Introduction to Biomedical Imaging Systems
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Lecture - 32
Ultrasound_Intro_Phys

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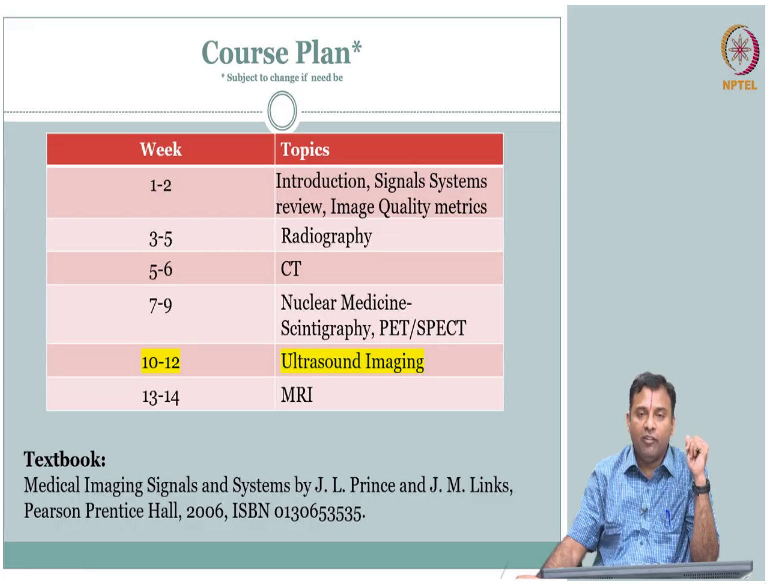

Ok. Welcome back to the next module. So, what we going to cover in this is going to be another modality which is Ultrasound imaging. Like I have said earlier, in this course, you could actually take ultrasound; it is a standalone piece. You could actually take up ultrasound imaging module itself after probably the first introduction right. But, level of content is going to be at a introductory level. So, this cannot be a substitute for a full course in ultrasound imaging that can typically follow you know ah introductory material that is given here.

So, to the best extent possible, I will still cover along the lines of what is covered in the text book that we have been following. However, there are few material that you will find are also slightly different from what is described in this text book.

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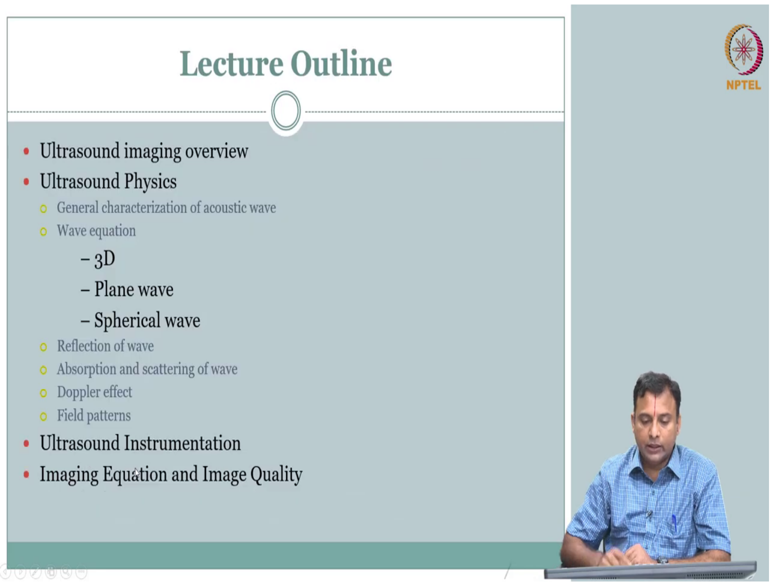

So, let us jump in. What is going to be the outline? The standard template for this course we will maintain that which is going to be we will have a quick overview of the imaging modality, followed by the physics of the modality right; in this case, it is ultrasound physics and after covering extensively the ultrasound physics part of it, we will move over to the instrumentation and imaging equation, image quality. In fact, I am very hesitant to say we will cover all this, we will cover; meaning we will touch and go all of this.

But, it is going to be really you know more of alerting you that these are the concept that will be involved and you may have to go back and really read if you want to work in ultrasound imaging. Perhaps, you should consider taking a further course in this. But, what will be covered in this order right, the level will allow you to compare probably if you have taken the previous modules right; the X-ray, nuclear medicine, you will be able to relate back to see how this is completely different from what was covered there ok. So, this will be only to that extent it is not going to be very involved.

Nevertheless, I hope you will find this interesting and the beautiful a modality which its own advantages and lot of challenges that I hope you will take it you know with lot more interest and study by yourself little more on this subject ok.

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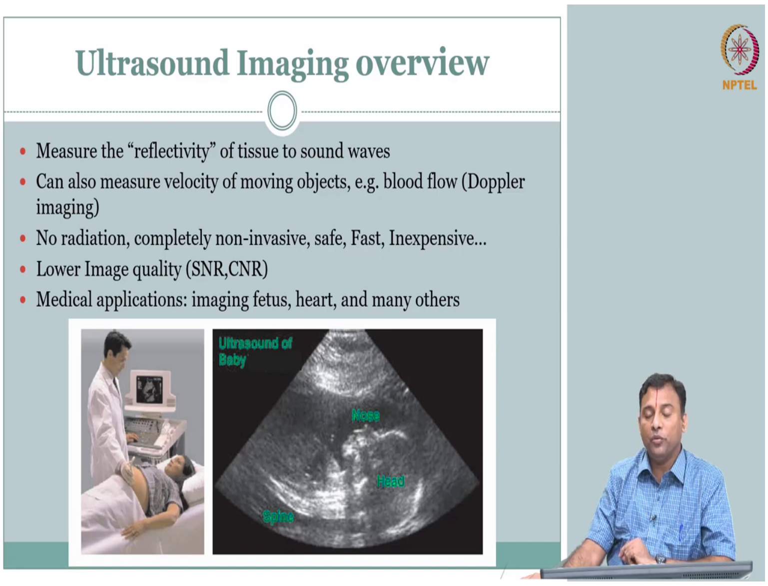

Let us start with overview. So, what we knew from before right in the subject right in this course, we have strived to answer this question. In each modality, what is it that we are trying to go after; the image right, what is the fundamental or the physical property of the object that is captured in each of the different modalities. In that sense, here loosely, we talk about measuring the reflectivity of tissue to sound waves.

Remember in X-ray based modalities, we talked about attenuation right. How the body attenuation property to X-ray energy, the distribution of that is what we were trying to capture. In nuclear medicine, we were talking about radioactivity right; how is the radioactivity captured and then said, where is the source for this radioactivity and how much is the radio activity coming from a location. So, we were trying to do a source localization of radioactivity in nuclear medicine.

So, here in ultrasound imaging, loosely we could call this as reflectivity of tissue to sound waves. So, we need to understand what is sound wave and then, we maybe we can you know define little more rigorously what do we mean by reflectivity. But, essentially you can appreciate here that this is this does not seem anywhere close to the other properties that we have covered so far. So, this is a new newer information about the material the object ok.

So, apart from this right, we talked about distribution of mu. In X-ray, we talked about structural imaging and when we came to PET, we were talking about nuclear activity and essentially, there also where the radio activity is coming from that was interpreted in relation to maybe in some physiology that is taking place. Here is an ultrasound, it gives you additional handle. You can also measure the velocity of the moving objects. I mean what is moving inside the body, mostly blood right. So, we want to see blood flow; can we visualize the velocity of blood flow right, distribution of the velocities, these are very powerful aspects right.

So, ultrasound gives you view using what we call as Doppler principle. So, we will cover a intro on Doppler principle as well in the physics. But, this is a unique aspect that we have not covered so far right, whatever we did in X-ray imaging and PETs nuclear medicine.

The two properties both for the structure or the anatomical image that you are going to see in ultrasound and some of these like blood flow imaging or blood velocity imaging, these are not really directly obtained in the in the modalities that we covered so far. So, this is a new information that we are getting that itself is a very powerful right. You are using a newer modality, you get to see right, you get to see new information about the target object.

So, again comparatively here, there is no radiation right. So, compared to your previous modality that we have covered so far, there is no radiation and also, it is completely non-invasive ok. So, you are going to sit from outside and see what is inside the body right. Of course, in some cases remember, we talked about sending a camera and like minimally invasive if you would call it through the natural orifice. Like that, here also you can send an ultrasound probe and you can do it; but it is still considered non-invasive ok. It is safe; of course, no radiation no ionization, so it is considered safe.

Fast; what do we mean by fast? Very qualitative, fast essentially saying that you can see the image as they are probing unlike your CT or X-ray, where we were talking about minutes that came down to seconds and then, you have a room adjacent, where you have to be safe and the computer generates the image. Here, actually the patient right, you could actually see your own image, while they are doing ok.

And so, it is also inexpensive; of course, this is in relative to the other modalities that we have talked so far. So, it has lot of advantages. It gives you new information right. So, nothing is you know if this can do everything, there is no need for the other modalities. But, you know like I told you the whole field, there is always a give and take there is a trade off.

So, not if you look at the image quality right in terms of your signal to noise ratio or contrast to noise ratio is typically much smaller compared to the radiation imaging that we covered so far. Of course, it is also much lower than the beautiful modality that we will cover later which is MRI.

So, this is one of the inherent disadvantages and another being, it is very local. Remember, we talked about whole body imaging in PET, whole body imaging in CT, ultrasound is still very regional. You look at a target, you do abdominal imaging or you do heart imaging or you do you know surface MSK imaging, you do breast imaging, thyroid imaging, prostate imaging; so, it is very local ok. That is one of the other disadvantages along with the quality.

However, there are some medical imaging applications for ultrasound that are unparalleled. For example, imaging the fetus right. So, baby inside that tummy, oh you are not going to give radiation, all those things; it is all risky. Maybe MRI also you know we did not cover it, but we have seen the photo of an MRI in our intro right. You see the patient dust or you maybe you have seen it in you know in person, when you have gone to some imaging centre.

You have to go into that big tunnel right like your CT. So, that may not be that comfortable for the mom right. So, imaging fetus is a very powerful application for ultrasound. Heart imaging, live beating heart you can see right. So, that is another powerful; echocardiography is something that you would have heard. There are several applications like that.

But, just the point is to bring out some of the things that you may be familiar with right. So, looking at the so this is noninvasive right. So, you are looking at the baby inside the tummy by pacing a probe on the outside. So, you can also see that this is regionally. He has it in the hand and is moving around. So, this is a typical scanner. So, it has besides the patients.

So, the patient can also see right. So, these are the advantages, the these are the new information that we are going to get using ultrasound. And let us jump in to really see more about what do we mean by this and how do we get this; what is this image mean, from a from a physics point of view right? What is this white mean? What is this black mean? What is this gray right? Why is this black and white, what does it mean?

Of course, we know the clinical ok; this is nose, head, spine that is fine. But, from our physics point of view, what is the meaning of this color right? What is the physical quantity; what is the units right? Those are the things that we will try to understand ok.

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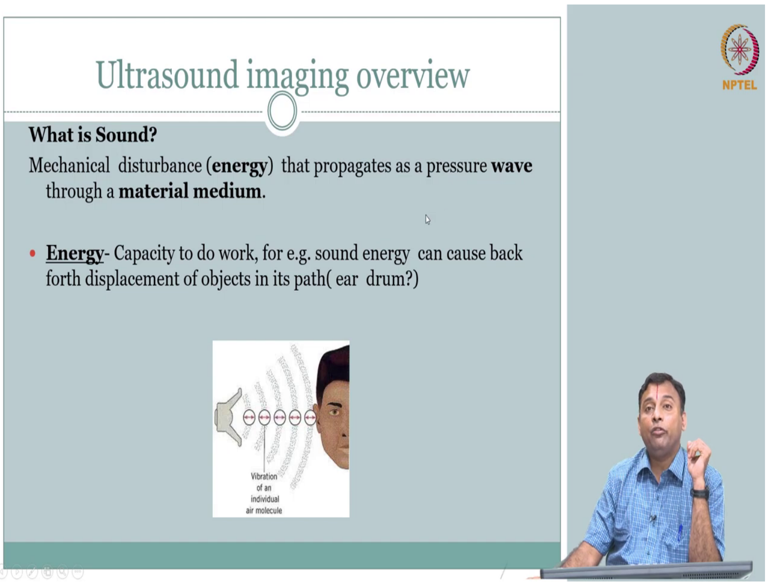

So, before we go into ultrasound, it might be productive to recognize that ultrasound is just one type of sound ok. Sound we are very familiar. So, we will start with what do we mean our understanding of sound and then perhaps, get to ultrasound ok. Sound is nothing but a mechanical disturbance ok. A mechanical disturbance, I have just put in bracket or bold few terms which we will go in detail.

So, mechanical disturbance or energy that propagates as a pressure wave through a material medium. So, there are key three key you know descriptors that are here, which we need to carefully understand. If we understand that understanding sound or defining sound is will become you know very formal.

So, let us take what is the energy right. Energy is capacity to do work ok. So, in that sense, what do we mean? Oh, it can essentially sound energy can do back and forth displacements of object in its path. So, something that we are familiar. When we talk about sound, we hear sound. So, what happens when the sound comes, when the pressure wave comes right? The energy comes right; there when the sound energy comes, your the ear drums are displaced ok. So, sound is a energy; energy means capacity to do work. So, sound energy can move back and forth a object that is in its path. In this instance, it is the ear drum that we can relate to ok. So, ok.

So, there is a energy that propagates as a pressure wave, what do we mean by wave right? We are all familiar, we use this is not electromagnetic wave. So, this is the thing, where we covered electromagnetic spectrum and we sometimes also called as electromagnetic waves.

But then, right we had radio frequency wave. So, we did not really microwave; but then you know sound, ultrasound, it is not going to be there in your EM spectra. This is a modality that is not there in that chart that we started out ok. So, this is different you know even from the fundamentals of which engineering program, background students are exposed right.

EM means electrical background students would typically be exposed to electromagnetics; whereas, the moment you talk about sound, pressure, it is always mechanical engineering. So, you see the interdisciplinary nature of the imaging systems and more so ultrasound starting with the fundamentals right.

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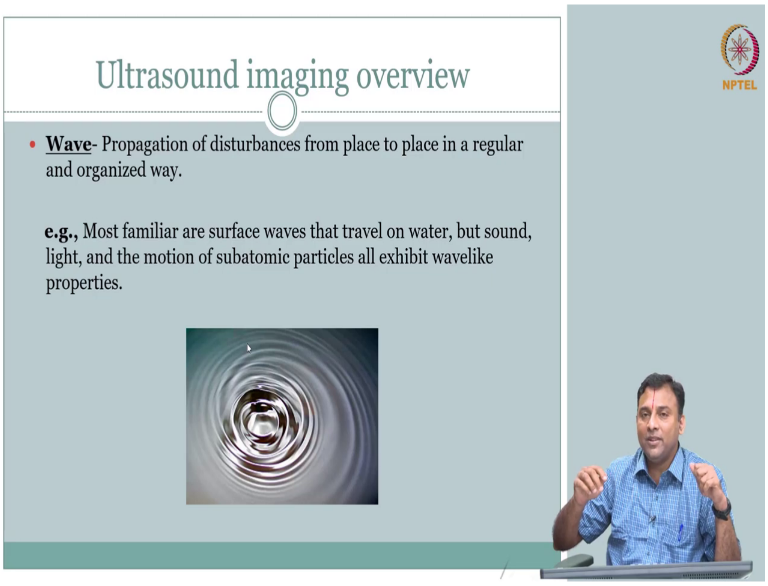

So, wave is a propagation of this disturbance, place to place the real nicety about wave described here is in a regular and organized way. So, even though it is disturbance right, it is a disturbance; disturbance is usually used in a term of connotation of negative sense right, something is disorderly, the disturbance means that is.

But, the beauty about ultrasound wave that we are going to cover or sound wave that we are describing right now is that this disturbance actually travels from place to place in a regular and organized way. What do we mean by regular and organized way? That means, as engineers, we are looking at it and saying look regular and organize means can I use a mathematical expression to describe the wave right. So, we are all very familiar with waves right.

For example, you throw a stone into the water right, you will see ripples, you will see the waves that are moving here in the surface. So, we are all familiar with waves ok. But what is the wave that we have described here? Oh, propagation of disturbance in a regular and organized way.

So, when you throw this you have seen the wave right, you would have seen this; but the beauty that we look at is oh not only we will describe oh you see the waves that are moving, we are engineers; what we will be able to do is we will be able to write a mathematical expression right. A mathematical equation and say oh, I can communicate right this wave in terms of this equation ok.

That is why it is regular and organized helps us essentially it tells you can come up with a mathematical equation to describe this. But, you know what the what a wave is right. So, let us jump in little more. The last part is going to be your energy wave and last one was material medium, through a material medium right.

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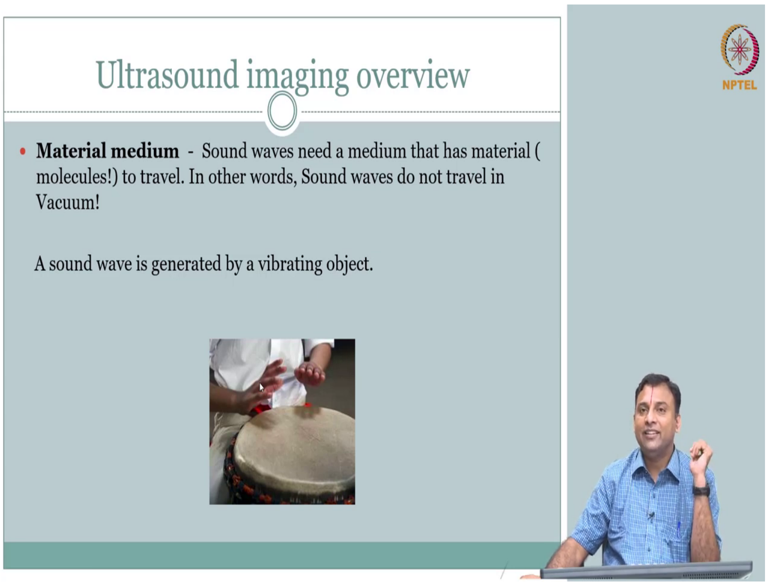

Sound waves need a medium, see this is the important. This medium has to have a material or molecules to travel. This is one of the first differences between what we have covered so far electromagnetics right, based modalities. There sound can I mean light can travel in vacuum; the EM can travel in vacuum; whereas, in ultrasound key difference is the fundamental signal right your sound wave cannot travel in vacuum.

It needs the material medium to travel that is the first take home message. This is something that is kind of overlooked right. But, this will play a important role ok. So, this is our material medium. So, clearly, how do you generate a sound wave? You need a material medium.

So, a typical example right, you have drums, you tap it, you vibrate; that vibration right, you can it comes and it moves the energy comes your ear drum; your ear drum vibrates right, you interpret that.

So, sound wave can be generated by vibrating objects. So, you need a material medium; otherwise even if you tap, if there is no medium, there will not be any sound that will be generated ok. So, this is an important idea. So, if this is the case, how are the different ways right when we talk about ultra, what is now we talked about sound waves. Now, we need to our interest is ultrasound ok.

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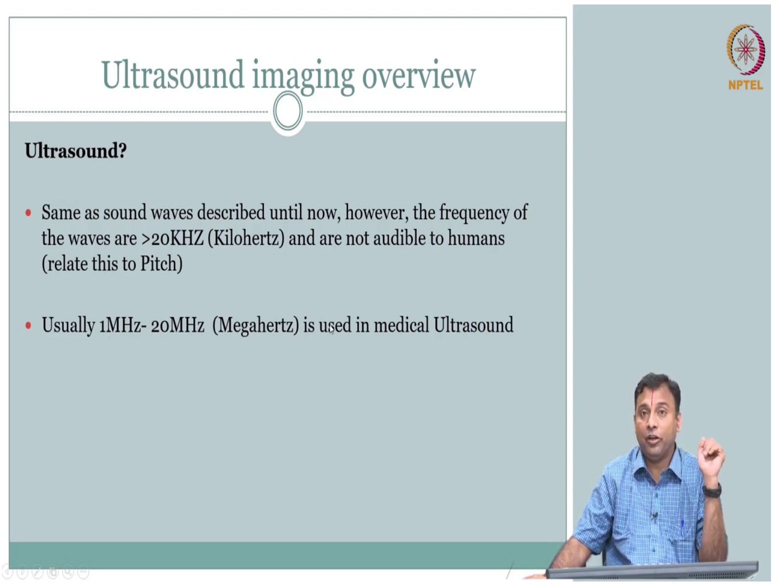

So, what is ultrasound? Same thing as sound that we covered right. All the understanding that we did using sound is ultrasound is just one specific class, where the frequency right, when we talked about sound, we talked about ear drum we can hear. So, the only difference is ultrasound you have your frequency of the waves greater than 20 kilo hertz. Why is that?

That is beyond your audible range. So, relate this to pitch ok shrill pitch. So, there are so human beings it is 20 kilohertz; it is known that some of the animals for example have a higher range audible range right. So, you know dolphins they say, bats use ultrasound; even sometimes dogs can here little higher frequency that we are not able to hear.

So, sound and ultrasound, the difference is ultrasound is just operating at a frequency that is beyond human auditory range, which is greater than your 20 kilo hertz. So, typically in medical imaging right, you have several different applications of ultrasound; non-destructive right, non-destructive evaluation that they do in engineering firms for quality check for steel or concrete or you know the whole area of non-destructive testing or evaluation.

If you are from mechanical engineering domain, you probably would have heard that there are also they use ultrasound. But, typically, it is on the kilo hertz range; hundreds of kilohertz range; whereas, in medical ultrasound, typically you are talking about 1 megahertz to 20 megahertz ok. So, high higher frequencies.

So, much for ultrasound. So, now, we know what is an ultrasound. We kind of get you know what it means, but our objective is to slowly jump in and talk about mathematically describing this right.

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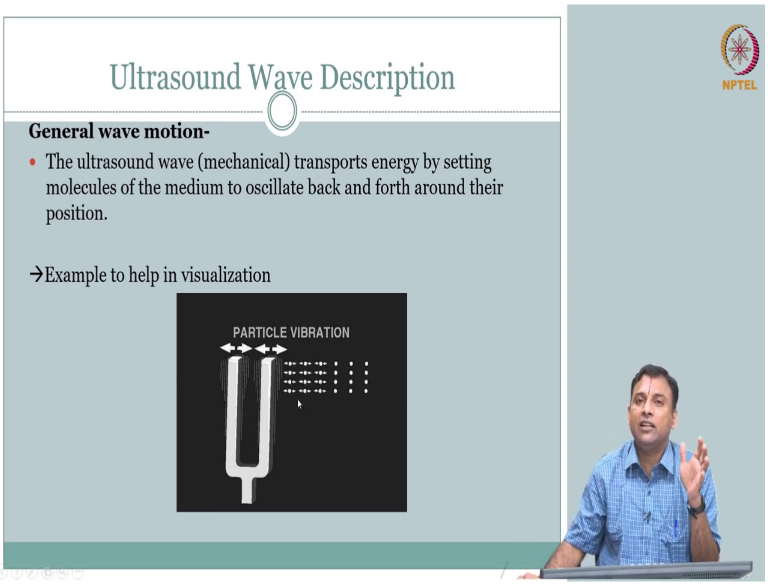

So, what we have seen so far? Ultrasound also exhibits a general wave motion right. What do we mean by general wave motion? It is a right my ultrasound wave transport energy by setting molecules of the medium to oscillate back and forth ok. So, there are several ways this. There is a relationship between your medium right; when we say medium, there are particles in the medium. The particles in the medium has to oscillate right and then, a wave rides on it. So, there is a intricate relation between the motion of the particle and the wave that is moving ok.

So, that defines the type of waves that are, there are different types of waves depending on the relationship between the particle motion and the wave motion ok. Simple way that you would have heard about generating a sound wave right, your tuning fork experiments in your high school right.

You take that tuning fork, you tap it, it vibrates right. When it vibrates, the particle surrounding the tuning fork also vibrates and whatever oscillation this is happening if you can hear that vibration proceeds hits your ear drum and you interpret the sound and you would have done different frequency; tuning pitch right. You will have different forks each one will create a different pitch right that is what.

So, this is a easy visualization that there is a connection between particle vibration and the wave that is travelling. This particle is not coming and hitting your ear drum. This particle is around the tuning fork. That particle is not coming and hitting your ear drum that you hearing. It is a wave that is coming and hitting the ear drum. So, the wave is transported.

The wave travels because of the support of this particles that oscillate around its position. So, the particle does not move right; it is moving back and forth in the same you know around it same location ok. So, that is the key. So, depending on the type of particle motion and the direction in which the wave moves, we can have different types of waves ok.

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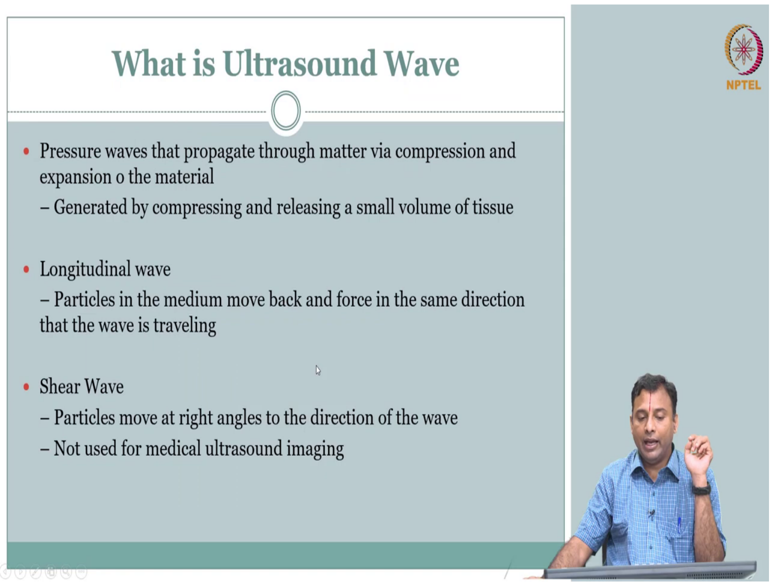

So, pressure wave that propagates through the matter via compression and expansion of the material right. That can have two different; there are several, at least two that we will recognize here, at least in this slide and then move on with only one. So, one is longitudinal wave.

A longitudinal wave is where your particle motion and the wave motion both are in the same direction ok and a shear wave is where the particle motion and the wave motion are orthogonal to each other; perpendicular to each other. Of course, we will have a small animation to explain this; but there are several different motion surface wave that we saw right.

Surface wave is another type of motion, where the particle is moving in one direction, wave can move in another direction. So, there are several different types of waves. We will focus on longitudinal wave right, where the particle motion and the wave motion are in the same direction. We will ignore shear wave because our body what is our body? Or we are mostly consistent of water.

So, we are liquid body; 70 percent water, so we are all fluid ok. So, essentially, it will not support shear wave or for all practical purposes, your soft tissues are water heavy and therefore, will not support shear wave. So, this kind of motion is not supported in biomedical tissues and therefore, soft tissues and therefore, we will focus only on longitudinal wave.

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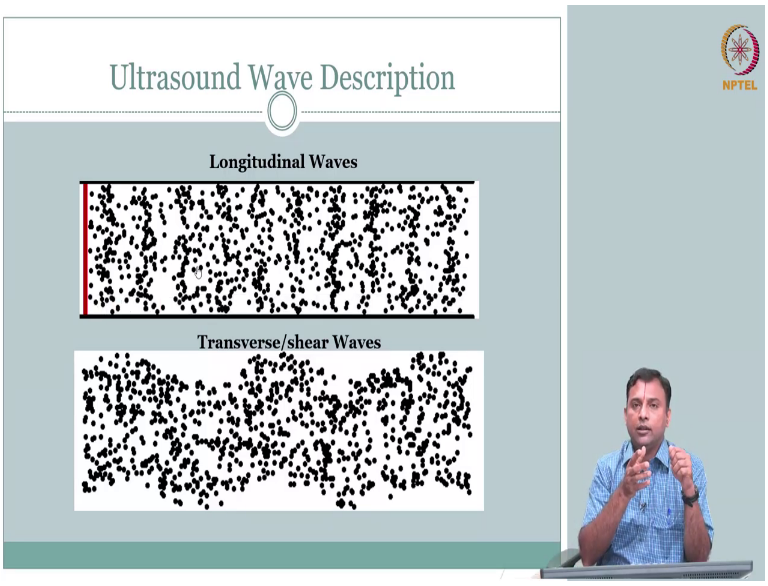

So, let us just take a couple of animations.

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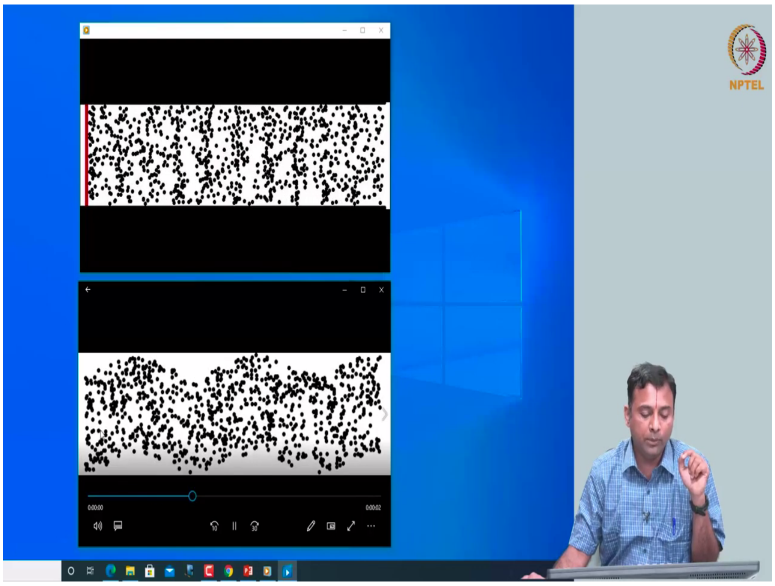

So, what do you see in the top right? Well, this is essentially kind of a piston right. So, it is this is basically trying to move this is a medium, these are all representation of particles in the medium. For an untrained eyes, what do you see? Oh, you look at it and you will say perhaps if you are thinking like most people, you look at see there is something that is going from left to right right? There is motion from left to right. Oh, that is your wave. So, you can clearly see that the wave is travelling from left to right ok.

Here, I oscillate this right, this red line that is basically vibrating, moving left and right and therefore, the particles surrounding that are starting to move and you see a wave. But, notice carefully, there is an illusion. If you sit at one particle right, localize it. Take one particle and focus only on that particle, that particle is actually moving left, right, left, right, left, right.

So, if you sit on any one particle, you actually see that with the time. It is actually moving only back and forth; it is not going from left to right till the end. But, they all are synchronously doing that; that you have a wave that is going from left to right; whereas, the particle itself is only oscillating around its position back and forth..

Here, the direction of oscillation back and forth and the direction of wave are in the same direction. This is called as your longitudinal waves; compressional waves ok. So, this is what we will exploit right in biomedical imaging; ultrasound imaging. Of course, the other one is your shear wave, transverse shear wave. So, here why is it transverse shear wave? Look at the particle motion. Clearly, you can see a wave is going from left to right right. So, you can just look at it you see that the wave is going from left to right. But then, you sit on this one particle right. Say let us take some yeah; you go up, come down; go up, come down; go up, come down.

So, the particle is actually going up and down around its mean position; whereas, the wave is going left to right right, perpendicular to it. So, this is a shear wave. So, the particle displacement, the particle motion and your wave motion are on perpendicular directions. Clear? Of course, this like I said is not supported in the body for the soft tissue that we have and so, we will ignore shear waves. We will focus only on what we call as compressional waves or longitudinal waves ok. So, we will only focus on longitudinal or transverse.

So, there is also a possibility of mode conversion that is you have one mode right, you have longitudinal. When you have an obstruction, some of which can get converted to transverse. But we will ignore all those in our material ok because that is not that dominant in biomedical applications ok.

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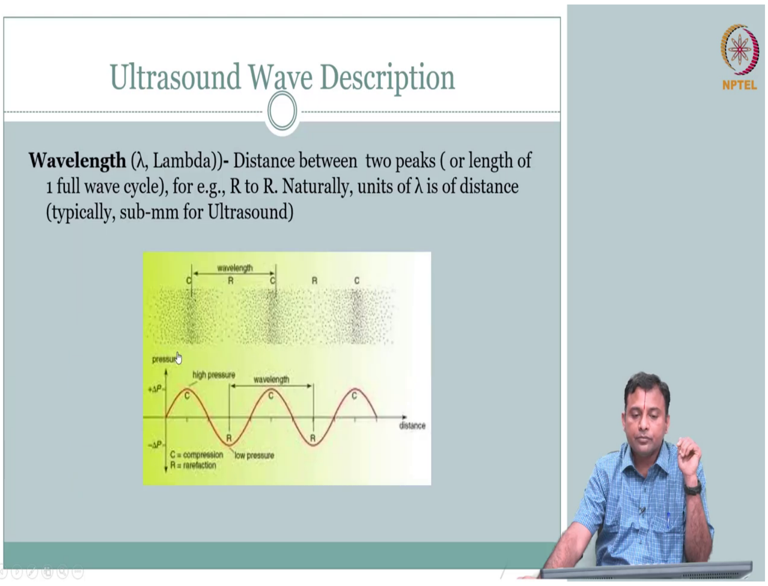

So, if we have to describe the wave, we need to have some wave equation; but before we set stage, we need to talk about an important descriptor for this wave right or wavelength ok. Wavelength or lambda, this is what we typically use. So, you have regions of compression and rarefaction. So, it is one cycle. You can measure it from wherever; peak to peak, trough to trough. What we are talking about is one cycle ok. So, that is your distance between or distance of one cycle is your lambda. Typically we get, so this is in length scale ok.

Typically, we get sub millimeter; this is an important parameter because this determines your, this is very related to the resolution of the modality ok and so, this is very important; sub-mm lambda is achievable in ultrasound ok.

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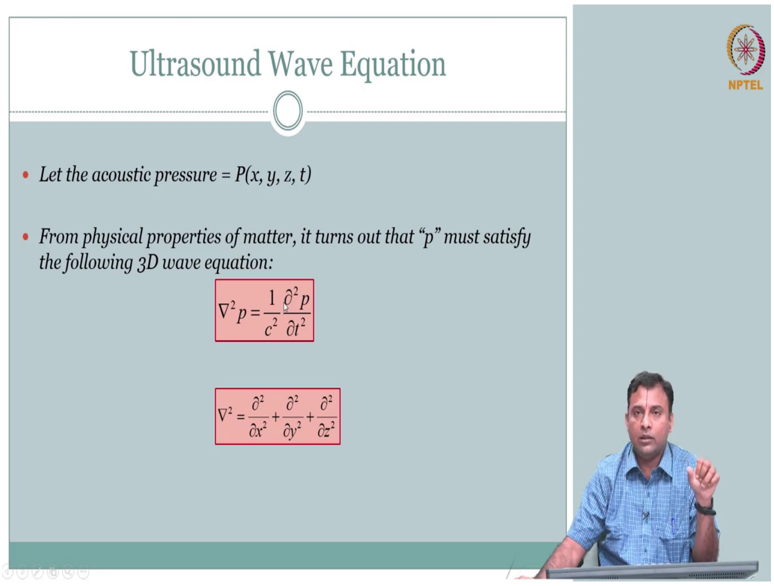

So, now, we will jump into talking about the wave equation. So, we say let us let the acoustic pressure be P of x comma y comma z comma t. What does that mean? So, from your everyday experience, you know say for example sound right. When you create sound, what happens ok? My voice box creates that pressure difference and it generates sound of a particular frequency. So, a sound wave and my mouth is the aperture right, I am controlling the aperture opening.

But, the point is a source, a sound is travelling, inherently what is happening? It is 3D. Unless I make an effort to make it go in a particular direction right, it is going to be three-dimension that is the pressure wave is travelling in three-dimension. So, if I have to describe the wave, then I need to describe the pressure changes in x comma y comma z and also, with the time right.

So, it is changing with the time. So, that is what we saw in the wave right. The wave it compression rarefaction. So, the, but you sit at one location, you are going compression or rarefaction. So, with it changes in space and time. So, let the acoustic pressure, we will denote it as P which is a function of x, y, z and t.

So, given the introductory nature of the material here, we will not go into derivation; but it is from the physical properties of matter, so we could relate this "p" right we could describe this "p", this in three-dimensions using a 3D wave equation which would be of this form.

What is this? Oh, this is your Laplacian spatial derivatives right, second second order. So, dou square by dou x square plus dou square by dou y square plus dou square by dou z square. So, this is your Laplacian. So, your pressure is changing in space and time. Clear? So, your pressure is changing both in time and space that is what we saw right.

You saw the wave travelling from left to right. So, if you freeze one minute right, one instance, each location has different pressure; some experiences high pressure, some experiences low pressure right.

And if you freeze time right, then you see same behaviour. So, both time and space or the pressure is oscillating or it is changing. So, you have if you have to describe this pressure, it is changing both in time and space. So, you can you get this you know wave equation that captures both the change in space. Of course, here we have three-dimension. So, we put the space as x square, y square and z square and time.

So, what is this? This we will talk about it little more detail; but this is your we talked about wave, we need a material right, medium; a material medium for wave propagation. So, this has to do something with the material medium. What is this? This particular thing written as c is your speed of sound. Because c, we use even for speed of light right.

So, c is kind of velocity. So, here c is velocity of sound ok, velocity of sound in that medium. So, this is actually a material property. We will talk about this in a bit ok. So, this is your 3D wave equation. However, like we said, this is fine. This is all powerful, neat; but then we are interested in making life easy with context of the application which is biomedical imaging.

So, in our case, even though 3D is there, we will try to look at waves at least two different types of waves that are applicable for imaging.

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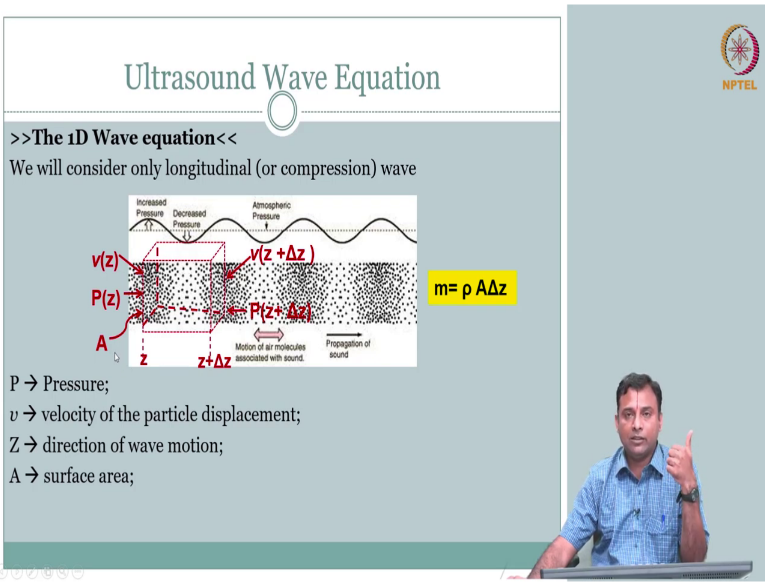

One is your plane waves. In plane wave, what is a plane wave? Oh, the waves that we talked about three-dimension x, y, z, pressure is changing. We will talk about in a particular plane it is all of the same pressure right. It is changing only in one direction. So, we will talk about 1D wave equation that is the wave is travelling only in one direction. In this case, we talked about it in z direction. So, the wave is moving in z direction, so in the x y plane right, in the x y plane all the points, all the particles in this x y plane experience the same pressure. So, this right; this is moving back and forth.

So, the wave is moving left and right. In plane waves, what we talk about? Plane waves, we are talking about the planes that are moving ok; that means, all of the particles on this plane, experience the same pressure ok. So, we will consider only 1D wave equation. 1D wave especially longitudinal or compressional wave. So, this is a setup right. Why? We are not going to derive, but just want you to have a look at the schematic, how we set the problem and then, we will straightaway write the equation in fact, we have the equation right. We will just list it for the dimension that is described here.

But, what you will see here is you could have these are the particles right. So, you could have increased pressure, decreased pressure. So, that means, the pressure is changing in z direction. So, we will have one pressure at one location, another pressure at P of z plus delta z. So, there is a differential pressure. So, there is this velocity of the particles. So, particles have at a velocity right. Of course, you can then particle, then you have a cross section area. Remember, we are talking about planes, so cross section area through a volume through which this is happening.

So, this is your area right; z is the direction of our motion. Of course, then you can write your mass in terms of density area delta z. So, volume A into delta z is your small volume, rho is your density. So, density into volume gives you mass. So, we could set the problem up like this right. We are not going to derive, but if you use the first principles of conservation of mass and conservation of momentum, you will be able to get to this equation.

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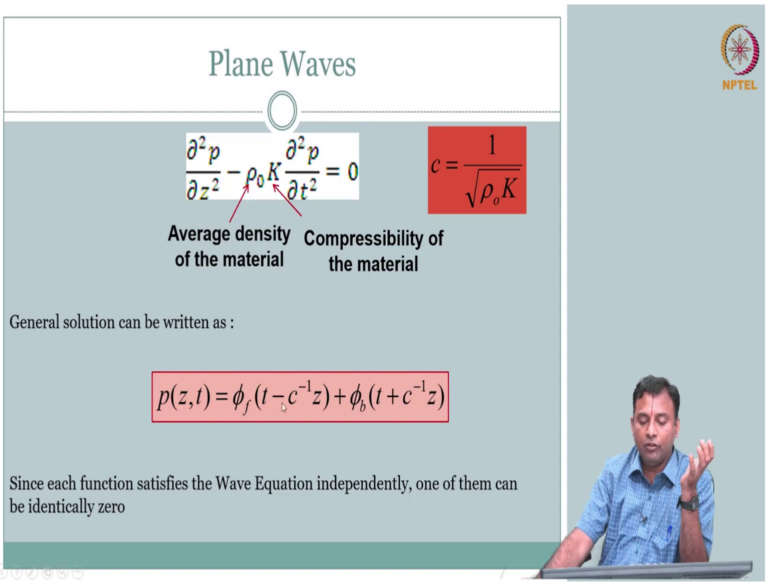

So, this is nothing but x and y are gone. So, this is one dimension, the wave is travelling in z direction. So, this is dou square p by dou z square right. So, dou square; well in Laplacian, we had x and y that is gone. We are considering only the wave in one dimension along the z direction and therefore, your pressure is changing only in z direction; dou square p by dou z square equal to right 1 by c square we had dou square p by dou t square.

So, I have bought that R.H.S to L.H.S. So, it becomes dou square p by dou z square minus rho naught k dou square p by dou t square. Of course, when we did the general thing, we talked about here what was there 1 by c square was there and I said it is related to medium property. So, this is actually medium property right; average density of the material and then, this is your compressibility of the material right. Particle is you are applying pressure it has to move. So, how compressible right; so compressibility of the material and the density of the material will determine the propagation of this wave ok. Clear? So, this turns out c is your because we had remember in a general equation, we had1 by c square.

So, essentially c is 1 by square root of rho naught K. So, we are not deriving any of this, given the introductory nature. But, essentially you see that the wave equation that we are going to deal with the 1D wave equation that we have dealt with is dependent on the material right. The material property of density and compressibility is related to c.

So, if I say c that is speed of sound that is actually material dependent ok. So, that is a very important aspect ok. So, we have this equation. What is our interest? What this says is a medium right, a medium can support a wave propagation. If it supports, this equation will be satisfied. But what can be p right? What is your p? Oh, this is just an equation. I need a solution for this equation right, I need to have a p.

So, what is the solution for this equation or in other words, what is the pressure function or pressure waveform that can actually propagate in this medium right satisfy this equation. So, solve this, you can have a general solution. Thus general solution for this can be written in terms of p of z comma t right.

Because we are dealing only with the z direction; pressure at any z comma t has either a general function of this form which we have a suffix or subscript here as f, meaning forward direction or b backward direction. So, these two are independent; each of them can satisfy the solution here of this form. So, general function which is having t minus c inverse z. So, this is called forward travelling, why? Oh, if you take the general equation, it is phi z of this thing right c z right; it is a function of c inverse z. So, at t equal to 0.

So, as t increases, what happens? Oh, you travel in positive z direction. As t increases, here you go in the negative z direction and therefore, this is called as forward or to the right right. So, this is called as your forward propagating wave and this is called as your backward propagating wave.

Because the basic function phi of f basic wave form is function of right basic wave form at t equal to 0 right if you reference that, this term is moving towards the right. So, this wave is moving in the positive z direction. This term, this wave is moving in the negative z direction as t increases and therefore, it is positive forward wave or backward travelling wave ok.

Of course, we are interested since both of them can independently they have to satisfy, we can treat one to be one of them is identically 0. So, essentially, we will drop them backward. So, we are interested in sending a wave into the tissue ok. So, therefore, we are interested in the wave that is going into the tissue and it can come back.

We are not really interested in when the tissue, when the vibrate, when the wave is generated. One goes into the tissue and one goes the outside air right or the other direction, we are not really interested in that. And therefore, we could without loss of losing the generality, we can treat only one term here, one solution here, one general solution here, which is of this form ok.

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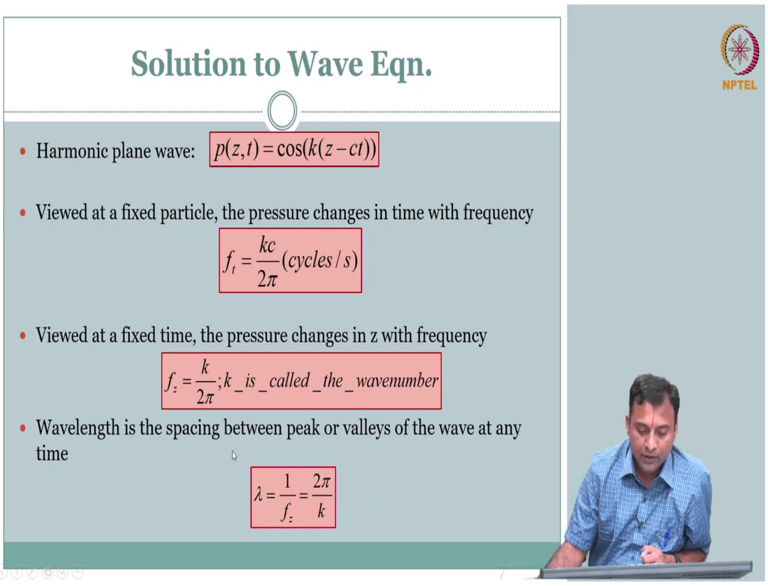

So, what could be an example solution, one solution of that form? You could have any form right; anything of this form is a solution. One typically used solution is the harmonic plane wave. What do we mean by that? p of z of t is cos of k z minus k of z minus c t. So, you could have delta function also ok. You can have any function. You can have a triangular waveform; you could have a square wave form; anything that satisfies this equation right, but typically we use a cos ok.

So, when you have this, what are the interesting relationship that we see? We have this can be a solution to the equation, if the argument right it has to satisfy equal to 0. So, what we can do is we can first get a if this is the solution, let us get the meaning of what is happening. If you sit right viewed at fixed particle, so if you are sitting in a particle, then what is happening?

The pressure changes in time right; I sit on a particle. We are interested in only compressional waves. So, what is happening? I am moving left and right right; back and forth along the direction. So, if I sit at one particle, with a time, I am moving to the left, moving to the right; rather I am having compression, high pressure, low pressure right that is what I am experiencing.

So, fixed at a particle, the pressure changes in time and how what is the frequency? Oh, frequency of this oscillation in time right. So, that is your temporal. So, f t is k c by 2 pi. So, this is your time quantity. This is your z is your spatial quantity. So, when you are talking about a particle, the particle is not moving in space fully right. The particle is oscillating about its location with time.

So, you have a frequency of oscillation which is your f suffix t which is k c by 2 pi from here right ok. So, then what happens if you are looking at it from a space point of view, viewed at a fixed time right? If I freeze the time at one particular instance, then the pressure change in the z right there is a pressure rarefaction compression, rarefaction function, it is moving.

When I hit the still button, froze it in time, you would have seen some location will be high pressure, some location will be low pressure. And essentially, it is going to be a cyclic form right and therefore, you have what is called as f suffix z which is your frequency or spatial frequency right. So, your f suffix z comes from k z here. So, it is k by 2 pi.

What is this k? Oh, this is an important quantity is called as your wave number. So, this we are very familiar, f t, we are very familiar; every one dimensional signal, we talk about time as the index right. So, therefore, frequency, we always say cycles per seconds. So, we are very familiar with that; whereas, now we are dealing with spatial quantity.

So, f suffix z is defined as k by 2 pi. What is this k? k is your wave number. Let us just complete this and I will explain you one more time. So, wave length which we saw. So, why am I talking about wave length? Oh, we are talking about here f z, we are talking about fixed in time the pressure changes in z with frequency. So, pressure changes in z, one cycle is what we called as wave length right.

So, therefore, there should be some relationship here. So, wavelength is the spacing between peak or valleys at any time. Now, I am saying f z is the frequency. So, naturally, these two should be related. So, lambda is nothing but 1 by f z which is 2 pi by k. So, lambda is your wavelength; this is your wave number. So, we can actually think about wave number k is equal to 2 pi by lambda.

So, wave number; what is wave number? Wave number is if you think about it, if you treat it is all distance quantity that we are talking about. So, 2 pi; if 2 pi is the distance right, lambda is also a distance right. So, number of lambdas that can fit in 2 pi distance is your wave number. So, number of waves right number of waves, one wave if you want to call that as one wave length one wave right, number of wavelengths that are there in 2 pi distance that is your wave number.

Number of cycles that are there in one second is your cycles per second, your frequency here. Number of cycles right, we also have time period 1 by f right, what is the time period? The time of one cycle. So, there we call it time period; whereas, in length scale we called this as wave length, one cycle in distance and therefore, your wave number is an important concept. It is number of wavelengths in 2 pi ok.

So, of course, we can also then relate right these two ok. I do already see that I have lambda is equal to 2 pi by k, but what is k? I can look at here and say look k is nothing but 2 pi f by c right; k is 2 pi f by c. So, 2 pi f by c will become top. So, 2 pi, 2 pi cancels this is your we know this from before. In electromagnetics also, we related this; c is equal to f lambda right, same relationship.

So, you get the generic c is equal to the wave length, speed of sound and frequency relationship is very similar to your electromagnetics ok. However, the interpretation right, the interpretation is going to be different here because the wave length, the c is a medium dependent property ok. So, if I increase or decrease frequency, what will happen? In a given medium, the medium is not changing. The wave length will adjust itself right. Wave length will adjust itself to maintain c as a constant. That is a very important take home message ok.

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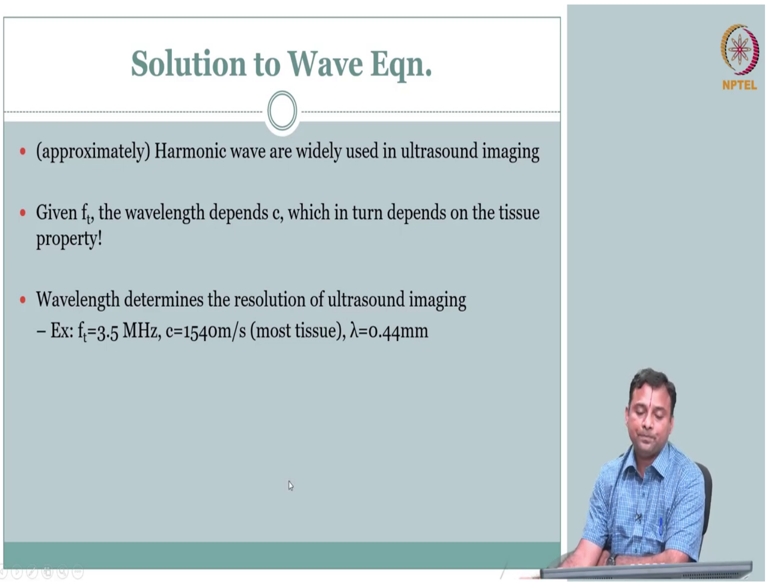

So, why are we interested in this solution? These are very widely used in ultrasound imaging. So, typically, we send three or four cycles of cosinusoidal right that is what we send. And so, given f t, the wave length depends on c. Of course, which in turn depends on the tissue property. So, this is the key, you can send any frequency from outside, you can generate the sound and send inside. But the speed depends on the material property and the speed that material determines also the lambda. A typical wave length right for given configuration say I have a frequency transmitted is about 3.5 megahertz.

Remember we said 1 to 20 megahertz; 3.5 is a very in a range that is very popularly used ok. So, 3.5 five megahertz is f. If you send this and assume the medium to have some speed of sound which is 1540 meters per second is a very common velocity that is that most scanners are calibrated.

So, f t is this, c is this, what is your lambda? 0.4 mm; meaning, sub millimeter I said right earlier. So, this is 0.4 mm. So, this is very important factor because lambda turns out to be determinant right. This is closely related to the resolution capability of the system.

So, in what is the c of electromagnetics? Oh, 3 into 10 power 8 ok. So, what will be your lambda orders of meters right? c is equal to f lambda that is what we have used here also; c is equal to f lambda, I know c, I know f, what is my lambda? It turns out to be 0.44. So, the speed of sound because it requires a medium and the medium supports certain speed. This determine, this makes it a fortunate circumstance that the lambda is 0.44. If c is speed of light, you have meters right, your lambda will come in meters right. You do not want to image inside the tissue with meter resolution right.

So, clearly, this is a very fortunate circumstance that the speed of sound inside the tissue is only 1540 meters per second and therefore, you could use some typical megahertz range and get a wavelength that is small enough, that is small enough as in compared to the dimensions of the objects that you are going to. So, your resolution is closely related to your lambda ok.

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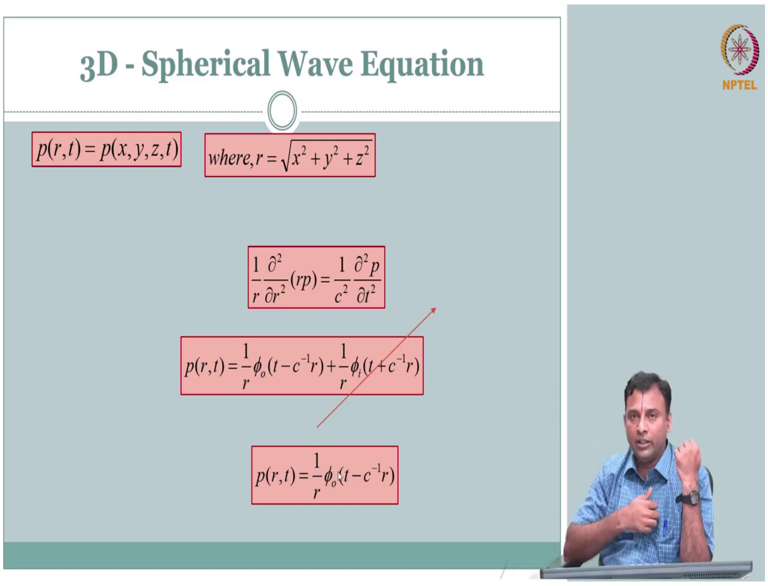

So, now, we will jump back to the 3D equation. This is in coordinate system that we use x, y, z; Cartesian coordinates. Clearly, the wave equation maybe can also be described in another coordinates. A natural coordinate that is very intuitive and very the mathematically you know straightforward is your spherical coordinates. Because I am talking now the pressure wave is going in all directions right; up, down.

So, it is actually in that sense, if you look at it my voice box is a point source. It is sending waves in spherical all directions. So, it may be lot easier to describe this wave in the natural, you know coordinates of instead of x, y, z of radius. So, this is 0 comma 0. All I can say is the pressure is changing in the same radius, it is the same thing right.

So, I could write p of r comma t instead of x, y, z. So, r will be your square root of x square y square z square right; spatial coordinates. So, if you do this, we can manipulate our wave equation that we had before, write it in spherical coordinates like here ok. So, if you it is very similar right.

Right hand side is actually no big change, your pressure is changing with the time; changing in space and time, so you get the second order. So, dou square p by dou t square 1 by c square. So, there is no change here. Here instead of x, y, z the Laplacian that we had we have written it in terms of just r. So, 1 by r dou square by dou r square r p.

So, if this is the wave equation spherical coordinates, what will be a solution? Solution can also be of this form pressure of r t will have two general solutions; one is what is o here is outward right. So, when the when the waves are going right, it is expanding and shrinking. You can think about expanding and shrinking; when it is expanding, so you can have a wave that is going outwards right.

What do we mean by outwards? Your radius is increasing here or inwards, your radius is close. So, in practice, what is going to happen? Your inward wave is not that intuitive and it is not we are not going to encounter that physically. And therefore, what we are really interested is if the two general solutions that you have, it is the outward propagation that will be used ok.

What is one difference between what we saw in the x y z coordinates and this thing you have a 1 by r tau right. So, with the distance, with distance, the pressure actually the amplitude of the pressure drops down linearly; 1 by r. So, that is something that we need to keep in our mind, back of our mind; when it comes, we will recognize that ok. So, we will deal with p of r comma t in spherical coordinates, you can write it as 1 by r of phi naught t minus c inverse r clear.

So, why is this important? This is important even though we may want to start with only plane waves, given the wave, so now, we know how to describe the waves right that is what I said organized. Because now, I do not have to visualize anything; I can just write this equation and the equation captures right, mathematically it completely defines the wave.

So, the natural extension is in our understanding, if I can understand this wave, what happens to this wave when it is sent in to the body that is the key. So, I want to send the wave into the body and the wave has to interact with the body and I have to capture the waves that are coming out of the body, after the interaction one way or back right, we had that through transmission or on the same side right that subtle difference we will make.

So, first is you have wave, we are able to describe the wave. Now, this wave has to interact with the tissue which will be our next focus of attention and then, we will talk about the configurations, whether you want to send the wave from one direction and detect it on the other side or you want to send in one direction, receive it in the same direction. All those new answers, we will talk about ok.