Introduction to Biomedical Imaging Systems
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Lecture - 35
US_Beampattern

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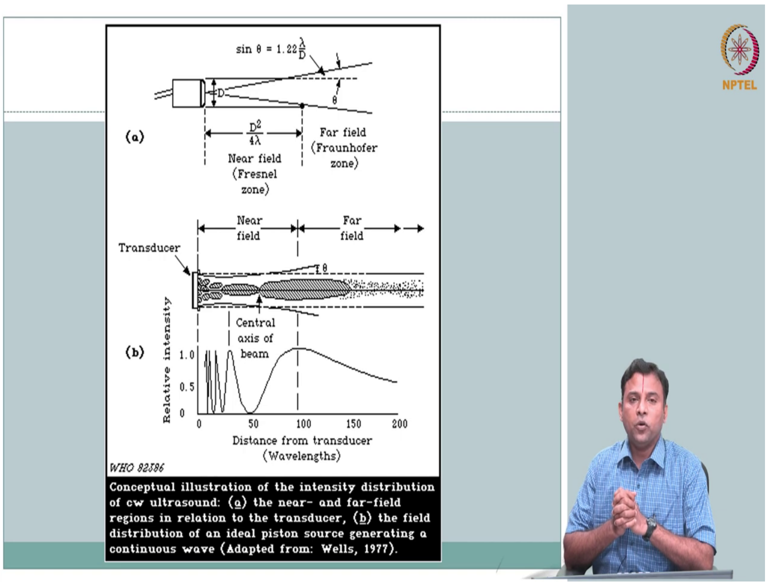

Ok. So, what we have covered this far is we talked about started with instrumentation and one of the major component in the instrumentation of an ultrasound system is your transducer which acts as we saw, it acts as both source right; you could generate pressure waves as well as it will receive the pressure waves. So, on one side, you can supply electrical quantity, it creates a pressure wave and when on the reception, we talked about how we will be using it as a pulse echo mode that is what is predominantly used for medical imaging applications.

So, when you have the pulse that go in, it bounces back. We saw about all the properties about reflectivity right as impedance mismatch. So, a portion of the signal or the echo of the signal that you sent comes back and the same transducer, the crystal the piezoelectric crystal, now when is hit with the pressure wave, it gives you a voltage waveform and that voltage waveform is taken and process further and you know you get all the image display signal processing put them together or organize in an image and display.

So, what we need to focus like we the style that we have adopted, we did instrumentation and then we go into imaging, but right. But the imaging equation or the image formation; but I told you slightly I have switched the style here because the last part of the imaging equation right that we typically cover in physics, I did not cover; I said we will cover the transducer or start with the instrumentation and then, bring in that equations ok.

And so now, we are at a stage, where we understand that we can use a transducer and we could give harmonic excitation and that could give you a pressure wave. So, the challenge that we are now going to address is what is the shape of this pulse right; what is the shape of this wave form.

So, I have a transducer, I excite a transducer and the frequency is dependent on the thickness that is what we saw. The thickness of the crystal determines the resonant frequency. But then, we also want to know why there are different size and shape of transducer right.

So, that means, what we need to understand now is when you have a transducer, when you have a transducer of a certain size and shape, when you excite it, where all does the pressure go in the field; how is the pressure distributed in the field or how is that wave that is propagating for which we got the wave equation right.

Is it just going ideally; just within that region or is the pressure wave having any because it is diverging right; is it having any weird locations where the intensity is high or low or what; so what we call as beam pattern that is what we want to study. So, in that context, I had this displayed earlier. As you will notice why this was displayed is if you have a transducer, inherently it is going to diverge right.

But then, it is going to retain. So, if you have a size of a transducer say for example, this is the transducer phase right may pump. So, it is going to get excited, it is going to oscillate back and forth and because of that on one side of the; so, if this is the back end, where you have the electrical circuit; this is the front end that is going to send it to the tissue. I excite this crystal, it is going to be oscillating.

This is in a thickness mode that we call. It is going to oscillate in the thickness direction because we are interested in compressional waves. You do this. Now, the question is if you do this in inside the medium, is it just going like the same way right. Unfortunately, it is not that straightforward. It is going to start to diverge as you go right; not just diverge, behavior of points near the transducer is going to be different.

So, here is an example of the pattern that you would see. So, you could see locations where there are high pressure, locations where there are low pressure right and even along the centre line, along the centre, you have high, low fluctuations. Of course, after some distance, it is little smoother. So, this is along the direction of propagation; this is across. So, when we do imaging, if we are just doing one dimension this is ok.

We know depth thickness crystal lambda determines your axial resolution ok; but then, the challenge in imaging is you have the other dimension. So, how does this wave spread in the at least the imaging plane perpendicular to the depth axis that is an important concept so, that is what we will focus.

I advice you please take a paper, pen, pencil, whatever you are comfortable start to write the equations as we go along. Next 7 or 8 slides is just going to be Brute force equations. So, my job is to kind of make you feel the physical meaning of how these you know equations come up. But otherwise, it is going to be heavy duty with respect to number of variables, you will have you know four integrals with so many terms.

So, please make sure that you have a pencil, paper, pen, start to write out you start to write out and then, listen to this commentary and then go read about it and then, think about it. Only then, I think you will be able to appreciate that the content that we are going to do.

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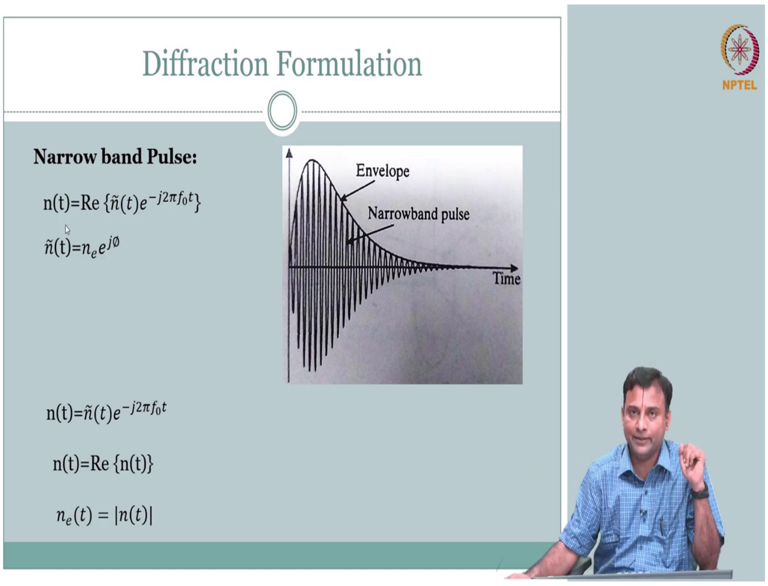

So, with that heavy dose of caution, let us jump into the subject which we call us diffraction formulation. So, to just give you a feel right, before we go in your optics, you would have done this Young's double slit experiment and stuff right, where you would have had ok, you have one hole, how does the light come through the other direction.

If I have two holes, you do not get two, two beams, you get interference. There is going to be fringe effect right and you change the plane, you will see how what happens to the light. So, similar effect that is for light; similar geometric effect is also applicable to sound and that is where you call this as diffraction formulation.

So, otherwise, there is if you have no diffraction, then it is just that ok I have this phase of the crystal that is oscillating, I get the pressure wave and the it is all. So, this is cylinder this is a disk shape for example right. If this is disk shape, if this is moving only along the disk right as a cylinder column, only those particles will all nicely move back and forth, if everything are ideal.

But that is not the case. You have edge and you have a finite size and therefore, you are going to have some spreading and then, interactions of the waves from different locations. So, you have to develop what is called as diffraction formulation. So, we will start with what we want to operate. We want to operate in pulse echo mode. So, ideally, we want to use a narrow band pulse ok. So, how do we formulate this?

So, let us write a narrow band pulse to be n of t which is of course, these are all acoustic is a physical system. So, you are going to have the real part of n tilde t with a frequency term here. So, this can be thought of as a right, you are going to write the wave because it has a frequency right of the crystal resonance frequency.

So, we going to write that resonance frequency; but we are also going to write it in terms of the envelopes of this tilde; n tilde of t is nothing but the envelope of the signal and there is a phase of this envelop. So, essentially, we are talking about a pulse echo. So, this is the pulse that you are going to send into the tissue. So, the crystal is going to oscillate right in this fashion ok.

So, you have you can look at it as a carrier frequency and then, that is modulation whichever way you look at it right. What you have is a RF signal with the centre frequency related to your resonance frequency and then, there is this envelope. So, why is it narrow band? If this envelope right, this pulse length; remember the pulse length, I asked you to kind of pay attention to when we did the axial resolution.

So, we want this pulse length right, narrow band. What do we mean by that? So, this pulse length should be large compared to the number of cycles you have here right compared to the time period the lambda ok. So, we will take this. So, we will pretend going forward this is (Refer Time: 09:31).

So, typically, what happens is the we send about 3-4 cycles of this know say for example, a Gaussian modulated sine wave with 3 or 4 cycles is a typical pulse form that is sent in for your pulse echo imaging ok. So, what we will do is let us pretend, we are going to send this guy right and we are going to have a real part of this n of t and your envelope is having so magnitude of this n of t.

So, you are sending this signal. So, this is going to be the signal that we think we are going to hit at the phase of the transducer. So, now, our objective is. So, you can look at it. So, if this is what is happening at the phase, if the phase is going to have this vibration right, this pressure wherever is created at the phase of the transducer.

Then, what is going to happen after it leaves the transducer and goes to a medium; how is this pressure distribution going to change or how is this pressure pulse going to propagate? So, with the time this pressure pulse right, this is a wave that is going to go. So, at different time instances, where is the pressure distribution; how is it distributed in space. That is what we want to go after ok.

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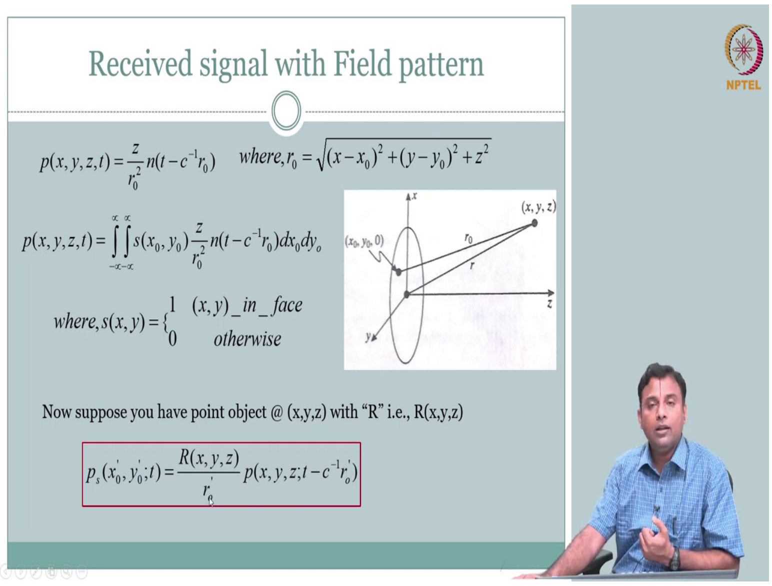

So, what we will try to do is set up a formulation. Again, for simplicity, there is a circle here, but this can be any shape. Remember the thickness mode, we are not worried now. The thickness is going to determine your resonant frequency that part is taken care. So, pretend you have some thickness; so there is some fundamental frequency, resonance frequency.

Now, we are interested in the shape and size of this transducer crystal ok. We are going to now if you use a particular says and shape and size and excite it with the resonance frequency f naught as the pulse that we saw, how is the pressure; what is going to be the pressure field in front of the transducer that is what we are going to see.

So, we take a shape. So, for simplicity, it is a circle here, but you notice it is essentially can be generalized to any geometry. So, pretend this is your transducer phase, you have your x naught y naught 0 right. So, this is x naught y naught 0. This is your origin. So, this is your point in the field right at some x comma y comma z and there are some respective distances right.

So, what we will do is recognize your pressure right pressure at x y z; what is that? That is this location. So, what is the pressure experienced at this location as a function of time? Well, you are sending n t right; this is the one that you are sending. So, if that pulse, it will take some time for the pulse to come from wherever it is generated till this location. So, that is going to take time.

So, but so when the pulse comes there, it will probably have the same shape as the pulse that you sent. Of course, the you see this term right here; what does this say? There is a small difference. How is this wave generated? This is generated from a for example, you take a point right, you have a circular phase here.

So, if it had been a sphere that is going to oscillate right this, then you have spherical waves; whereas, what is so? So, that was that we call as monopole, monopole source; whereas, what you have here is a disk. So, in that sense and you are trying to do it back and forth. So, it is not opening up in all the directions right. So, it is not a spherical wave for that reason.

Even though if you take the centre point right, you could pretend this disk or this area is composed of several collection of several points right; area is collection of points. So, you have multiple points, each point is going to send out a wave which is n of p of c inverse r that is your wave that is travelling. So, if it had been a monopole source, you could have just written a circular wave equation right; 1 by r.

But given this situation, this is called as a dipole model ok. Because of that you get z by r naught square of this guy. This is how the pressure field comes out at any location x y z and this is from a point ok. So, that is what is experienced from a point; but then, what you can? Of course, these are just giving you the dimensions ok, r naught a square root of this guy. Clear? This much is straightforward.

Because we know this is the pulse the pulse has to travel and at some location x y z, the pressure experience will be the pulse that you sent. Of course, the there is a you know ratio here which is because of which is not just 1 by r naught for example right. It is not just 1 by the distance as it would be for a spherical wave.

So, this is going to be slightly different because of the dipole model. That is fine. So, what happens? This is at one right. So, what we can do is we can write for a collection. So, if you have this is the surface of the transducer right, if one point because of one point at p x y z, you are experiencing this pressure right; we could then talk about p of x y z t from all the points because you are exciting the whole surface right.

So, you can if that is done, it is a net sum from all of the s, s of x comma y naught is your transducer right. So, we talked about size and shape. So, we are just leaving it here as the transducer function. So, here in this case, you see s of x comma y is 1. So, that it is circle. So, you have the source right this is your source.

So, what you receive here is from all the points in the source that is what this integral is saying; very straightforward right. So, one point contributes one point, wave is generated, you receive that. But in reality, the whole surface is oscillating. Each point, there is actually creating a wave and therefore, what you experience here at arbitrary location is going to be sum of all the point.

So, this is Huygen's principle ok. So, we make use of Huygen's principle and super position and we get that. So, this is fine. So, this is on the transmit part of it right, you transmit. So, this is how the pattern, this is what is going to be experience. What are we interested? We are interested in pulse echo.

So, what is going to come back; what is going to come back? If I pretend now this point has some reflectivity right, they have some reflectivity here, then if what did we see? Reflection coefficient; you go, you send a signal i; what you get back is i naught, i with the reflection coefficient right p r by p i receive right that is your reflection coefficient.

So, p r is going to be incident times reflection coefficient. What is incident? We this is the pressure that is experienced. So, what are we going to get back? It is going to be whatever is incident times the reflection coefficient. If it is going to reflect more, then you get more amplitude to come back. If it is going to reflect less, depends on the mismatch right that is one thing.

Other thing is this is I say there is a reflection coefficient, what is the size of the object? If I pretend this is a point object right; that means, we already talked about it in scattering. If it is a point object, if this is going to be hit with the wave, what is going to happen? It is going to radiate back or scatter back a spherical wave. So, this is going to be a point then and therefore, you are going to get a spherical wave. What is going to be the spherical wave that is coming out?

Whatever was hit that reflection coefficient time is what is coming out; when it is coming out, it is going to come out as a spherical wave ok. So, your P s which is you can think about it a scattered right, scattered wave what is going to come and hit a location is reflection coefficient times whatever you sent in ok reflection.

Of course, this is notice, here we had z by r naught square because it was a dipole model; whereas, here it is a point scatter. So, you are going to have a spherical wave and therefore, you are just using your spherical. So, whatever is hitting p of x right, p of x y z, this was this is hit that location. It is reflection coefficient, so this r into this guy is what is coming out and it is coming out of spherical and therefore, you have 1 by r naught dashed of this guy. So, that is what is your scattered wave ok. So, this is from one point response.

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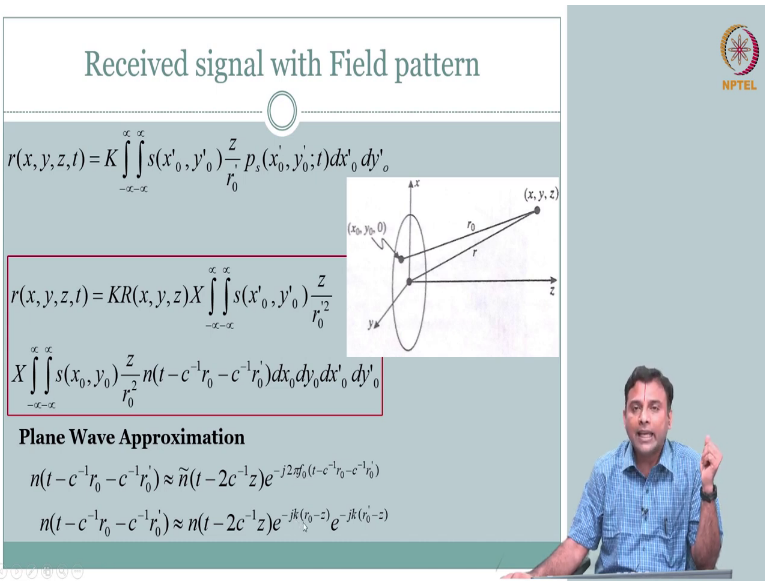

So, what do we would like to know? So, what are you going to receive? See the point is you have full; when you receive also, you are going to receive it all the points. When you are going to transmit also, all the points are transmitting and therefore, not just this point, what happens if we had one another point here?

That would also behave very similarly right the wave would have hit, come back, what if there was a point here? So, again, so we will say on receive is the medium right whatever you going to get back, you are going to have a distribution of reflect distribution of points ok and therefore, what we can try to do is your received signal.

So, now, see what we are writing? This is our received signal x y z of t correct is whatever is falling on the plate dx naught dy naught is your surface here. So, everything that is falling on this. When it is reflecting, what is happening? Spherical wave comes. So, this is my transducer phase; a spherical wave comes, this is my spherical wave for example that I am keeping this is as.

So, when it comes, the spherical wave is travelling. When it comes and hits the crystal back right on the receive side, it is going to now oscillate and then convert to electrical. So, this is going to oscillate right pressure wave. So, whatever this spherical wave that is coming out, wherever is hitting the surface of the crystal, all of the excitations in the crystal in the reverse is going to be summed, is going to be contributed.

If it is not falling on the detector, that wave we just went somewhere; we do not care about it. So, the received signal is nothing but what came out right the spherical wave that got reflected and came out, for that r reflection coefficient went into this some constant right. That has to do with essentially your efficiency of conversion for example from your electrical to pressure and pressure to electrical, all those things contributes all of that is some constant k.

So, you have absorbed that reflection coefficient also there. So, essentially, what you have is your signal that came back times wherever the surface area all of this is contributing. So, all the collections you get and this is for one point ok. So, now we can expand it and say look that is for one point right.

So, what do I do? I have a collection of points right you could just use simple superposition theorem, just each point is contributing maybe they are distributed differently right. A collection of that is what you are going to see. So, you can write your r of x y z of t essentially as full I mean that is why you have to be cautious because there are so many terms here right. This is into, this is into.

So, one is the why, what we have done is we have expanded see we had this p s right, scattered signal. I have just expanded that scattered signal as well ok. So, that is why you get. I told you right that is going to be lot of even four integrals here. So, you had this right, you have this and then, the p s was scattered signal collected over s x naught of whatever came back.

So, why is this two integrals? A surface area is contributing; surface is a collection of point. So, dx and dy comes there, then that field pattern goes hits and the reflected signal comes back or the scattered signal comes back that is again exciting the crystal. So, that is another two integrals for the area at dx, dy right and therefore, what you receive is what you sent and what how much came back, the echo that came back; sum of all that is what is your received signal.

This is fantastic; all incompassive, but it is also complicated right. This is just Brute force substituting the wave equation and to the context of some transducer generating that wave ok. So, now, what we need to do is we need to make it little more practical right. So, we need to simplify it. So, certain approximations, we will encounter now ok. How do I describe this wave pattern right, the beam pattern field pattern so, that so this is capturing everything. But then, it is not easy to analyze with this equation.

So, we will make some approximations. So, that this reduces to a form which can be better explained better analyzed and we will put the context that time. So, plane wave approximation; what do we say in plane wave approximation? Remember, we are looking at some plane located, so some plane at z.

All we are saying is this envelope of the envelope of this pulse right that you are sending, they are all hitting right; all of them are arriving under z plane simultaneously. That is what we are going to say as plane wave approximation. Why? Because once we make that approximation, this guy, this is the pulse that you are sending right n t of this can be approximated to your t minus 2 c z.

So, what you are going; what you are coming right, so 2 z and you change goes to your the phase term. So, you can approximate. So, you can make a plane wave approximation, where you say basically the envelope of the pulse are all arriving in the z plane simultaneously.

If that is the case, we can mathematically represent that in this form and if you look at this, we can even reduce its little further. Why? Because you know this 2 pi f naught is there. But then, we are talking about wave number. Rememberm I said that it will be conveniently used because you are talking about in space, you are going to feel patterns in space. So, I would rather convert this time f naught quantity to the how the you know wave is propagating in the medium.

Therefore, wave number becomes convenient. So, we will switch instead of f naught right, 2 pi f naught we will switch that right in terms of k. What is k? 2 pi by lambda; 2 pi distance by lambda is your k wave number, go look for that ok. So, we are just conveniently writing that in terms of wave number and splitting it. So, now, what we will do is ok. So, we need to substitute this back. So, next slide is also going to be I mean this is a big equation, we have not done much; we will just substitute this back and then, start to reduce ok.

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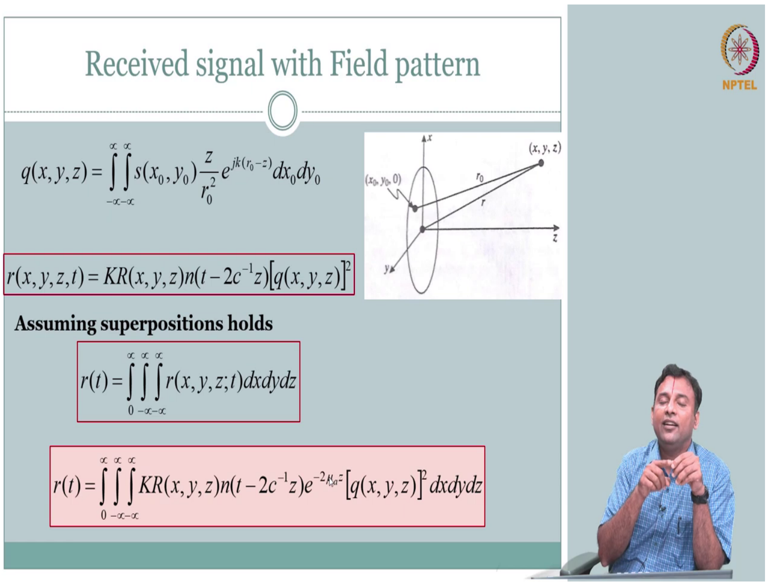

So, what we now call as q of x y z is just that term ok. So, you had two integrals, two double integrals; one of the double integrals, where we had we are making this approximation right. We can write, so this one right. You had two double integrals, there are four integration side.

Rather by doing this approximation, conveniently, splitting this as r naught minus z and r naught dashed minus z, what it turns out that if you put it back here? It turns out that you have you can simplify this into two identical double integral. The four integrals became two identical double integrals because of this manipulation.

So, the two identical double integral will be like this; s x naught y naught z by r naught square e power jk of r naught minus z. So, you had two double integrals. By making this plane wave approximation, we have reduced the four integrals to two identical double integrals ok. So, this is one of them ok.

So, why is this convenient? Well, at least this is only two integral ok. So, double integral, I can basically q square is what I will be interested in ok. So, what you are getting now is your receive signal. So, it is the same equation that we had in the previous slide. Now, I am going to break the four integrals into two identical double integrals, what called as q of x y z. So, other terms remain KR right and then, n of t z, then q x square ok. So, this is your receive signal.

So, it is already complicated. Mathematically, it looks too many terms; but conceptually, straightforward right. Whatever you sent right, the whole surface area is sending; it is a collection of points. So, the net sum of that is acting here and then, there are several points. So, what is received back is in the medium wherever there is point scattering right.

So, there is a reflectivity distribution, if you will you are collecting back wholesome of it that is what that is what this says. How is that distributed? You see it here from all the surface. So, this is how the shape and size of your transducer gets into the signal that you are sending and the signal that you are receiving ok.

So, now, what we will do is ok super position holds good. So, we can write your r of t right. So, this is for one we wrote, but then, we will assume superposition and therefore, whatever you are receiving at r right, whatever you see the r of t is from every location that see this is not one pressure at one location right, the whole surface area is oscillating.

So, it is creating a pressure field transmit. So, when you receive back for one point, we wrote this whatever is reflected at that point is r of x comma y comma z. So, if you have multiple points right and if you pretend superposition holds, it is it most of the time holds. Then, what you are receiving in general can written as r of t right.

So, if I have a transistor crystal, excite it and receive. I am going to get some signal right r of t and that r of t what is it capturing? It is capturing all this from the whole region in front of the transducer, where all this pressure went right; everything on the surface of the crystal is going to be summed.

So, you are going to have a r that is a function of t because the wave is travelling. So, the echo is coming. So, there is a time axis, but x y z is fused. So, what you are getting you do not know where it is coming from. All you know is it is coming after certain time interval. It is coming from the whole region that got insonified in this beampattern ok. So, that is what we get r of t. So, this is fine.

But even this is very good at a you know intuitively address, but then, you have several different terms. So, we need to figure out ok if you say that r of t is this and t as you can notice even though it says t axis, if you get time, what is it saying? It is going deeper in the tissue right.

What the echoes will come earlier instances if it is nearer the transducer, the echoes from the regions that are boundaries that are further down deeper will come late. So, even though, we write r of t we know that what this is saying is the echoes are coming from different locations along z direction ok.

So, now, the question is ok, if we talk about locations at z direction, then can we make some analysis to find out how the equation can be simplified or is it the same equation or can we simplify the analysis of whether if z is close to the transducer; can we what are the effects, when z is far away from or the point right along the z depth?

If it is far away from the transducer, what is the pressure field ok? So, even though, it is written as t you should kind of make that one extra interpretation that r of t that you are receiving right, you know the time axis is going to be along the direction of propagation in some sense ok fine.

So, we can write your r of t in full form because I just substituted this small r right. This is what you are getting. This is the fundamental signal equation for your ultrasound. Clear? So, this is not of course, I have conveniently you know introduced this term which was not there earlier.

What is this? e power minus mu a z is your attenuation. Of course, I have a 2 mu a. Why is this 2 mu a? The signal is going. So, it is attenuating e power minus mu a z, then it is coming back again. So, when it comes back again, it will again get attenuated by e power minus mu; it is the same material, mu a is its attenuation in that region right.

So, this is a pulse echo this is an important piece of information; the signal right, the time and distance. You had to be always careful because the time is going to be two times, it has to go because the signal that you are receiving is an echo. So, the signal lash the pressure wave would have travelled to that location, get reflected the echo comes back.

So, you have to wait for two times the time for the distance. So, distance and time of are related like you have to do half the you have to travel twice the distance right; go once, come back for if you get your echo at time equal to t; that means, you have to have a factor of two. It has travelled twice the distance in that t ok. So, that is a catch, very important, tricky.

I will highlight that in as we as we go how we use this for measurements and stuff ok. But that is a reason for introducing this conveniently. So, there is echo is going, echo is coming; let us also include the attenuation there. The two is there because it is always echo is going to be travelling one time further transmitted is going there, it is getting attenuated, coming back, again it is going to be attenuated. So, it is two times attenuation ok.

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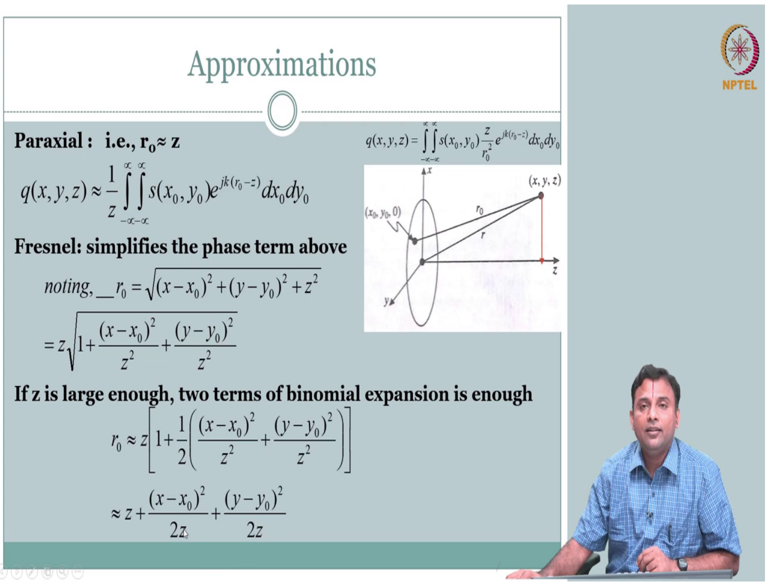

So, what we will do now quickly is further approximate, understand whether we can analyze this further. So, we will take this guy. This is the main guy right. Your q of x y z this is in some sense, this is also the directional function that is referred sometimes ok. So, we will or the field pattern that you really are looking at right.

So, this is determining the field pattern, the shape right this is this integral. So, we will make it little straightforward to analyze this. So, what we are going to do is first is we are always interested in something at the centre of the right transducer because in the end, everything is falling on the surface is going to get captured.

So, if I have to register that, I will have to register that at x naught comma y naught comma 0 for example right. So, so you are going to say that is where the transducer was located, when you got this time trace of the signal which is depth. So, in some sense, we are interested in the pressure that are close to the z axis. I am not really interested in something that is off the axis that much.

So, we can do what is called as paraxial approximation, that is your r naught is absolute approximately equal to z. So, if you look at our q, what that helps is that probably helps only the amplitude part; the closer you are right, z equal to r naught if you approximate that, you can reduce the amplitude part. You do not get anything with the phase path right. So, you can write that.

So, first approximation you can do that. So, you can bring your 1 by z out and you have this ok. So, this looks good. Can you recognize what is your form here? I have a amplitude term. So, there is this phase term. So, if I have to simplify, I have to somehow simplify; if I can simplify the phase term, maybe I can simplify this ok.

So, next we talk about Fresnel approximation. So, first to kick in, we want to simplify this phase term. So, we will use Fresnel approximation. What does that say? We note here r naught right; r naught is your square root of x minus x naught y minus y naught plus z square.

So, we want to simplify this. How do I simplify this? Because this is what we are going to go after r naught minus z; how do we simplify this term. So, we recognize r naught as this and therefore, we can quickly write this as you can take the z out. And you can write it as a fraction.

So, far is just mathematical manipulation, but the assumption comes or the simplification comes is we are interested in knowing whether you are close to the transducer or away from the transducer right. We also showed at the beginning of the module, I showed you the pattern is going to be weird; if you are close to the surface or away from the surface, far field near field.

So, we are going to say look if this is the case, there is this x naught y naught and all are going to determine say you are talking about. So, your z is the depth the distance. So, if you look at this, if you can have reasons to believe that you are looking at z that is far away right.

So, if z is large enough, z is large enough, then in this I can just use I can ignore the higher order terms and approximate using only the first two terms for this square root expression ok. So, if you do that, your r naught becomes the square root of one; this I am simplifying retaining only the first two terms. So, I have 1 plus 1 by 2 of this guy plus this guy. Of course, therefore, you can write z plus x naught minus.

So, where are we going to use this? This r naught, where are we going to use this? This r naught I have to substitute here. When I substitute, what do you see? , there is a z here, there is a minus z here; those two will cancel ok. And then, you will be left with only this x minus x naught by 2 or x minus x naught square by 2 z and y minus y naught square by 2 z, you will be left with only that ok.

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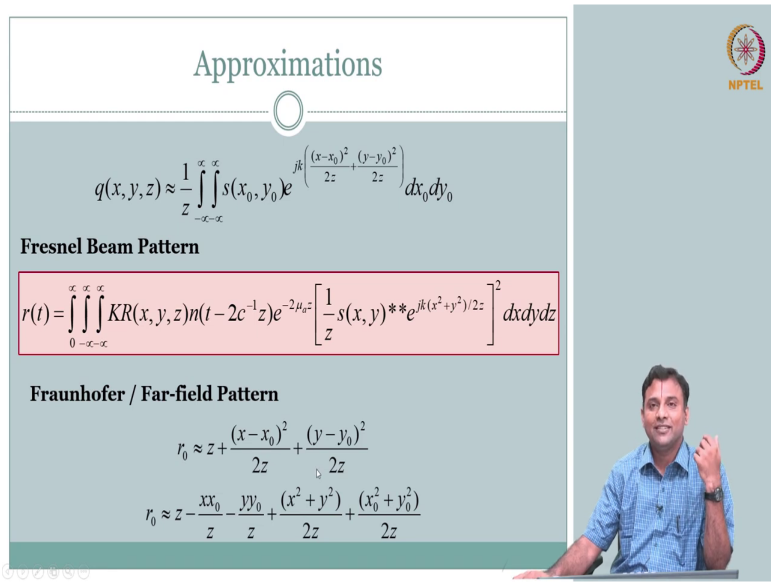

So, x minus x naught the whole square by 2 z; y minus y naught by whole square by 2 z. If you look at this right, I know this is only q, this q has to be substituted in your receive signal r of t, it has to go in. So, first we are trying to work on this field pattern right or so that is determined by this q ok.

In some sense, they also plot this and they call directivity function right; this direction it is going. But how do we simplify this further? Do you see anything here that rings a bell; have you seen this formulation somewhere? At double integral, you have a exponential with some of course this is just dummy variable; x naught y naught. In your formulation usually, you have x and y and shift will be x naught y naught. It is just a different variable; but conceptually, what is this capturing ok.

So, this is your Fresnel beam pattern. We will just a recognize that this is akin to your convolution two-dimensional convolution right. So, what we have done is we are recalling this r of t same as before, only the place where we had q of x comma y comma z square right, the q, I have simplified using Fresnel.

How I simplified? I have simplified by making this approximation in that expansion in the phase term. I have used only the two first two terms in the binomial expansion. If I do that, I get this form. When I get this form, I recognize this is nothing but s x comma y convolution with this two-dimensional convolution.

So, in some sense, instead of writing two integrals and doing 2 d x naught y naught, I have reduced this to a written it recognized as a convolution operator ok. So, this is called as your Fresnel beam pattern. This is holding good in the near field. What you have to recognize from here is, if you are close to the transducer.

So this is your phase of your transducer right, this is the phase of transducer, this is your depth right; if you are close to the transducer right, not only do you get the effect of this guy, but you also get the effect from the neighbors, the corner points as well right. So, as you in the near field essentially when you are close to the transducer, you are going to have contributions from all the surface along with the point which is the closest.

So, as you start to move, move, move, move, move, the influence of all of the others start to diminish ok. So, Fresnel beam pattern is complicated. Fraunhofer or Far-field pattern, what does that mean? Same thing, we will have to make see the phase term, we made one approximation. Can we approximate or that expansion that we wrote; can we write it some other way?

So, we started with r naught z plus this guy, can we you know expand this right? So, this is what we had right; we had this r naught minus z. So, this went out and we were left with this term in Fresnel. So, now, we go back to the same square root whatever we had, we would like to expand this further. When you do that, you have x minus x naught by 2 y minus y naught by 2 and this terms ok. So, this is another level of expansion that we have done here. Why are we doing this?

Of course, if we expand this, you have to substitute back in your r minus r sorry r naught minus z, only this z will go. So, now, instead of two terms, you are going to have four terms; exponential of this, exponential of this, exponential of this, exponential of this ok or here you have some of two terms or exponential of sum of four terms ok. So, that is what will happen.

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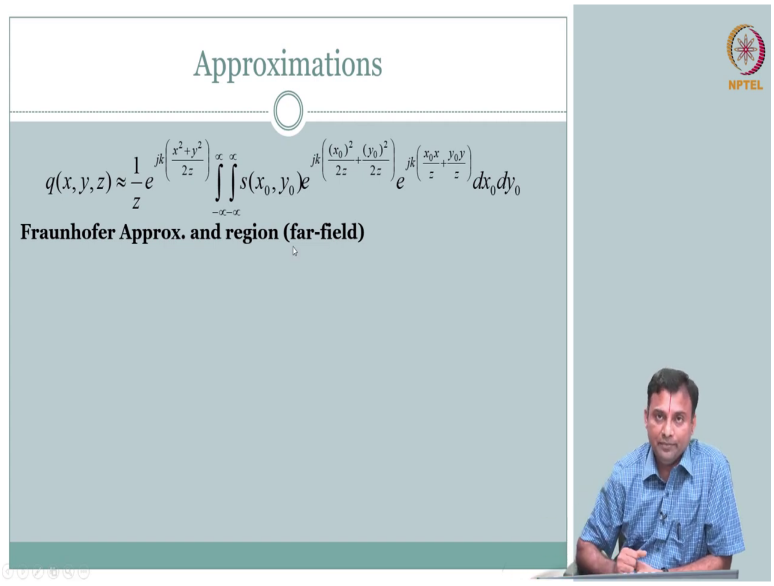

So, we will have exponential of four terms, instead of that we are writing it as two exponentials of two terms; you know convenient split, rest of that does not change ok. So, why do we conveniently write this? Again, we are going to always look at it how do we reduce this double integral or at least how, what is the meaning of this double integral of some function that is there operated on another function right.

You see this k, this k, so there is some wave number which can be related to also frequencies. So, Fraunhofer approximation and in the far-field what we are going to do is recognize that if you are far field, what do you mean by far field?