Introduction to Biomedical Imaging Systems

Dr. Arun K. Thittai
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 33
Ultrasound Phys_ Interactions

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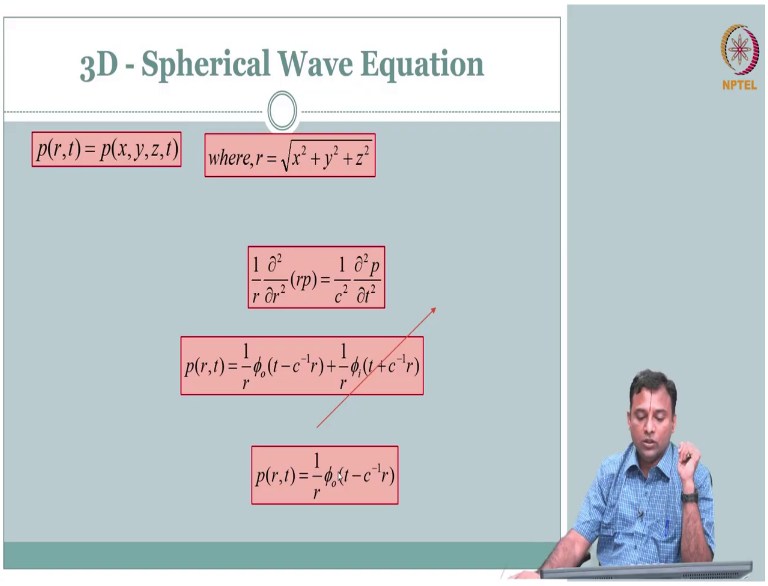

3D - Spherical Wave Equation

$p(r,t) = p(x,y,z,t)$

$where, r = \sqrt{x^2 + y^2 + z^2}$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rp) = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

$$p(r,t) = \frac{1}{r}\phi_o(t - c^{-1}r) + \frac{1}{r}\phi_i(t + c^{-1}r)$$

$$p(r,t) = \frac{1}{r}\phi_o(t - c^{-1}r)$$

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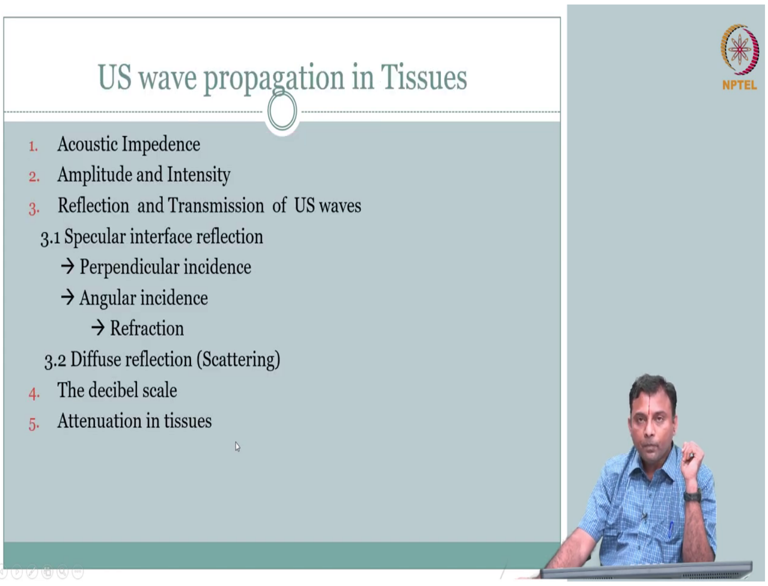

US wave propagation in Tissues

1. Acoustic Impedence
2. Amplitude and Intensity
3. Reflection and Transmission of US waves
   3.1 Specular interface reflection
      → Perpendicular incidence
      → Angular incidence
         → Refraction
   3.2 Diffuse reflection (Scattering)
4. The decibel scale
5. Attenuation in tissues

So, the natural topic is we know how to describe a wave how does it interact with the material medium ok. You need a material medium to support the wave ok. So, you can generate the wave then it is air and then you have to send it into the body then it becomes tissue. So, you need to know how the wave is going to interact wave the as we described in equation how does it going to interact when it is sent into the body.

So, we will first introduce couple of terminologies right and then we will talk about the interactions ok. So, we will start with impedance and then you know amplitude, intensity remember very similar to how we covered X ray also first you know the flux photon flux, fluence right, intensity that is how we defined and then we use that in our interaction similar thing we will do here.

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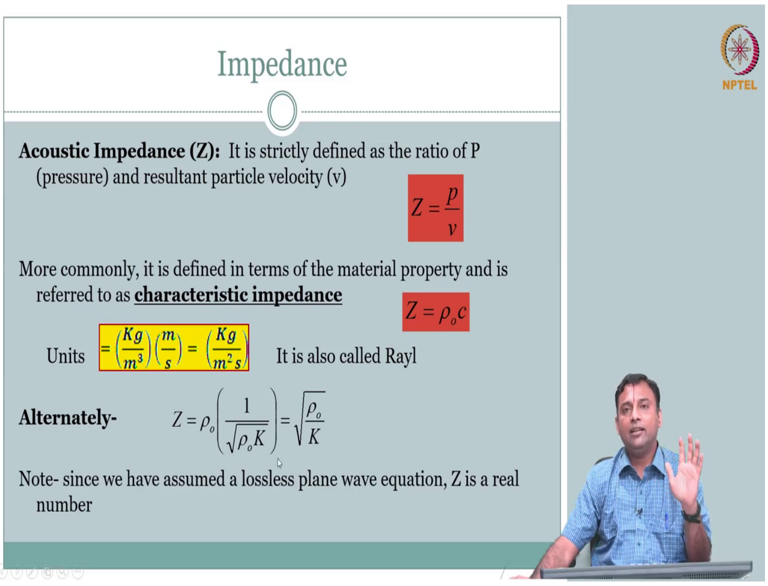

Impedance

Acoustic Impedance (Z): It is strictly defined as the ratio of P (pressure) and resultant particle velocity (v)

$$Z = \frac{P}{v}$$

More commonly, it is defined in terms of the material property and is referred to as **characteristic impedance**

$$Z = \rho_o c$$

Units $= \left(\frac{Kg}{m^3}\right)\left(\frac{m}{s}\right) = \left(\frac{Kg}{m^2 s}\right)$ It is also called Rayl

Alternately- $Z = \rho_o\left(\frac{1}{\sqrt{\rho_o K}}\right) = \sqrt{\frac{\rho_o}{K}}$

Note- since we have assumed a lossless plane wave equation, Z is a real number

First is your acoustic impedance. What do we mean by acoustic impedance ok, what is the acoustic impedance? Right. It is strictly defined as the ratio of pressure to resultant. So, what does it that they are doing? You have pressure, you are applying pressure difference and the pressure difference is giving raise to particle velocity ok. So, impedance means, there remember your analogous to your mechanical right analogous to electrical system that you may be familiar your pressure is a driver here.

So, in electricals it will be voltage that is the driver the voltage difference drives current. Here the pressure difference drives that gives raise to velocity. So, what do you have the connection between v and i that is your driver and what is flowing? v and i is your resistance right. If it is having frequency it is right you can talk about impedance.

So, here you have pressure and the particle has to vibrate right, particle as a resultant velocity. So, impedance is very similar. So, your Z is your acoustic impedance in this case is the ratio of p by v, the driver and the velocity clear. So, in some sense you could get a feel for why it as defined that way, why the name comes as impedance, but it is a acoustic impedance ok.

So, P by v is your acoustic impedance and this is a very important quantity. Why? Because it is you can get a feel for it right, you can already sense that each material may behave differently right. So, your Z in some sense is related to the inherent material property ok, but we already talked about material property we already use some other terms as well right.

So, most commonly it is defined in terms of material property and is referred to as characteristic impedance. Why is it characteristic impedance? Because if I tell you the impedance value it will relate to a particular material that is why it is characteristic impedance, it is a material property ok.

We talked about something else as well, c ok. We talked about c as a another material property right and c was related in terms of your density and compressibility. So, you look at the different properties all related, but these are all characterizing the medium. So, acoustic impedance is a parameter that is capturing the material property as well. So, you can clearly relate your Z to rho naught c ok. So, your why is this characteristic impedance? Because it is of a particular material, so, Z is your rho naught c ok.

So, if you do that it has its own units, right. So, K g per meter cube is your density, c is your velocity, s this is meters per second. So, you have K g per meter square second; however, it is mostly called in rayl. What is this rayl? This is in honour of a pioneer Lord Rayleigh right, Rayleigh theory of sound. So, Lord Rayleigh's contribution is immense to this field and in fact, it is you know a very old subject in that sense.

I think he did the is one of his classic books came in 1809 1885 or 1895 or something like that. So, it is you know close to more than 140 years or 130 years ago. So, it is a well established also. He is done Lord Rayleigh contribution is immense and therefore, this units is in a honour of that called Rayl ok. So, this is one way of writing it alternatively because we have the c we also encountered in our wave equation and we saw that 1 by c square you wrote it in terms of compressibility and density right.

So, I can substitute for that. So, I can write as Z is rho naught, c is 1 by square root of. So, there is a alternate form also you can write ok. But I think this is a very standard way to remember. Z is equal to rho naught c is a characteristic impedance is a very handy relationship ok. So, that is your impedance.

So, it can be actually you know we have assumed a lossless plane wave equation, right now we have just talked about a plane wave there is no loss. So, Z is a real number ok because of this assumption, but reality that may not be the case. As we will go we will see like with other cases you know energy is going to be lost in one form or the other there is going to be some loss right.

We call as attenuation right. Here also we will have attenuation and. So, in reality it is not lossless, but for now the way we have done it is ok Z is a real number. It can be complex in reality because of the attenuation ok.

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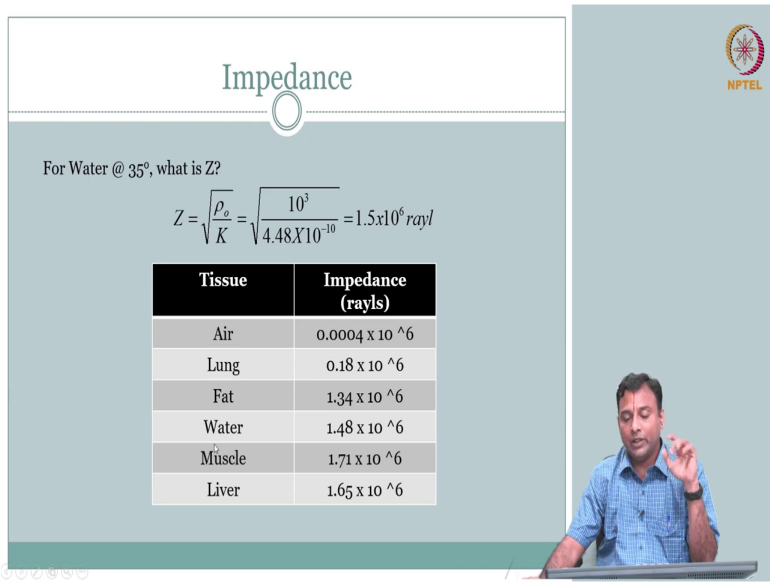

Impedance

For Water @ 35°, what is Z?

$$Z = \sqrt{\frac{\rho_o}{K}} = \sqrt{\frac{10^3}{4.48X10^{-10}}} = 1.5x10^6 \, rayl$$

Tissue	Impedance (rayls)
Air	0.0004 x 10 ^6
Lung	0.18 x 10 ^6
Fat	1.34 x 10 ^6
Water	1.48 x 10 ^6
Muscle	1.71 x 10 ^6
Liver	1.65 x 10 ^6

So, impedance, now then the question is what is the feel for what is this number. So, for water right we will be said human body we are about water bodies. So, what is the impedance value of in water? So, if I try to send the pressure wave in water what will be the impedance encountered right. At some temperature at 35 degrees what is your Z ok?

Take the log book you will be able to find the density right. What all do we need? We need speed of sound, we need density right. These are there in the log book. If you take that you can get your Z to be 1.5 into 10 power 6 Rayl by itself really this does not convoy much apart from its a factual information. Where it becomes interesting is so, if Z for water is this much right what are the typical property what are the typical materials that, we are going to encounter in human body? Fat, muscle right, water you already have, bone right all of these different things, air ok.

What are those values? It turns out you see that apart from these two fat, water, muscle, liver all of them are about similar right, second decimal first decimal changes, 10 power 6; so, very small difference between the different materials. What is completely different? Air orders of magnitude difference, lung is slightly different, but then notice where do you get air in the body, inside the lung right. So, how do you get to the lung from outside? You have to cross skin fat muscle and then you get to the right then to the lung.

So, now the question is this tells you a important information. We will not cover more than that here because we need to talk about the interactions in more detail, but you recognize here that because of this impedance difference between the various tissue types right, ultrasound is inherently useful only for certain clinical applications.

In fact, it is useful for last majority of the clinical applications, but you will notice that some of the applications for example, lung imaging or seeing inside the lung it is usually is chest X ray is what is done to detect lung nodules or whatever.

Ultrasound is not that popular ok. So, you are going to see the effect of that that is because of you see this huge impedance difference between lung, air and the other material that are there in the body ok. So, we will just leave it recognize that here, but we will come to it in the subsequent slides ok.

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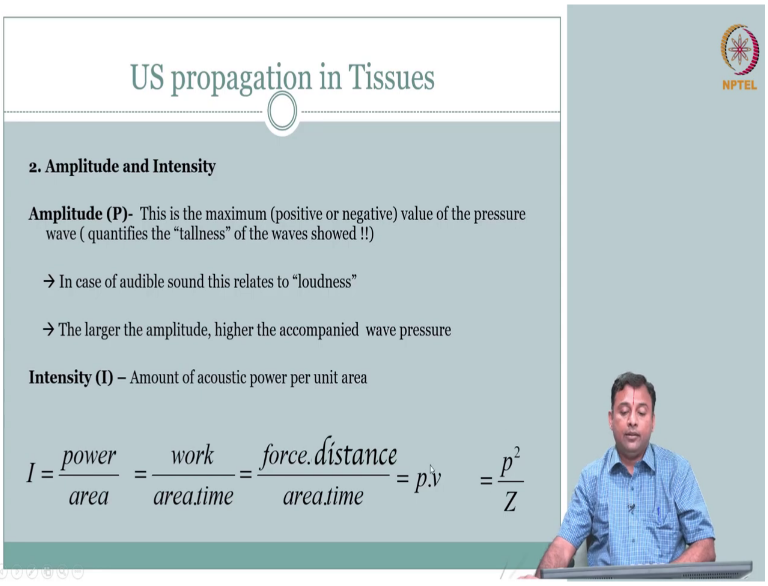

US propagation in Tissues

2. Amplitude and Intensity

Amplitude (P)- This is the maximum (positive or negative) value of the pressure wave (quantifies the "tallness" of the waves showed !!)

→ In case of audible sound this relates to "loudness"

→ The larger the amplitude, higher the accompanied wave pressure

Intensity (I) – Amount of acoustic power per unit area

$$I = \frac{power}{area} = \frac{work}{area.time} = \frac{force.distance}{area.time} = p.v = \frac{p^2}{Z}$$

So, then how do you; again we talked about this number of photons and photon intensity in the previous X ray base modality. So, here again you have amplitude and intensity. What is the amplitude? What we have been dealing with? Pressure amplitudes. So, pressure it is the maximum or right, the value of the pressure ok. So, here this what does this do?.

It quantifies the tallness of the waves that were showed. So, in some sense you can relate. If you talk about sound not just ultrasound sound right because ultrasound is just frequency is different. So, if you take a amplitude of sound you will relate it to the loudness, increase the volume decrease the volume right that has to do with your amplitude.

Whereas so, larger the amplitude higher the accompanied wave pressure that is why you increase the too much volume you know your eardrum you may get pain because it is trying to push the eardrum back and forth right. Your frequency is not changing, it is the amplitude ok, so, amplitude. What is the intensity then? Intensity has to do with some square term per area right.

So, intensity is the amount of acoustic power per unit area. So, you had a amplitude, intensity is your power per unit area. What is your power? Ok. I have my intensity as power by area. What is my power? Power is you know we I will just trying to I can just give out your formula, but then I think it is a good exercise to start from the definitions and then build your way. So, that there is no confusion in the end ok.

So, amplitude no issues that is your P that we have been talking about the pressure amplitude. When we talk about intensity we are talking about intensity is nothing but your power per area. Power is nothing but work per time, how much work can you do per time work per time. But, what is work? Work is force into displacement or distance right force into distance by area into time.

Now, if you conveniently look at it we have certain terms of interest force per area and distance by time. What is force per area? That is your pressure. What is your distance by time? That is your velocity. So, I can essentially get my intensity is nothing but pressure into velocity.

Notice that this velocity is not the speed of sound velocity not the velocity of the wave, this is of the particle. So, this is a another the there is a velocity of the particle there is a velocity of the wave. These two are two different things, do not confuse ok. So, I is pressure into velocity, this is your intensity ok.

So, from what we have done right we already covered impedance. So, using that definition we can write p we can change the velocity as p by Z. So, you get p square by Z. So, your intensity is pressure square by Z there right. So, intensity is you have a amplitude square of your amplitude divided by Z your impedance.

So, this is a handy that impedance itself right. Relating particle velocity and pressure itself will be used very conveniently in the description of how this pressure particle velocity when the wave is moving to describe its interaction ok. So, this is intensity.

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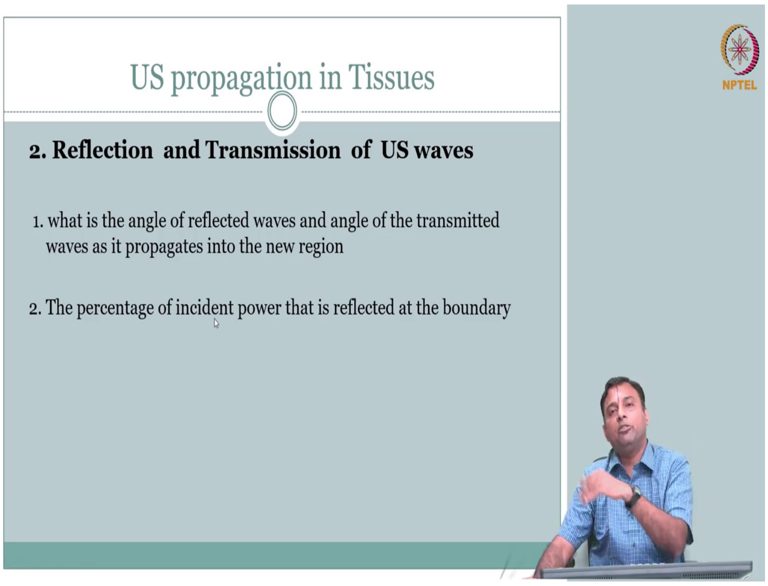

US propagation in Tissues

2. Reflection and Transmission of US waves

1. what is the angle of reflected waves and angle of the transmitted waves as it propagates into the new region
2. The percentage of incident power that is reflected at the boundary

So, now the question is ok, I have those wave, I know how to describe this wave, I have used certain terms now, describe the different medium; medium 1 different medium 2 they can be different in terms of Z or c, Z is your acoustic impedance or your c speed of sound density compressibility right. So, now, the question is what happens if I encounter a different medium.

So, I send the wave and now the wave is going to go from one medium to the another medium then how do I describe it, what are the interactions and how do I describe? So, you have going to talk about reflection and transmission of ultrasound waves when it is going from one medium to the another medium ok.

So, when we talk about reflection and transmission the two things we have to answer; what is the angle and how much is the amount. So, I have a pressure wave or the sound that is going in one medium I have another medium I am encountering which so, I have something that is coming back something that is going to go further down.

How much is going to come is one aspect, how much of it is going go forward right that is one aspect, where is it going to come, what angle is it going to come, what angle is it going to go further that is the another aspect. So, there are two details that we need to know right. So, the angle of the reflected waves, angle of the transmitted waves or the and the percentage of the incident power that is reflected back into the same medium and the percentage of power that is going forward ok.

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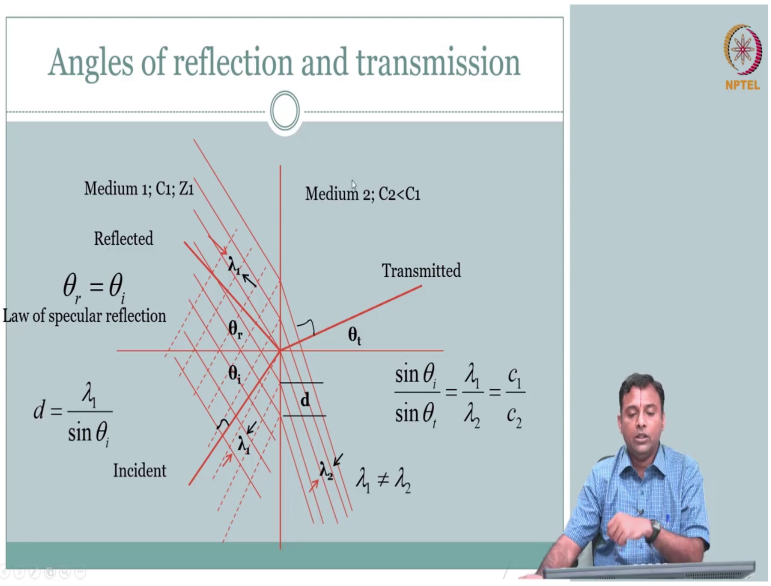

Angles of reflection and transmission

$\theta_r = \theta_i$

Law of specular reflection

$d = \dfrac{\lambda_1}{\sin\theta_i}$

$\dfrac{\sin\theta_i}{\sin\theta_t} = \dfrac{\lambda_1}{\lambda_2} = \dfrac{c_1}{c_2}$

$\lambda_1 \neq \lambda_2$

So, let us take two mediums, medium 1 and medium 2. Again our interest is not going to be in the derivation ok. We are just going to recognize what it is and maybe you can get a feel for it when we relate it to something that you probably know from your high school with optics ok. Medium 1, medium 2, whatever I have drawn here these are the plane waves plane wave fronts right. Just to determine describe these are plane wave, the solid line that you are seeing are capturing the plane wave front.

So, for example, this line captures all of them at high pressure right. So, this is the next high pressure. So that means, distance between the two high pressure peak to peak is your lambda. So, in between it is all going down and up, but I am just representing because it is a plane wave, I am just putting this line to describe that plane that is propagating ok.

All of the phases are also travelling like that only, so, no problem with that. So, your lambda, so, when it is going like this right it is going I have an interface right. I have an interface, it is hitting the interface coming back in the same medium and some of it is going forward.

So, I have a wave that is coming wave that is coming, I have an interface, on the same side wave is coming back in the forward direction some amount is propagated that is what this plane waves here it is the dotted lines are the reflected waves the solid line are transmitted waves in the medium 2. So, you have law of specular reflection which states that theta i is equal to theta r. So, I hit at theta i angle of incidence angle of reflection. Theta i equal to theta r.

Of course, you can talk about from the geometry you can talk about the d, d is here. From the geometry here you could talk about d is lambda 1 by sin theta i that is on the incident side, but d is the boundary. So, d has to be matched on the other side as well. So, d is should also be related to lambda 2 and sin theta 2 theta t ok.

So, if you of course, your lambda 1 and lambda 2 are not same because these two are two different mediums correct. Same frequency this is C 1 this is C 2, therefore, lambdas are to be different right because these are two different mediums. C is equal to f lambda remember. If f is same right C is different, lambda will be different. So, that is what. So, lambda 1 is not equal to lambda 2.

However, at the boundary of this or the interface of the two mediums you cannot it has to be same it has to match right, otherwise the interface will give away. So, this has to match and therefore, what you can get is this relationship. This I am sure you would have seen. Where would have you seen this?

Sin theta i by sin theta t is equal to C 1 by C 2 is very similar to your Snell's law in optics that you would have studied, same Snell's law. So, when you are reviewing this Snell's law when you are reviewing this you will also understand there is one another important angle. What is that? Critical angle ok critical angle above which, what will happen? There will everything will be reflected bad back total reflection same concept holds good here as well ok.

So, the so, this is so, the angles we know theta i equal to theta r and we can relate the incident angle to transmission angle in terms of these parameters ok, so, much for angle.

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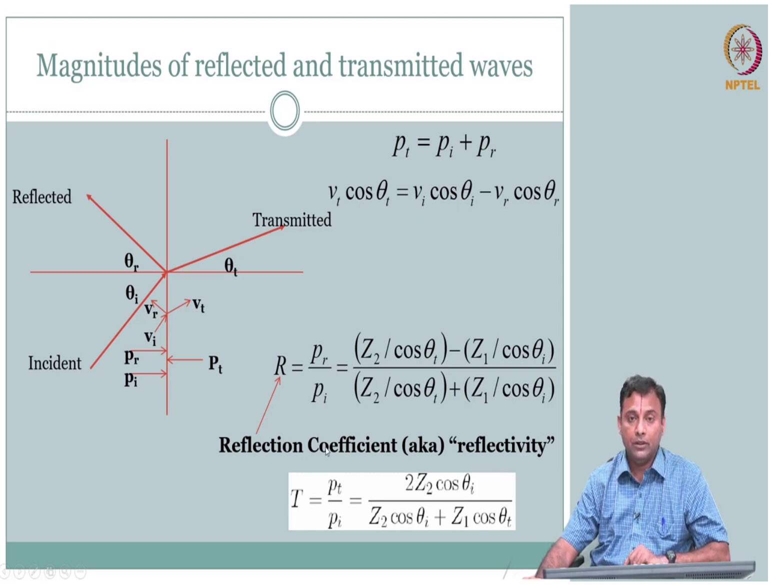

Magnitudes of reflected and transmitted waves

$$p_t = p_i + p_r$$

$$v_t \cos\theta_t = v_i \cos\theta_i - v_r \cos\theta_r$$

$$R = \frac{p_r}{p_i} = \frac{(Z_2 / \cos\theta_t) - (Z_1 / \cos\theta_i)}{(Z_2 / \cos\theta_t) + (Z_1 / \cos\theta_i)}$$

Reflection Coefficient (aka) “reflectivity”

$$T = \frac{p_t}{p_i} = \frac{2 Z_2 \cos\theta_i}{Z_2 \cos\theta_i + Z_1 \cos\theta_t}$$

What is next? Ok. How much is coming back right? Magnitudes of reflected and transmitted waves, how do we get that? Again we will draw a simplified version. Here I have taken the liberty to just draw one line. This is just perpendicular to the plane wave fronts right. This is just the line direction that is perpendicular to the plane wave front. So, I have just reduced I have taken all the dashed line and solid lines to make it little clear, but essentially what you have seen here is I have a theta theta i I have a theta r.

But now what should be the case? I is look at this, this is the boundary condition. So, the whole idea is the medium is going to support the wave propagation; that means, at the interface the interface cannot tear apart right. Some amount should go back some moment is going forward, but you cannot the interface cannot tear apart; that means, it has to satisfy a boundary condition.

In fact, two boundary conditions, one is in terms of velocity of the particle the medium right that particle that is there at the interface that has to be satisfying certain constraint, so, that it does not tear apart. Likewise the pressure, you have two pressures alright and one side you have one pressure the other side you have another pressure. So, pressure as to balance. If there is no pressure does not balance, then also you have a chance of rupture of the interface.

So, what are the boundary conditions? Your pressure on both sides should match out at the interface. So, your p t should be equal to p i plus p r at the interface and then the component of velocity right. That what is the component? Here is your cos theta t. So, v t cos theta t, that is trying to pull it this side should as right v t cos theta t v i cos theta i minus v i cos theta i minus v r cos theta r.

So, here one is velocity is going this direction, velocity is going in the other direction reflected. So, you get the negative because of the directionality right. So, you have v r v i cos theta minus v r cost theta r should match out v t cos theta t right. So, that has to be balanced then the pressures on either side also has to be balanced ok. So, if you do this then what is the amount that is reflected?

Reflected amount is nothing but whatever is reflected by whatever incident right. You can talk about reflection coefficient, a reflection amplitude reflection coefficient because this is just pressure. So, these are pressure amplitudes. So, amplitude reflection coefficient is the pressure that is reflected to pressure that is incident ok.

So, you can then do certain rearrangement, use the right we know the relationship between pressure, velocity and impedance right. One side it is Z 1, the other side it is Z 2 correct. So, we could use that relationship and write it in this form. I think you there are several ways of writing it. I think the textbook they are writing it as Z 2 cos theta i minus Z 1 cos theta t divided by Z 2 cos theta i plus Z 2 cos theta t ok.

So, it is just different forms of writing, but essentially you get what is reflection coefficient, it is the ratio of reflected pressure to incident pressure. Likewise, what is your we will just observe this and move. Basically, you get the feeling. We talked about some reflectivity right. You get what it is coming from, where it is coming from, some reflection is there.

So, we will kind of see how this can be seen as the reflectivity that we are going after. What is going through is your transmission which is p t by p i, you can get this ok. Of course so, this is with respect to reflection and transmission coefficients of amplitude ok.

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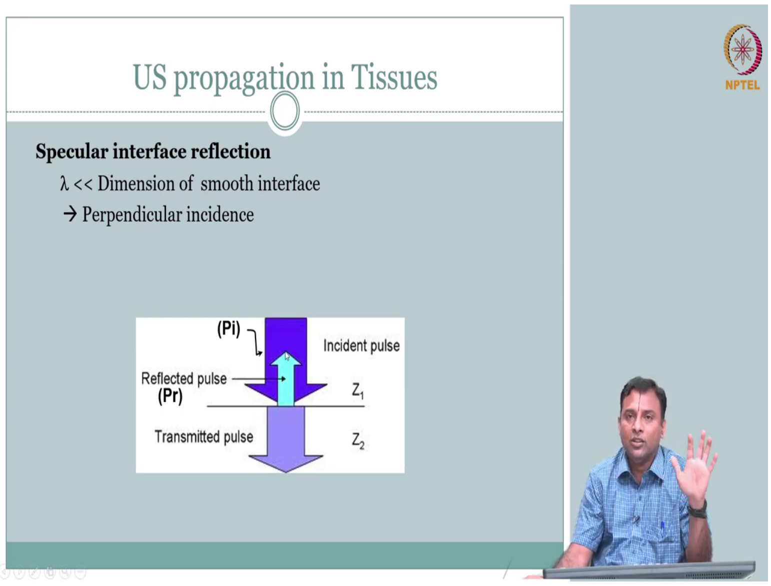

US propagation in Tissues

Specular interface reflection

λ << Dimension of smooth interface

→ Perpendicular incidence

So, what other possibility is there? What we talked about? You have your lambda right, we talked about a specular. There is another thing where it is called as specular interface reflection where your lambda is much much smaller that is this interface right the interface that we drew. You notice the interface that we drew like a line there was no roughness ok. It was very smooth line that we had.

So, when the lambda is far less than the dimensions of the smooth interface we encountered what is specular interface reflection. So, we had theta i equal to theta r in a. So, what happens if you are 90 degrees perpendicular incidence right? So, perpendicular incidence is a simple case where you have interface which is a smooth interface, you p i, p r. So, what we had the equation is in terms of theta cos theta t cos theta i here we have 0 degrees, so, cos 0 will be 1.

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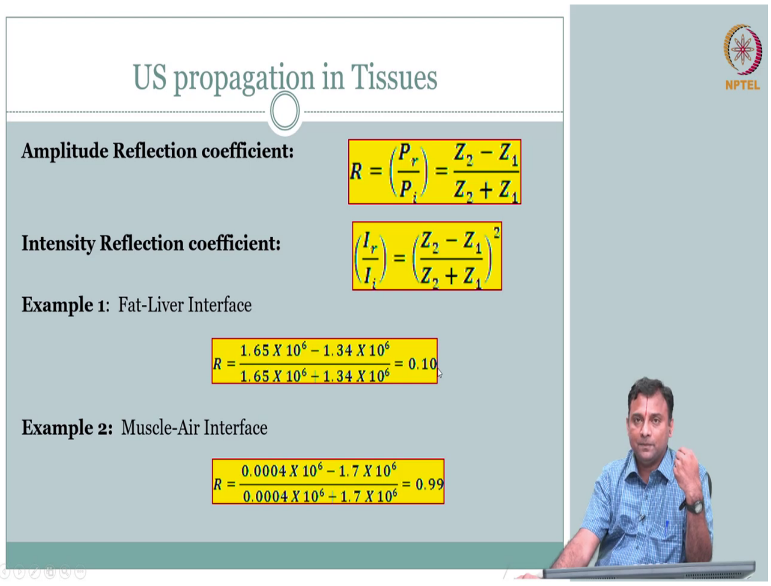

US propagation in Tissues

Amplitude Reflection coefficient: $R = \left(\frac{P_r}{P_i}\right) = \frac{Z_2 - Z_1}{Z_2 + Z_1}$

Intensity Reflection coefficient: $\left(\frac{I_r}{I_i}\right) = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$

Example 1: Fat-Liver Interface

$$R = \frac{1.65\ X\ 10^6 - 1.34\ X\ 10^6}{1.65\ X\ 10^6 + 1.34\ X\ 10^6} = 0.10$$

Example 2: Muscle-Air Interface

$$R = \frac{0.0004\ X\ 10^6 - 1.7\ X\ 10^6}{0.0004\ X\ 10^6 + 1.7\ X\ 10^6} = 0.99$$

So, essentially you will talk about amplitude reflection coefficient in case of perpendicular right, reduces to a simpler form. R is Z 2 minus Z 1 by Z 2 plus Z 1, this is your amplitude reflection. You could have your intensity reflection coefficient as square of this, intensity reflection coefficient is square of this. Likewise you can do the transmission as well ok. So, that is not a big deal. Let us take a example just to get a feel for what it is.

If you have a fat liver interface, so, send the sound waves are going they are travelling in fat and now they come encounter a liver. So, what happens at the fat liver interface? Well, we know the values for fat, we know the values for liver right. If you look at the table that was shown few slides ago we had Z for liver Z for fat Z for water right your acoustic impedance.

So, now what is the R? R, amplitude reflection coefficient is going to be Z 2 minus Z 1 by Z 2 plus Z 1. Z 2 is your liver, Z 1 is your fat. So, what is going to be the reflection coefficient right? R is Z 2 minus 1 by is only 0.1 ok. Before we comment about this little bit further let us see what happens if we have two other materials.

For example, muscle and air. Same thing, go to the table that we had, if it is muscle air interface then you have perpendicular incidence. You have Z 2 minus Z 1 when you substitute what you get? You get 0.99. So, now the question is ok, one is 0.1, the other is 0.99, what is good, what is bad, what is desirable, what is not desirable, what how do we put to context this numbers that we are seeing right. So, there is a thing. There is a good and a bad. So, what does that reflectivity says?.

I send sound wave in. If the wave interacts when it goes to the fat liver right it goes through the fat, it hits the liver. When it goes that happens R is 0.10; that means, only 10 percent of the amplitude, the pressure actually reflects back to the medium. So, in this case for example, I am sitting outside, I am sending this sound waves in. It goes through the fat, it encounters liver, only 10 percent of it comes back.

So, if I have to capture this what is coming back and then say you came back from this location or there is an object there is an interface at that location then this quantity is going to be very small quantity that I am getting that is the bad side. The good side is; that means, there is 90 percent of the amplitude right that is still there to go further.

So, I can come with fat liver interface I will have enough pressure to actually go inside the liver and perhaps come outside the liver and therefore, when I am sitting outside I can see through the liver. I can see through from fat to liver and the through the liver and maybe liver to the surrounding muscle at the back side of it.

So, the good news is little reflection happens and therefore, you have enough strength enough pressure wave to go further down go deeper ok. The downside is I am getting an eco back, but I am getting reflected signal back, but the quantity is less ok. So, in relation to this what do we interpret?

So, here what is happening? I have muscle air interface is 99 percent. Here I get a lot of signal back. 99 percent comes back when it enters when it encounters this interface. Just to make it vivid let us say I have air muscle interface right. So, air is outside, muscle. So, when I start with the skin and I get muscle. So, when I send the sound wave I have a way to generate my sound wave ultrasound.

I am putting it into the body ok. What is going to happen? I am going to encounter air muscle interface right. What happens when I encounter air muscle interface? 99 percent will come back; that means I generate the sound wave ultra sound wave try to put it into the body. But because air is on one side that is outside the body and muscle is inside right two different Z's, I will not be able to send sound inside the body. Then how will I be able to see what is there inside the body?.

If I cannot send 99 percent comes out at the skin it just bounces back right. So, this is a troublesome thing. So, what typically they do is when you go for a if you have been for an ultrasound imaging, what they do is they put some acoustic gel coupling gel they call. So, they put a acoustic jell that displaces the air then we have this wave generator that will be placed in that gel so that the sound that ultrasound that is generated goes through the gel and goes through the tissue.

Therefore, you do not have air in the in between. If you have air in between you will have air muscle interface and you will not be able to penetrate inside the body. This is the reason when you go towards the lungs right when you go towards the lung it is there inside. So, muscle when you go towards the lung one side is inside is air outside is some other soft tissue. So, you are not able to penetrate the lung. So, this is a very practical I should say constrain that you will encounter.

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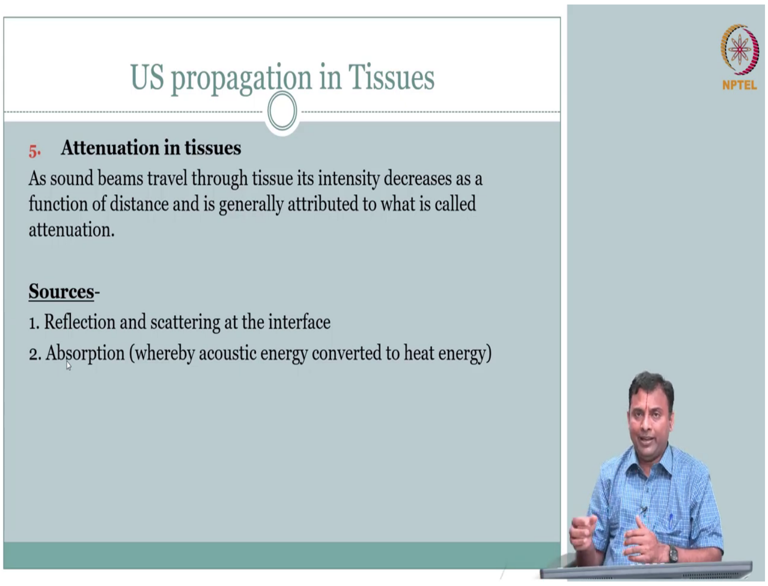

US propagation in Tissues

5. **Attenuation in tissues**

As sound beams travel through tissue its intensity decreases as a function of distance and is generally attributed to what is called attenuation.

Sources-

1. Reflection and scattering at the interface
2. Absorption (whereby acoustic energy converted to heat energy)

Last, but not the least is your attenuation right. What is attenuation? Anything when there is a loss, this is what we have been talking about right, anything when there is a loss. Here what are the losses possible? You can send sound and the sound can go in different directions right or it can get observed. You are talking about particles that are vibrating. So, maybe it will take some energy and right it will not be able to there will be some loss, absorption ok.

So, sound beam travels through the tissue, its intensity decreases as a function of distance ok. So, we do not care about what it is rather there is a loss and this we call as attenuation ok. So, what are the possibilities? Sources for the loss; a reflection and scattering at the interface. So, I am sending the signal. Some signal we just calculated right, R comes back.

So that means, if your downstream right as you go further in depth the signal is reducing because some amount is reflected back because you have you had some interfaces scattered, right. So, you could not just come at one direction you could go in several direction. We will talk about that in a minute. Absorption, where again you know this acoustic energy is getting converted to heat energy after all you have particles that are mechanically moving. So, they are going to be friction they are going to be lost due to absorption ok.

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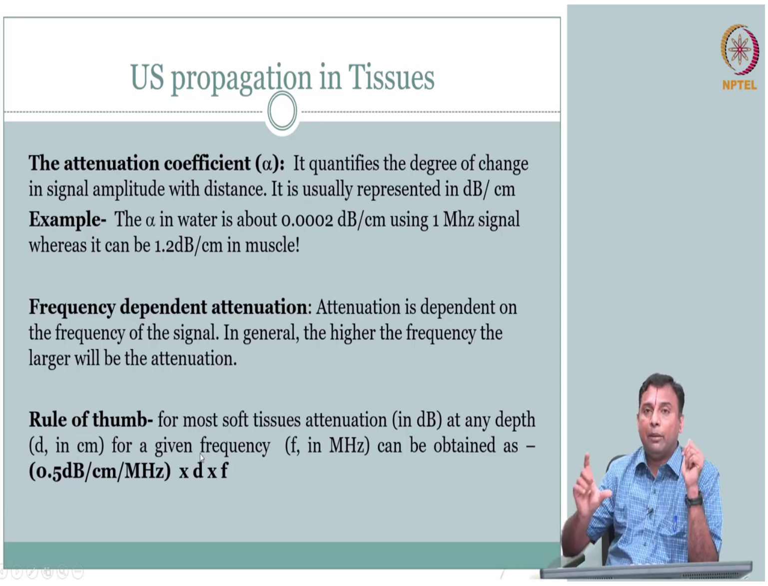

US propagation in Tissues

The attenuation coefficient (α): It quantifies the degree of change in signal amplitude with distance. It is usually represented in dB/ cm

Example- The α in water is about 0.0002 dB/cm using 1 Mhz signal whereas it can be 1.2dB/cm in muscle!

Frequency dependent attenuation: Attenuation is dependent on the frequency of the signal. In general, the higher the frequency the larger will be the attenuation.

Rule of thumb- for most soft tissues attenuation (in dB) at any depth (d, in cm) for a given frequency (f, in MHz) can be obtained as – **(0.5dB/cm/MHz) x d x f**

So, typically what we call as attenuation coefficient? It quantifies the degree of loss right degree of loss in a amplitude with distance. Most commonly we call it as dB per centimeter decibel loss per centimetre. It is represented as decibel loss per centimeter; however, it is also known right. So, the alpha is dB per centimeter using a particular frequency ok.

So, in a material for example, water, it is very less 0.0002 dB per centimeter whereas, if you go to muscle it is 1.2 dB per centimeter. So, now, the you see the challenger. Alpha is also using, this is reported per megahertz or using 1 megahertz ok. So, depending on the distance you are travelling of the tissue type the signal is going to get lost, reduced ok.

So, now you couple this with the small r that we are going to get from a typical soft tissue fat, muscle, right very small r. So, you see the quality. The signal level that you are going to pick is going to be very small ok. Why we said using 1 megahertz? Because depending on what megahertz you are using right we call as frequency dependent attenuation. So, if I increase the frequency right if I increase the frequency the attenuation will also the amount of loss is also going to increase.

So, it is dependent on frequency that is operated. Why would I want; so, the question would be why would I want to then use higher frequency because now signal is reduced. I want to probe deeper into the body. So, why do not I just use a smaller frequency, why should I use higher frequency? Remember the relationship between f and c and lambda right? For a given medium, if I increase f what is going to happen? Lambda is going to reduce right.

So, if I decrease frequency, lambda is going to increase. And we talked about lambda being something related to resolution, the smaller it is the better the resolution is ok, sub millimeter we calculated one example as well. So, the idea is if I want to increase the resolution I will increase my frequency, but if I increase my frequency the depth through which the signal will go right is going to be reduced or attenuation is going to come into picture.

So, the signal that you can capture and measure is going to become less and less with depth ok. So, this is a inherent trade of in ultrasound that you will face, higher the frequency lower is what we call as the penetration depth ok better will be your resolution ok. So, rule of thumb for more soft tissue we can get about half a dB per centimeter per megahertz.

So, depending on your problem acceptable level of image quality acceptable level of image quality in terms of S and R, in terms of resolution, you pick a frequency of choice. So, depending on that is why you see we will do that in instrumentation as well, but you will see that when you go for a abdomen scan they will use a different transducer.

When you go for you know some other say thyroid or heart echocardiography they will use some other transducer. So, all this changes or frequency they use is different because depth which they want to interrogate is different ok.

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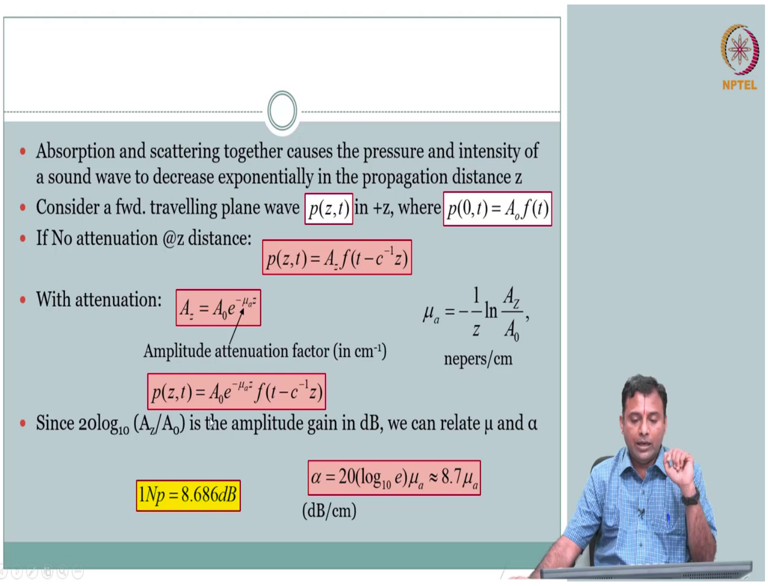

- Absorption and scattering together causes the pressure and intensity of a sound wave to decrease exponentially in the propagation distance z
- Consider a fwd. travelling plane wave $\boxed{p(z,t)}$ in +z, where $\boxed{p(0,t) = A_o f(t)}$
- If No attenuation @z distance: $\boxed{p(z,t) = A_z f(t - c^{-1}z)}$
- With attenuation: $\boxed{A_z = A_0 e^{-\mu_a z}}$ $\quad \mu_a = -\frac{1}{z}\ln\frac{A_Z}{A_0},$

Amplitude attenuation factor (in cm^{-1})

nepers/cm

$$\boxed{p(z,t) = A_0 e^{-\mu_a z} f(t - c^{-1}z)}$$

- Since 20log$_{10}$ (A$_z$/A$_0$) is the amplitude gain in dB, we can relate μ and α

$\boxed{1Np = 8.686dB}$ $\quad \boxed{\alpha = 20(\log_{10} e)\mu_a \approx 8.7\mu_a}$

(dB/cm)

So, absorption and scattering together cause the pressure intensity to decrease; again we are modelled this as exponential decay very similar to what was done in our X ray as well ok. So, as you propagate with the distance there is the exponential decay loss. So, if you have a forward travelling wave right p of z comma t which is what we want to consider.

When you have that, so, at z equal to 0 at source right at z equal to 0 you start with an amplitude a naught of a pressure function right; a sinusoid, cosinsoid that we started. So, you can have some amplitude of that, but what happens after it travels certain distance right? If there is no attenuation is straightforward, your p z of t will be whatever is the amplitude A z of t f of same wave question that we had right.

So, it will have, but if you have attenuation what is going to happen is this oh this has to be A naught this has to be A naught ok. So, z will have the same A naught whereas, when you have attenuation with a distance you are going to have a loss. So, you are going to have A z will be A naught whatever you started with distance you have lost in a exponential format, amplitude attenuation factor in centimeter inverse ok.

So, your p z of t is including your attenuation is A naught whatever you started, you have some loss e power minus mu A z of this may form that you started with. Of course, there is another units right they play with which a because your mu a can also be written right you have exponential. So, you can write this in terms of natural logarithm; 1 by z ln of A z by A naught. So, here which is called as nepers nepers per nepers per centimeter.

So, clearly you can see a relationship between your mu naught this natural logarithm, but we also talked about attenuation right. We talked about this is a amplitude attenuation factor, but just before we called something attenuation coefficient ok. So, there should be a relationship between those two. So, let us see.

Since 20 log 10 of A z by A naught is the amplitude gain in dB a straightforward definition A z by A naught 20 log 10 right because it is amplitude. If it is 10 log then it is power at power loss, it is 20 if it is amplitude. So, 20 log 10 of A z by A naught we know right is in dB. attenuation we know, attenuation coefficient alpha was in dB. So, now, I can relate. Here I see this is in ln natural algorithm.

So, I can relate these two as alpha is 20 log 10 base e of mu a. So, this is approximately 8.7 mu a. So, your alpha which is your attenuation coefficient in dB per centimeter is nothing but 8.7 mu. What is mu a, is your amplitude attenuation factor, you can have nepers per centimeter. So, if you have that 1 neper is about 8.687 dB.

So, you have to be very careful when you read the question right, make sure the units are appropriate whether it is in dB or nepers you have to have this conversion that you have to do ok. So, that is with respect to your attenuation.

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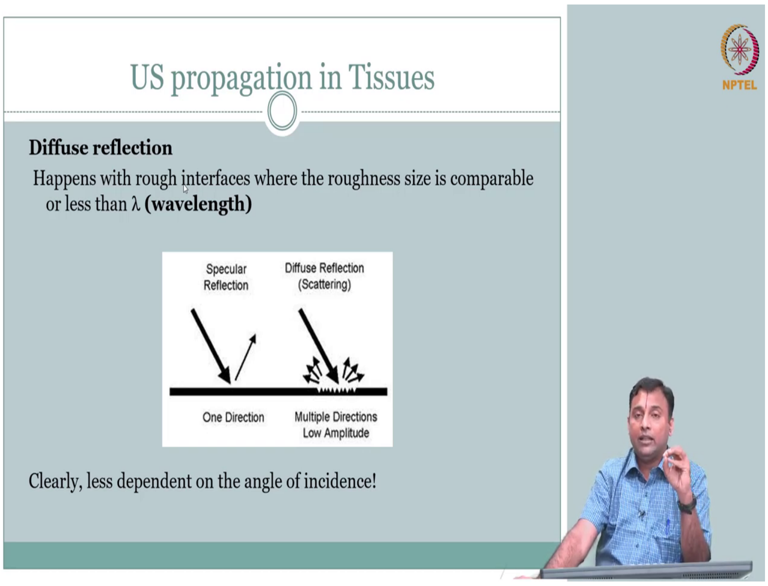

US propagation in Tissues

Diffuse reflection

Happens with rough interfaces where the roughness size is comparable or less than λ **(wavelength)**

Clearly, less dependent on the angle of incidence!

Last, but not least very important is we talked about scattering right, we talked about reflection right theta i equal to theta r and smooth interface. So, now what happens if you have a rough interface? What do we mean by that roughness is size comparable or less than lambda wavelength?.

So, everything is a object. You have a object whose size is much smaller than your lambda. Lambda is also length scale right. So, object dimension is less than your lambda. So, what we saw in the smooth interface in the specular case was the object size right, the interface was much much larger than your lambda.

Whereas what happens if your object size is much much smaller than your lambda? That is the case what we call as diffuse reflection or scattering. Two things that you see, this is what we saw theta i theta r. Whereas, here notice theta i no theta it is going all that is why it is called scattered. Notice here it was flat whereas, here you have imperfections, you have roughness.

So, the object size here is that roughness is you can think about radius of this roughness right, length scale that is comparable or less than your wavelength if that is the case then you have scattering or diffused reflection. This is very important what you vividly see is, it has plus and minus. What is the advantage? Advantages is less dependent.

So, if I send a signal, if I send a sound wave I do not need to know the orientation of my interface. See if it is a smooth interface like this if I send a wave at certain angle I need to know what angle I send. So, that theta r I can receive the signal that is coming back by placing a receiver at that angle of reflection. So, I need to know this how will I know what is there inside that is the whole idea I want to see inside. So, that is going to be challenging.

Whereas, here what this is saying is no matter it is angle independent. You send it in if such an object is there which is come less than your lambda then it is going to reflect in all direction. So, no matter wherever I place I will get some signal. So, the advantage is it is angle independent, but the disadvantage is it is scattering all around. So, wherever I am picking I am going to pick only a small portion of. So, I going to have a very limited signal right.

So, it is going to be very low amplitude in multiple directions unless I pick all the direction I am going to essentially have little signal ok. So, this itself is important. Scattering is important because it turns out when we talk about organ, we talk about the you know outer interface that we talked about when we calculated our R that is only specular reflection is ok.

But, predominantly the object itself right liver or all the tissue itself it turns out that you can think about it as having very small acoustic scatterers are spread in the body. So, all these tissue are nothing but collection of acoustic scatterers ok. There are small imperfections interfaces. It is not like one homogeneous medium surrounded by one wall it is not like that right. Because you have cells, you have collection of cells right.

So, so the idea is you have small interfaces meaning, small acoustics scatters small in relation to the lambda that we are using about 0.5 or 0.5 mm is the lambda right we saw. So, you have acoustic scatters that is distributed in our body ok.

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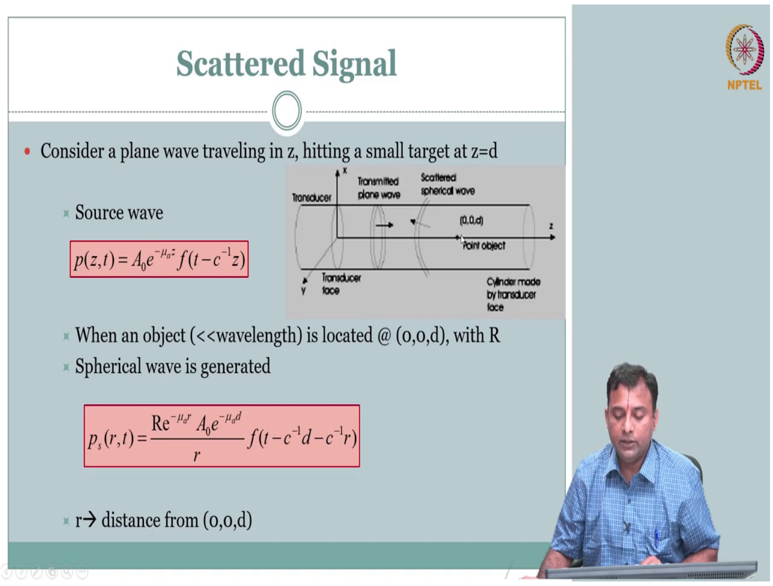

Scattered Signal

- Consider a plane wave traveling in z, hitting a small target at z=d
  - Source wave

$$p(z,t) = A_0 e^{-\mu_a z} f(t - c^{-1} z)$$

  - When an object (<<wavelength) is located @ (0,0,d), with R
  - Spherical wave is generated

$$p_s(r,t) = \frac{\mathrm{R}e^{-\mu_a r} A_0 e^{-\mu_a d}}{r} f(t - c^{-1} d - c^{-1} r)$$

  - r→ distance from (0,0,d)

So, our interest is if you have object which is a point object at sub location d and I send my plane wave which I know how to write right what is going to happen here? It is going to go as a plane wave, but when it is going to hit the point right it is going to scatter all around. What does that mean? That means, we can now treat this point is going to reflect everything and the waves a scatter everything right and the waves that are going to come back are more close to spherical.

So, I send a plane wave. This point object as converted that plane wave into a spherical wave. How much of it is coming back? That depends on the reflection coefficient of that object right, impedance mismatch between that point that point objects material characteristic compare to the surrounding right.

So, we can actually write our plane wave equation which we know. We can actually write our wave equation in spherical coordinates which also we know from before. But, how do we relate those two? By relate those two in terms of the reflection coefficient R, right. So, we can write your source wave which is your plane wave. We know from before you have A naught e power minus mu a. So, this is the one that is coming and hitting ok.

So, when an object is much much less than your wave length is located at 0 comma 0 comma d which has with a reflection coefficient right. It has impedance mismatch and that impedance mismatch gives rise to a reflection coefficient, so, with R right. So, spherical wave is generated. You can write the equation for the spherical wave as p r of t is this is your R reflection coefficient exponential of mu a r. So, after it comes it is decaying with r. What is that? This is your source right.

So, r f t minus c; so, it is gone in one direction, it is coming in other direction right going towards d. So, your r small r is nothing but distance from this point 0 comma 0 comma d, this acts as you source now. So, now, this is what you are you are going to capture. So, I am going to send in a wave that is going to be converted by point into a scattered signal. The scattered signal can go in any direction and if you are going to pick up that reflected signal at any location, this is the signal that you are going to go after ok.

So, this is a very important concept ok. Most of the time you would have seen an ultrasound unlike your chest X ray. In ultrasound the you know you have a probe we will talk about it when we go to the instrumentation, but in general what you have observed right, you have a probe which generates the signal and it also acts as a receiver.

So, you keep it, you are only doing on the same side. You are not doing through transmission. You would not have seen one probe sitting here and the other probe or some instrumentation on the back usually right. You if you have gone for ultrasound scan you would have noticed that. So, it is on the same side.

So, the whole idea here is I send a sound signal that gets bounced back because of the material property, how the material is distributed how the acoustic property, how acoustic scatterers are distributed in the body based on that I get my echoes back. I send the acoustic pulse. I get echoes of that pulse. So, typically we operate in what is called as pulse eco ultrasound imaging.

So, this reflected signal or a scattered signal right it can be reflected or scattered whatever you are going to get the eco ok. So, this is how you can write that.

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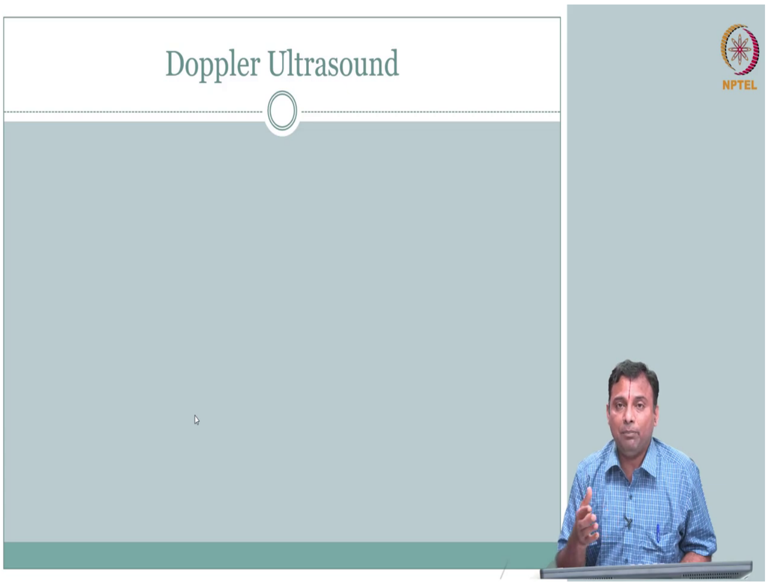

Doppler Ultrasound

So, we will stop here. This is good with respect to a structural part reflectivity. We can get the distribution of the reflectivity, which encapsulates the density, compressibility which can be talked in terms of either speed of sound or your acoustic impedances Z right particle. So, all of this is captured in the R ok. So, now, we will have to talk about the Doppler principle which can be used for our blood velocity which would be left to the next lecture.