we could actually do Hankel transform ok because of this condition.

So, go, look back at Hankel transform. So, essentially, we can use Hankel transform to get your h of r, your h of x comma y instead of that, you can get h of r because it is circularly symmetric so, r is your square root of x square plus y square. So, this is your blur function ok. So, you can calculate the blur function from your inverse Hankel transform clear so, fantastic.

So, resolution we talked about is always can be modeled as a blur function and this is how the two practical constraints of what you are designing right, what is your detector size and or detector response and your window function that you are operating, this is how that enters the blurring ok.

So, these are things that you could play with when you implemented right in any of the examples like I said Lenna right, you can take any image data, you can play with this blurring function, you can see what is the effect when you change the window size sorry window shape, hanning, hamming, ship log on whatever, detector function you can give it some point spread value and see what is happening ok good so much for resolution.

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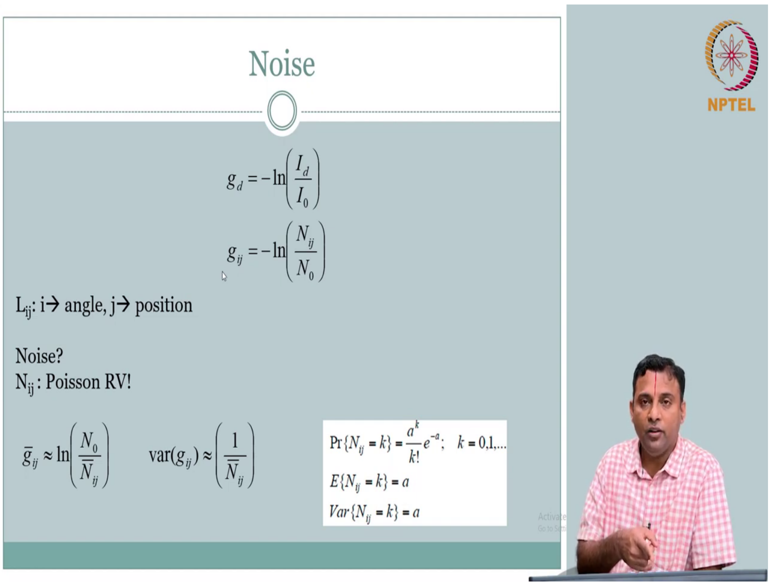

Then, noise so first we will cover a noise, but then, we will say noise is not treated in isolation, it has to be treated with respect to contrast right or the; or the signal ok. So, where is the noise coming from? So, before we do that, what is our measurement? Our measurement is g of d, at the detector location you get, but we would not I mean we would not play with this because this is fine, we made one assumption right, what is that?

I do not want intensity. If I have monoenergetic, then I do not have to worry about intensity, I can essentially say we are operating at a equivalent energy so that you have only one equivalent energy monoenergetic case. So, you have photons into energy per photon.

So, we will say instead of I d, if we treat a monoenergetic with the equivalent energy, then I can actually write this in terms of number of photons ok. So, your g d instead of detector now, because we have i and j, why? Because you have a collection right, you have a collection from different lines, parallel lines. So, i is your angle, j is your position. So, you can essentially get g ij equal to minus lon of N ij by N naught. So, much for what you have measured.

But what you have measured, where is the noise coming from right? From the physics we know N ij, the number of photons that are hitting the detector remember we talked about this the burst, the time arrival could be statistical and therefore, we also said one of the random statistics that is useful, random variable useful characterize this is your Poisson. So, your randomness comes in your N ij right. So, randomness when we say, then there is a statistics, mean and variance, variance or the fluctuation is your noise right.

So, where is the noise coming from? Noise is because of this guy, N ij, number of photons that are hitting the detector ok. So, each time a burst is happening, each of the detectors probably are not getting the same number of photons because of that. So, it actually comes out from a distribution called Poissons, it forms a Poisson distribution which we covered earlier right. Why is this important? The mean and variance are same value ok. So, what is mean? The variance is also the same value that is the idea of this guy Poisson that is a characteristic of the Poisson random variable.

So, in our case, we can then say ok the randomness or the noise from N gets translated to the measurement noise and therefore, you have a mean of the measurement and a variance of the measurement just because of your N right, this is the random variable. So, you have your mean ok. So, this is your mean and variance ok this is fine, but what are we really interested? We are not interested in mean and variance of the measured guy; we are interested in the mean and variance of the image, the noise in the image.

Image is what we are going to characterize right, analyze. So, where is this noise, how does this noise relate to the image that we want? You take this because this is the projection and then, you go to your reconstruction algorithm so, this goes into your reconstruction algorithm, how?

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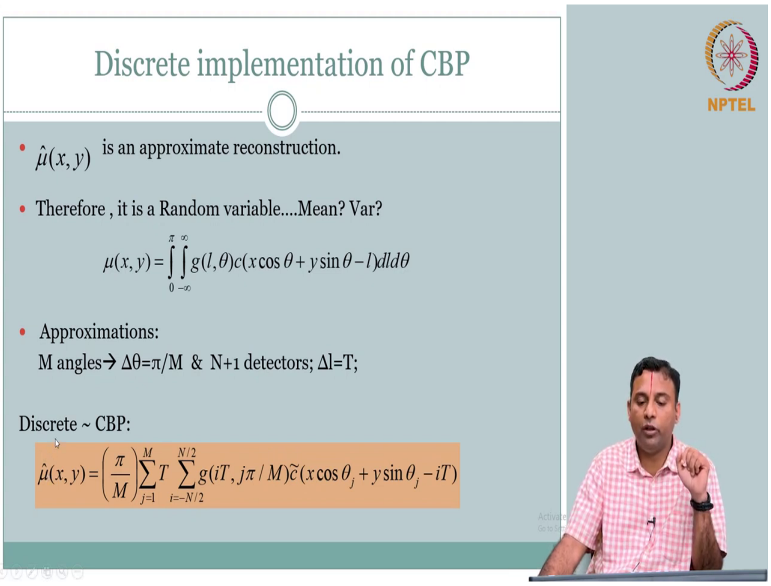

So, we will talk about you are implementing a convolutional back projection for example, so, you start with some noise because of the g and how does that affect your estimate? So, this is your estimate, this is your you are ending up doing what in CT, you are trying to get how the attenuation coefficient mu is distributed in space that is what you are going after.

So, you get an estimate of that using some convolution back projection right. So, it is an approximate reconstruction and therefore, you are going to have. So, you start with some noise and then, you have some approximations because you are implementing it so, you get the noise in the image.

So, how do we get that? So, if this is also right because it is an approximate estimate, what goes in is a random variable, this is also going to have random fluctuations. So, if we say random fluctuations, then we need to know ok so, this is a random variable. If it is a random variable, how do I characterize that I need to know the mean and variance ok.

So, we will start with our formulae right, back projection algorithm. So, you have your mu of x, this is your formulation that we derived, go look back is the same equation, you have your projection g of l comma theta, you have your filter function right c and then, dld theta ok. So, this is fine, but then, this is the ideal right. So, we if we implement, you have to make some approximation so, you have to implement it so, you have to make discrete implement way implementation of this.

So, what do you do? You will have to first start about the approximations that you have to do. First is when you say different view angles, you are going to have discrete number of view angles right. So, you are going to have say M angles you are going to take that means, your theta, delta theta is going to be pi over M so, that is your step angle, it is not continuous right, you are going to stop, take.

So, number of views you are going to take is finite so, that is going to be one approximation and then, number of detector elements right, detector has a size and therefore, for a given width, there can be only so many detectors right. So, your width of imaging or the detector length to the individual detector or the field of view length to the detector length right, this is another. So, if you have N plus 1 detectors, each of the detector you can say has a T width for example.

So, that is a approximation right, it has some finite width. So, those two and then, if you do this, integrals become summation ok. So, what we can have is an estimate so, this becomes an estimate pi by M summation over T right, T is your and then your i so, you can look at right, this is your g, your filter function, everything now in discrete steps ok.

So, this is your discrete convolution back projection. So, you are made an approximation so, how does the noise random variable in this translate to the random variable here or the randomness, the mean and variance in g, how does it translate to the mean and variance in mu because this is your image, the final image you are looking at is this distribution of mu of x comma y. So, what is the noise there or what is the signal to noise ratio there that will be of importance.