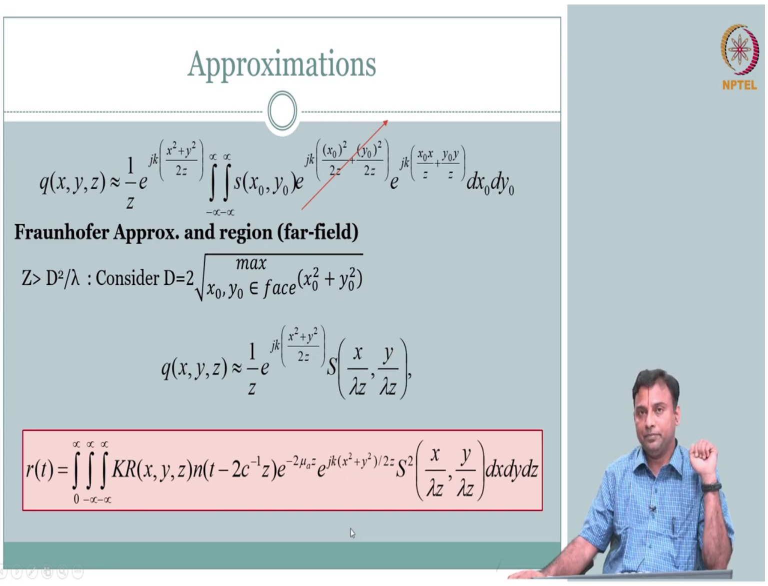
Introduction to Biomedical Imaging Systems
Dr. Arun K. Thittai
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture - 36
Approximations

(Refer Slide Time: 00:15)



How far are you? So, we should have some condition under which we should say you are far all this in relation to where your transducer phases right. So, when you say transducer phase you can clearly intuitively guess the influence of this phase right is going to be the distance. If I have a bigger phase I will probably have its influence farther region, if this is a smaller phase probably I will have the influence over a smaller region.

So, this near or far you can intuitively guess is going to be in relation to the size we are talking about right. So, I mean you know in a very colloquial example if I have a long arm right, I can reach you. So, your near me means if I can hold you right. So, it is in relation to whether you are near or far depends on what is my reach right. So, like that if it is a near field or far field depends on the surface. If this is the larger surface maybe I will have more influence over a deeper region right.

So, in that sense, you have to define your far field, it terms of it will in terms of the size of the aperture ok. So, your Z the distance if it is greater than D square by lambda ok, what is your D? D is just a dimension, so if it is circle for example, then it is a diameter. If it is a rectangle you could take the you know the largest diameter right. So, here phase basically this is to do with your dimension of your aperture. Circle is simple, I can just use the diameter. If you have something else you can still consider that is it is a maximum of the dimension of the phase that you are using the transducer shape, ok.

So, if that is the case why do we why are we interested in this approximation? If Z is far greater than your D square by lambda in such a situation turns out that this guy goes to 1, this exponential of this goes to 1, ok. So, then what happens Oh. You are left with S naught and then this exponential. Not only that look at this exponential little carefully what do you see? k ok, k is what? Oh. k is your wave number is your 2 pi by lambda. Oh. So, e power j 2 pi by lambda.

So, quickly you will recognize you have a form where x naught y naught is there, you have exponential, your running variable is also x naught y naught here. So, you have a x naught y naught here. So, this 2 pi by lambda is j 2 pi right x. So, this x this lambda can go in right, you can 2 pi by lambda j 2 pi x by z lambda y naught y z y by z lambda. Does not this look very familiar to you? We have done this several times. What we have done is oh f of x comma y exponential of j 2 pi of u x plus v y d x d y. What was that?

Oh, your Fourier transform, 2D Fourier transform. So, this is nothing but a 2D Fourier transform of this guy that is what this is, fantastic right. So, you will quickly recognize what you have here is 1 by z this guy times this whole thing is nothing but a Fourier transform right, only thing is your u and v you have to recognize. So, this was x naught and y naught.

So, your x by delta x by lambda z y by lambda z. So, you can you have a special frequency u x v y is what we wrote in our general formulation right.

So, you can recognize what is your u, what is your v 1 by lambda z 1 by lambda z is here right. So, what you have in some sense is very revealing. I mean this what this is say, what is this S, what is this S? Oh. S is your aperture right shape something to do with your shape of the transducer. What this is saying is your directivity what you are seeing in the far field the pressure distribution in the far field is going to be related to the Fourier transform of the shape.

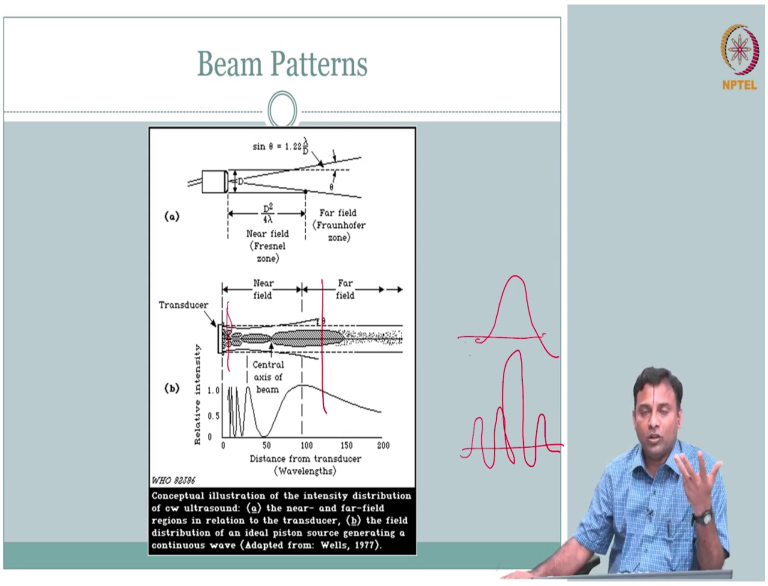
So, if I have a particular shape I know the distribution that is going to happen in the far field, it is going to be following a Fourier transform of this. That is a very powerful you know powerful concept ok. So, please spend some time to appreciate what is happening here. You may I have to read it multiple time prefer to get that joy as to ok I do see what is happening. So, I will spoil your party by showing some examples, but clearly at this point you should be comfortable that we talked about thickness mode that determines your frequency and then the shape and size both have a role to play.

The Fourier transform of the shape is going to give you the far field pattern right, the pressure distribution in the far field the shape of that right that pattern is mathematically described by as the Fourier transform of this shape. And the size of this is also going to determine where this is going to be applicable whether you are in the near field or far field ok.

So, you see the size and shape of the transducer determines how this pressure is going out and how where all you are getting the reflections from and the sum total that you are getting ok. So, just to put that back in our parent equation r of t is all from before I have just written this because we had a q square remember. So, the q is now we recognize that as, so you have your S square ok.

So, this is a very powerful equation. The far field it is little straightforward because you know the distribution is going to be determined by the Fourier transform of the shape of the aperture ok, otherwise all are the terms are from before ok.

(Refer Slide Time: 07:36)



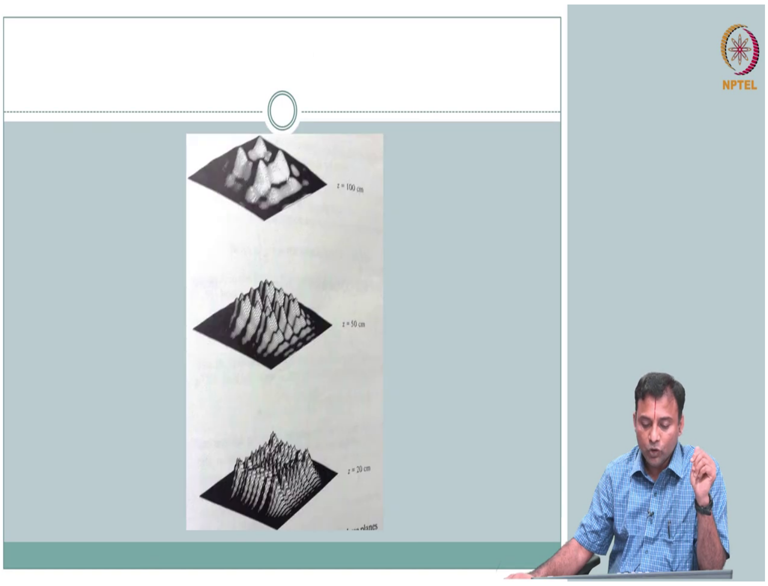
So, just to circle back complete this idea, so, now, you understand what I meant. So, when you when you when you have this excitation from this crystal here say in this example it is a circular transducer. So, you notice the beam pattern. So, if you take a cross section right if you take a cross section, so, if I take a cross section there right if I am to do the intensity there you are going to have. So, I am just doing a rotation, so that is easy for me to draw.

What you are seeing is going to be predominantly here you are going to see intensity map like that. This is in the far field. If we do that in the near field what is going to happen? Sorry, right. If I do that there how is the profile going to be? Oh. it is going to have up, down, up, down, up something like that right, clear. So, if I take the profile along the center this is going to be the fluctuation. So, that is what we had right. The this is captured in a equation here we have a plot of that ok good. So, what we will do now is recognize two other important ideas, one is this divergence right.

So that means, it is diverging is what we. So, angle of divergence is a parameter. And what is the idea in this beam? See what is the disadvantage in divergence, right imaging what are we supposed to do? Oh. In imaging we are supposed to know the lateral also we should have good right. In the axial direction is fine where the high frequency wave is going but, in the lateral is related to your beam width. So, we need to, so divergence right we should be able to know what is this divergence.

So, divergence is not good ok because you are loosening up your lateral direction if you have a point it will spread more in the lateral direction. Point spread is to you are going to relate your both contrast and your resolution per say.

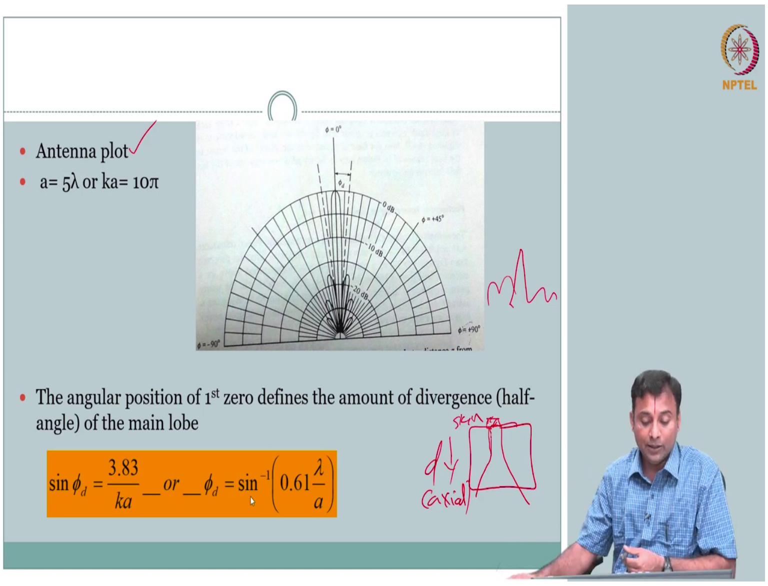
(Refer Slide Time: 09:58)



So, what we will do is we will just quickly show some. So, this is your from a rectangular transducer. What I should before circular this is the rectangular and you see the similar trend in the near field. What is a near field or far field? Remember, go back we just made some formulations right. If its circular disk D square by 4 lambda; so, 2 times that. So, D square by 2 lambda if it is a rectangular transducer shaped transducer D square by lambda right. So, we had all this in the previous module.

So, when you go you see this general nature. There are lot of fluctuation in the near field or close to the transducer as you go farther and farther it becomes much smoother field ok irrespective of the shape of the transducer the general behavior is same.

(Refer Slide Time: 10:51)



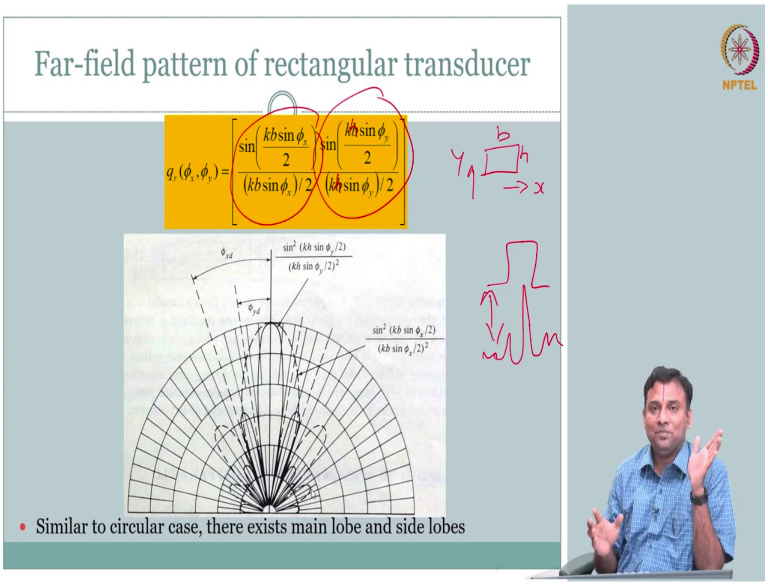
So, this is one another important way of explaining or communicating what we just covered which is what is called as an antenna plot. So, typically because you are talking about divergence you are talking about angle, 0 will be exactly straight of the transducer right. But more importantly it is not just doing 0, it is spreading out in plane and you know elevation the perpendicular both, but in imaging at least we are interested in plane how it is spreading in the other direction.

So, if you see an image right if you see an image if this is your depth d right skin this is depth, this direction is fine that is your axial direction, but what we are interested right is spreading in the other direction. So, we are talking about this direction. So, if I have to just plot right, this is my transducer right. So, typically how do they represent? They represented in terms of antenna plot where you see that there is a angle of divergence. You can calculate that, you can calculate the angle of divergence using right. So, you have your main lobe.

Remember, how I showed there is a fluctuation in the near field right. You had main lobes and side lobes. So, you can see all that here. So, this is your if I am trying to insonify if I only front of the transducer unfortunately because of the size and shape of the transducer you get different pressure experiences in the field. So, there is a pattern to it, so we call its field pattern.

So, your sin phi of d. So, phi d angle of divergence is sin inverse of; so, what you recognize? What is this a? a is your radius, d is diameter, a is radius of the of the transducer. So, the you notice there is a inverse relation. So, if I have a larger a, I will have smaller divergence right. So, that is something that is a important message that you should remember. So, if I have a larger aperture, larger transducer size then I can probably have less divergence I can right that is an important aspect.

(Refer Slide Time: 13:26)



So, that is for circular similarly, for your rectangular you can get your function to be like this. What is this? Do not get intimidated, it is actually very you know very powerful that we already covered an important. What was this q we talked about? Oh. It has to be some Fourier transform of your x sorry of your s. When you look at this what do you see? Oh, sin of some quantity by that quantity. Have you heard sinc sin theta by theta, oh yeah yes sinc. So, this sinc as so, this is rectangular transducer.

If you have a rectangular transducer with different dimensions right, so, your b is your width for example, or square transducer. So, how does what is your; so, this is two dimensions square means. If it is a rect function, what is your Fourier transform? Sinc right. If you have a rectangular in one right in time domain in frequency domain that will be a sinc function. You have done this Fourier transform before. So, now, you have two surfaces right the surfaces two.

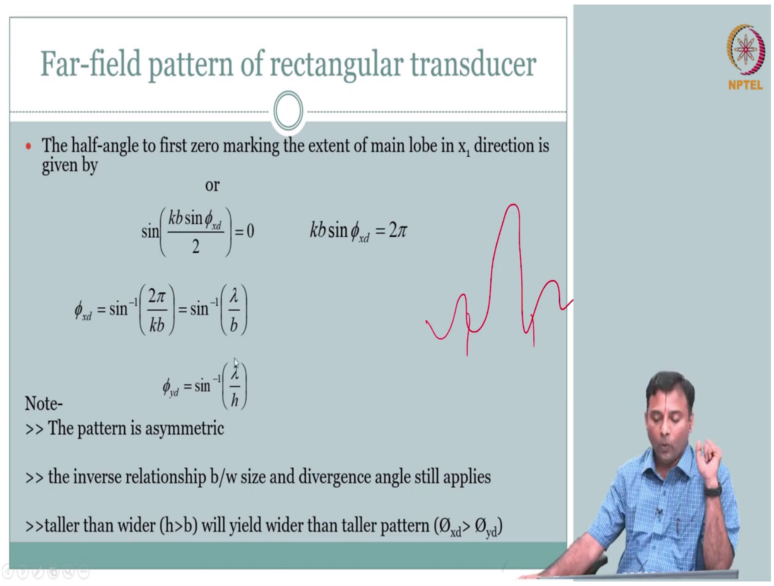
So, you have rectangle in one rectangle in other. So, that is what is your square for example. Then you have to have sinc for one dimension sinc for the other dimension because Fourier transform ok. So, it turns out that that is also, so, here that is why I think I made a mistake. So, one has to be h. So, the x direction is b y direction is h ok. So, this is your x direction, this is your y direction, z is in and out of the right the plane that is coming in. So, clearly if you have a rect function right if you had a rect function you have sinc right, sinc is your transform Fourier transform.

So, here you can see this as sinc in one direction sinc in other direction that is why it is oh I have a rect here sinc square right. So, look at the beam pattern. So, whenever it is right you have two of them because one is for one sinc function the other is for the other sinc function. You notice the divergence is different in the different directions. So, in plane right.

When it comes out its 3D right, if you do its 3D. So, you can have lateral and elevational ok. So, each of that how it is determined by that respective dimensions h or b. So, to summarize this also exhibits a similar case to circular with main lobe and side lobe. So, the problem with all of this I will talk about this further when we go to the imaging aspect explicitly, but the side lobe is undesirable.

I am thinking I am sending pressure in front of the transducer, but when it is leaving the transducer, it is not just hitting the object in front of the transducer it is also hitting right with some pressure levels, it is also hitting at different angles ok. So, when you are receiving it you are going to get a cos from because of these pressure it is hitting in some other object in a different direction. But you have no way to tell that your what you get is a surface integral ok.

(Refer Slide Time: 17:20)



So, you have to recognize here as well we could get the you know the divergence angle we talked about right. So, we can here also we can get, but for two directions. So, you can get for x direction, you can get for y direction. It is a similar idea, it is a sin inverse of lambda by h or sin inverse of lambda by b. So, clearly this pattern is a symmetric in circular case. It is symmetric and therefore, we just got sin inverse of 0.68. So, something that I probably missed.

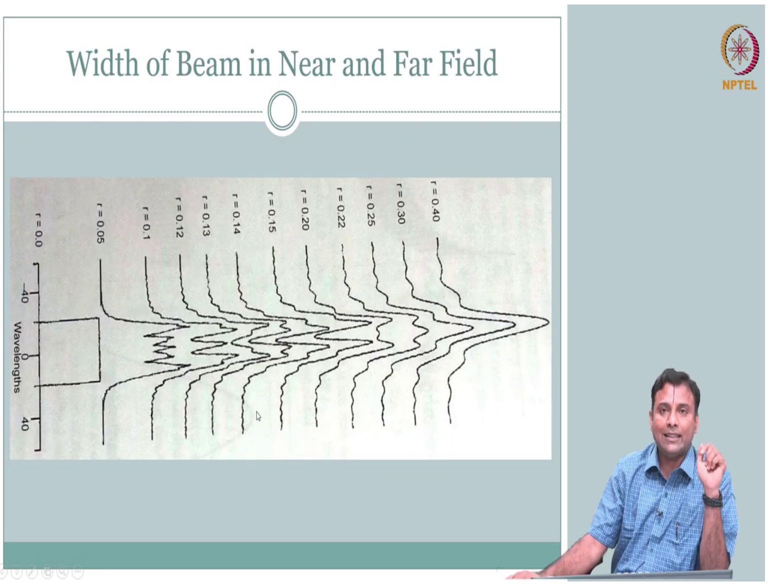
Circular also same right, it is when I said rectangular you got sinc. When I have circular disk the Fourier transform of that is a Bessel function. So, Bessel function also has a standard right form. So, you can calculate what is the 0s. The first 0 crossing in your Bessel function right, first 0 crossing you can define as your main lobe ok. So, that is how it was done there, but it was circularly symmetric. So, I did not mentioned that I forgot. So, here it becomes, so it is symmetric whereas, here it is not symmetric.

The the divergence is different in the different directions because the dimensions are different, one is b, the other is h. So, the inverse relation still holds good that is the larger the dimension less divergence in that direction ok. So, that is important. So, in this case for example, taller than wider transducer shape will yield a wider than taller pattern ok. So, this is very importance take home message is if I want divergence to be less I should have large aperture, but is there downside to it.

Well we will when we cover imaging we will explicitly handle that right we will address that. But even before that in some sense you should intuitively start to think ok what is the good if you have large we can probably divergence is less. But what is the disadvantage of having large transducer? Well, we talked what we are receiving is a surface integral. So that means, you are going to get signal from a larger volume right. What is our objective? Oh, eventually we should be able to precisely say which scatter is located where.

The r distribution should be as fine as possible right, but if you are going to use this you are going to get one net sum right. So, in some sense you can already start to feel the problem ok. I want it to be small, divergence should be less, should I have divergence less I need to go for a larger aperture, but if I go for larger aperture then even if it is not diverging there we the volume that it is going to insonify is large already. So, you will get a signal from a large volume which is integrated and you get one value. So, in some sense there is a trade off ok. We will explicitly cover that as we go forward.

(Refer Slide Time: 20:45)

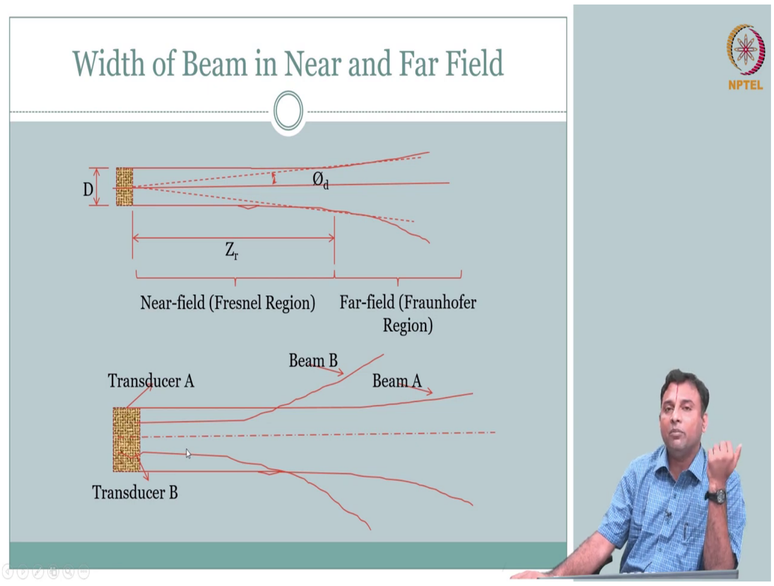


One last aspect again if you have a rectangular. So, you can think about the transducer if you take one cross section. If you going to have a hard right transition very similar to your windowing concept that you know in your probably first level signal processing course right when you do filter design.

So, if you take a rectangular window you will have more ripples right. So, what do they do? Oh, they do some windowing handing, hamming you would have heard those steps right. Similar thing is applicable here as well. So, if I have a transducer which has oh it is only crystal is here and it is cut such that there is nothing outside. So, it is a hard boundary. If you have a hard boundary you are going to have lot more ripples. How do I smooth it? Window it, shape it right, you can do window it shape it.

So, we will we will talk about that two things to notice here is the fluctuation is reduced actually. We saw this earlier also, but this is little less collected. So, you in the near field right this is your wave length this is your transducer this is your how far are you away in z axis with respect to with your dimension. So, if you are close you have a lot of fluctuation as you move away in the far field it becomes a smooth pattern ok.

(Refer Slide Time: 22:18)



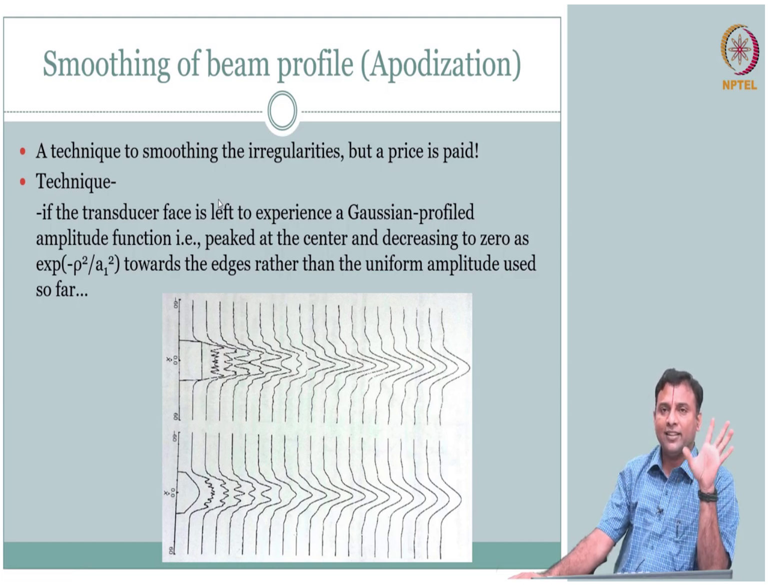
So, we will get back to this first slide where we had ok you have a (Refer Time: 22:23), D is your diameter of your crystals, so shape and size. You have a beam that is retaining its with same as your D until certain point after that it diverges we can calculate phi d. So, based on this we define a near field region or a far field region. So, now if you have another transducer with the larger say because that is what we covered inverse relation.

So, if I have a larger, so, if I have another transducer transducer A and transducer B, if a transducer A is much larger oh the divergence is less right that is what we saw so far. So, smaller transducer diameter then it start to diverge, so divergence is more. So, divergence is less divergence is more is ok, but then you notice if it is divergence is less right here. You notice that this extra space so, so much volume you are going to insonify.

So, your ability to look because at the end whatever you are going to get as r of t you are receiving you are going to say came from location 1, location 2, location 3 along the depth that is what you are going to say. So, it will be good for you to also localize it in this direction, but if you have a less divergence right having a larger dimension transducer then you are compromising on that localization ok.

So, we will we this little bit further, but now what we will think about is ok, given this transducer can I achieve both. Can I have a larger transducer, but I can I also have a smaller beam width? If I can do that then that solve my problem ok.

(Refer Slide Time: 24:26)



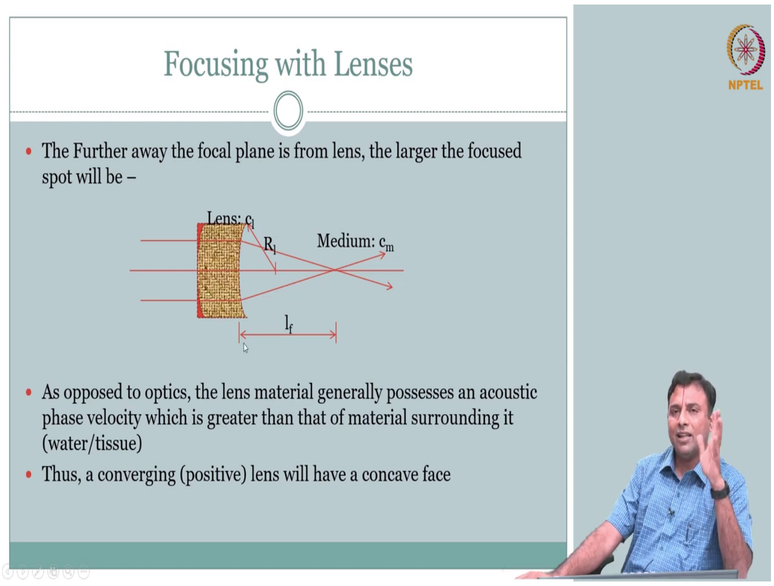
So, we need to talk about focusing. Before we do that let us just complete this beam profile that I was talking about right, your rectangular function hanging hamming window. So, you could do that here. So, what we can do is have the crystal, but make sure that the crystal is not excited the you have some weighting. So, the center point gets the maximum weighting as you go towards the end you have less weighting and this how much weighting you give can be determined based on your window function ok.

So, you could come up with the material. So, basically this is sound. So, I want only the center to contribute sides to less contribute less. So, I can have a sound absorber, more sound absorber at the corners and less sound absorber at the center and I can have a profile of that. It can change from no absorption to high absorption in a say for example, exponential fashion which is kind of your windowing ok.

So, if you do that you can make, so instead of this rectangular I can profile it right, I can come up with a material say sound absorber right. So, is that is a easy one to visualize at least for you. So, you can come up with some material and front face of the transducer, where it will preferentially allow only more from the center then you are going to smooth this out if you smooth this out you notice the profiles are lot smoother here than here.

So, this is a technique which we call as apodization ok which is very similar to your windowing that you would have studied is just the jargon is slightly different because the context is different.

(Refer Slide Time: 26:14)



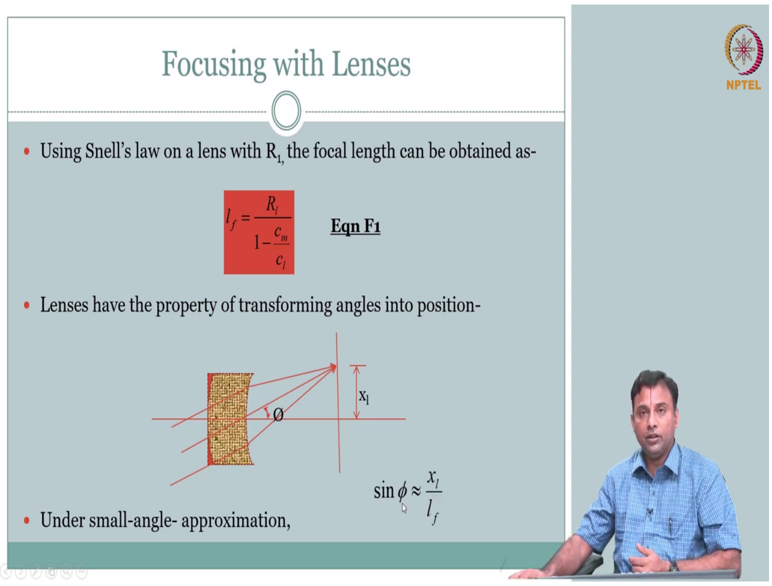
So, now the last point that we need to address is can I then make the beam profile or beam pattern as narrow as possible. So, can I how do I reduce the width of the beam ok? How do I reduce the width of the beam? So, I do not want divergence at the same time I want to reduce the width of the beam. How will I do that? Well, you could use a lens, a focusing lens is very similar to how you do lens in optics right.

So, we could have a lens in this class as well, but this lens we call as acoustic lens instead of optical lens you have a acoustic lens. So, what do we mean by that? Oh. This lens will have its own c you know material property. So, c l right and then you have this is your medium and it has a radius of curvature right radius of your lens R l. So, this is your transducer the pink or orange that I have.

In at the front of the transducer the flat describe it was a flat disk in front of that I could have a acoustic lens with certain curvature R l and that can be designed to where you want your. So, this is your length of focus where you want your beam to be as narrow as possible right that is your focal spot. So, there is a length of focus is the location z at which this lens is going to help focus the beam. So, all the acoustic pressure coming from the surface of the transducer all of them are going to be focus to a smaller region here.

So, it will become the narrowest right, the divergence we talked about. So, instead of divergence before it starts to diverge I want to make it to a as small region as possible in the imaging plane. So, the only thing is here instead of optics I mean as opposed to optics a shape if you look at it you are talking about a see there also it is the same thing right. We have phase velocity, but then here we are talking about acoustic phase velocity and therefore, this ratio and the shape converging means we will have a concave phase here a converging positive lens will have a concave shape here.

(Refer Slide Time: 28:51)



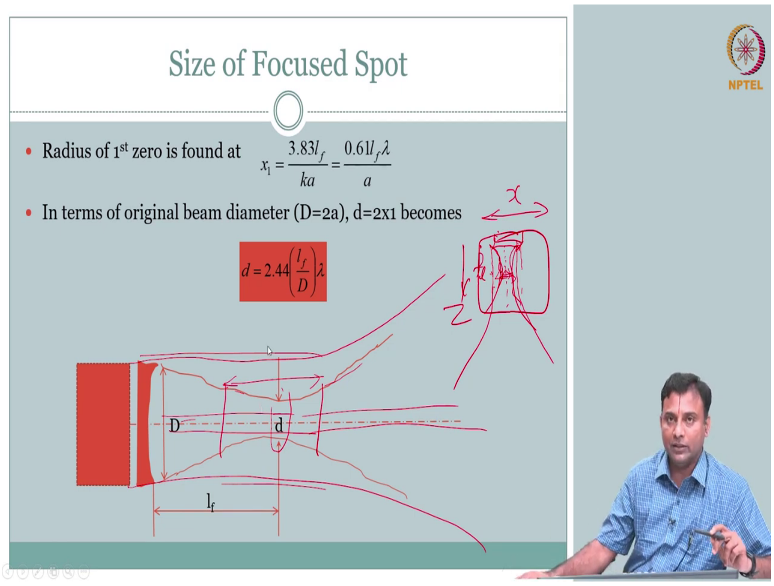
But otherwise the concepts are very similar. So, what this does is you can use a same Snell's law. If you do that you can get your l of f length of focus to be given the geometry right, you have your R l. So, you can get R l by 1 minus c m by c l, so this is fine. What are we interested? What is the at l f, what are we what is this? Oh, this is where the beam profile is narrow. What is your bream profile or beam pattern? Oh, beam pattern was my Fourier transform of the shape of the transducer right.

So, if you recall we had a Bessel function if it is circular we had a sinc function if it is rectangular right that is the profile at that z far field. So, if I want to do this what is going to happen? I could just substitute right. So, let us call this l f as some equation F 1. So, what does this l do sorry what does is lens do? Oh, it transforms position, more importantly for example, if it comes and hits at certain angle this angle is transformed to position that is what this lens is trying to do ok.

So, if we do that under small angle approximation right small angle approximation given this. So, this is your f l right. Ideas is going to spread reduce the spread. So, if it is spread out what is a lens going to do? It is going to bring it in right. So that means, it is going to kind of do a transformation of angles into positions. So, if you look at this geometry you can get your sin phi right, this is your l f, this is your x l. So, you can get sin phi is x l by l f.

So, if you do a small angle approximation we can essentially write right. We had our sin phi for a angle of divergence before same thing, but now we will substitute for sin phi to be x l by l f ok.

(Refer Slide Time: 31:09)



So, your x l by l f 3.83 by k a, this is what we had for the circular transducer ok. So, there we just left it like that sin phi d is equal to 3.83 by k a. So, phi d is equal to sin inverse of 3.83 by k a that is where we left for the circular. Now, we are having a lens and the lens helps us with the transformation. So, the sin phi d or sin phi can be approximated to x l by l f at the focal.

So, you can substitute, you can get x l is equal to 0.61 lambda by a. So, why is this important? So, we can now this is your x 1 is your azimuthal right. You have in the imaging plane in the imaging plane right this is my azimuthal or x, this is my z right. So, x 1 is the width that is what we are after. So, in terms of original diameter. So, a is what? Radius I said. So, diameters 2 a. So, your d width. So, your width right it is trying to focus. So, this was your transducer there is a lens this lens is trying to.

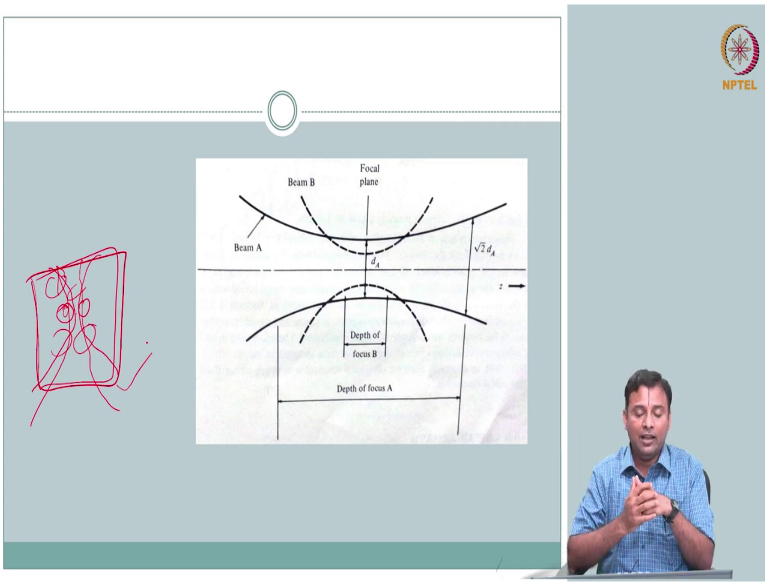
So, it is diverging right. It would have diverge before it will be like this parallel right and then it will diverge. This is what we had before. We wanted to have this divergence to be less by having a transducer of larger dimension, but if you do that you will not have you will have more area. So, what we did is we have a lens now which is reducing this, try to reduce it as small as possible the width. It will be as small at focal length right f l. At f l you will have the shortest width which will be d. So, because in the geometry if you see x 1 was only one direction, so, but here 2 times x 1 is your width clear.

So, if that is the case I can get my beam diameter. So, this will be by beam width. Some for circular transducer you can get beam width to be this parameter ok. So, the idea is if I can have a lens, I could reduce the beam width. So, I could have a larger transducer I can use a lens I can reduce my beam width. So, you may think ok maybe we have achieved both, so, we are good unfortunately life is not that simple. There is going to be a tradeoff we will just appreciate that trade off in the next slide.

So, what to complete what we have done? This is a transducer you have a lens acoustic lens in front of it. So, left to by itself without lens this would have been you know the beam would have travel like this and then gone like that. Now, because of lens I have tried my best to reduce this. Ideally you want only you know one line, this has to be as narrow as possible right is this without in divergence if you can go like this that is the best, but that is not practical.

So, we have tried to minimize this as much as possible. Of course, it is minimum and it starts to diverge again inherent. So, maybe there is a region around the focus where it is still less than what it would be without the lens. So, having a lens still helps this is the best case, but neighboring also there is some help. So, until right, so, this is called as depth of focus like how much can you retain the focus, the depth over which you can retain the focus that is your depth of focus.

(Refer Slide Time: 35:25)

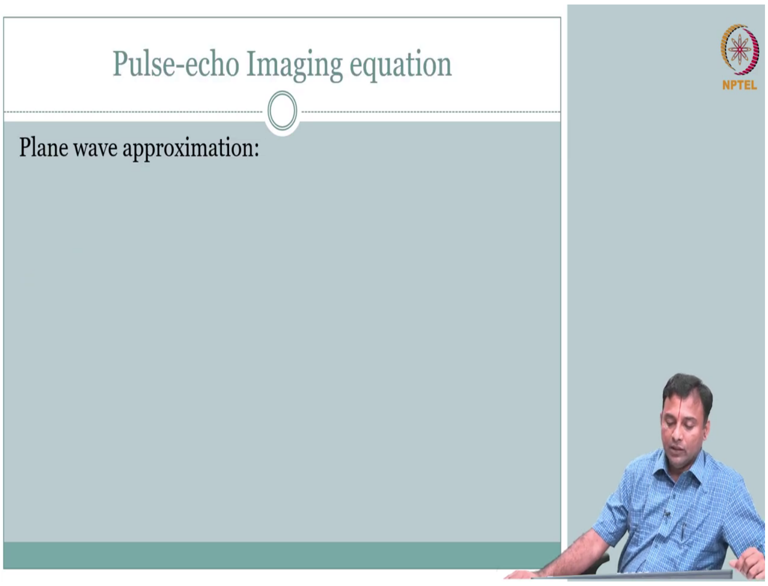


So, if you notice if I start be aggressive. Oh. I have a lens right, so I can make the I can change the radius of curvature right to make sure that I can I can have as small as possible. So, I can do all that, but then your depth of focus, so, that is the trade off. So, I could shape this up I could make it as narrow as possible. So, I could start with the big large dimension transducer and have a lens right, so that I can be very stingy.

I can make it as small as possible, but the downside is the more I push with the large transducer and acoustic lens to minimize get the tightest focus as you would say. The problem is it will start to the depth over which you can maintain that will reduce. So, now you see the problem there is a trade off. So, if I want to do imaging right, I want to do imaging it is a trade off right whether I want like this pattern right or can I live with little wider, but it will probably stay focus for a.

So, depends on its a tradeoff, it depends on what you are imaging. So, for example, I am interested only in a object two objects that are there then maybe I will use the tightest. But, if I have several objects at different occasions I would like to make sure that I see all of them reasonably well. So, it is a choice that you have to make ok, so it is set a trade off ok.

(Refer Slide Time: 37:17)



So, what we will do here is this kind of completes what I wanted to complete until instrumentation and the imaging physics part of it. So, you notice now we have not really covered the imaging aspect. What we have covered is I you will need a transducer and if you excite a transducer this is the field pattern that you will get. So, now, how will I use this transducer right which has some field pattern, how will I use that to image a region, how will I image this region that is what we are going.

So, we know what is the signal that we are getting right. We got the equations for the signal. Now, what we need to do is we will have to organize or material in the context of what do I want to image and how do I want to image ok. In that sense we will jump into the image formation image recon that will be our next target. Once we do that image quality in this module we have actually kind of combined it because lateral resolution, axial resolution we already are combining.

So, in some sense the image quality and then we talked about side lobe main lobe. So, we will not explicitly have a image quality of ultrasound separately. The next module right the next topic of how to use this understanding to do image formation in ultra sound is going to be kind of a very interesting because all these physics is fine how do you see and how do you interpret the image that you are getting your tune to see some ultrasound sonograms right.

What is that? I mean how do we connect whatever we have studied so far to what we think when we see ultrasound image looks like what does it mean there that connection is going to happen in the next you know couple of hours max right that is that is when we will anticipate to complete this topic. So, far we have covered as per plan the instrumentation as well ok. So, we will jump in to image formation next.

Thank you.