Introduction to Biomedical Imaging Systems
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Lecture - 04
Signals and Systems Overview

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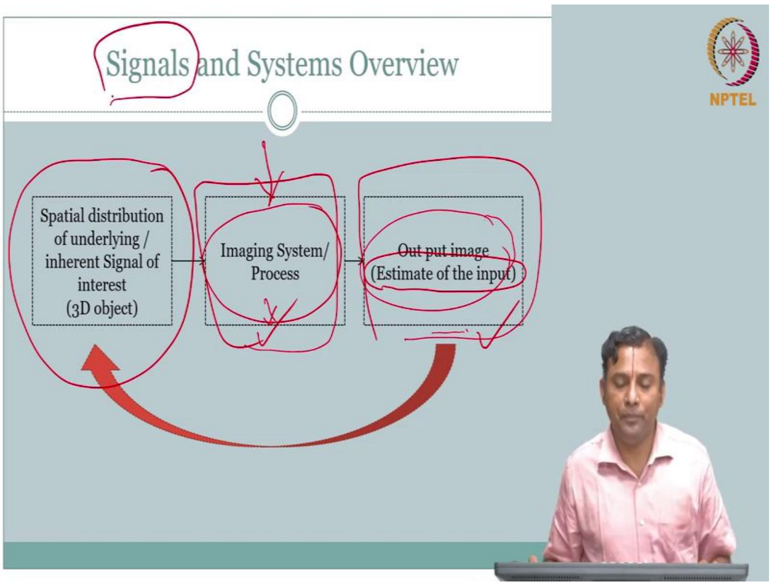

So, now, that we have done reasonably elaborate introduction right spanning over few lectures, it is time that we jump into the topics, but before as I warned earlier, we will not right away jump into a specific modality, but before we do that, we need to orient our self actually do some quick review of some concepts and recall that, we said each of the modality that we will cover can be actually treated as a separate module.

So, after the introduction right, I said you could actually go, take a particular modality and then, complete that as well right, it will be self contained. But before we do that, this part of going over the tools, the concept of how we are going to organize each of the different medical imaging system in the same format right we talked about physics, instrumentation and then recon.

So, here, we are going to we quickly review how we see the imaging system from a signals and systems point of view which kind of forces us to kind of brush up I am using the term brush up; because signals and systems you would have encountered by this time if you are in any typical engineering program right, you would have encountered signals and systems you know third semester, fifth semester, second semester depending on which you know which institute you are attending.

So, the point is you probably know, but then if you already know if it is such a you know basic material why even do a review right, you are supposed to reviewed by yourself. But that is not the case because although you know we are going to talk about images so, there is an additional dimension that is coming into picture even though mathematically it may seem straightforward.

I think if it could do you good to get a feel for it again and I will try to prod and make you know stretch your understanding of your 1D to 2D. So, this is in that context going to be a review that means, I am not going to jump in an do all the proofs and state the theorems like we do in the signals and systems when we introduce the topic so, that part I leave it to you that you should be able to go over ok.

So, how do we how do we start right? So, what is that overview right what is it that I am telling about having a template right having a structure to our discussion here, we will central to our discussion is going to be medical imaging systems etcetera; it is a imaging system that is central and let us get the easy ones out. What do you expect the imaging system to do? It is going to give you an output right which typically we know as the output image is what we call.

This much is straightforward at least without much the moment you thought about the course title right medical imaging system, biomedical imaging systems, this is something that you would have made up your mind, you know I have seen these scanners so, these are the medical imaging system, I know the output images that much is fine, but our interest is not merely taking the output image and doing some processing or saying ok.

If this is the scanner, this is the output image you are going to get that is not our objective, we want to understand this system right, the context of ok, I get this image, but what does this image mean right in the context of what is going into the system.

So, you have a input side so, you have an input so, we are going to treat the imaging system as something that takes input of some parameters remember that is the point that we were trying to articulate in the introduction as well what is it that you want to see inside the body right some parameter.

So, that has meaning,that has a physical sense that is related to underlying biological process right. So, we have spatial distribution of underlying or inherent signal so, that is the signal that we are talking so, it is not X-ray or it is not you know ultrasound pulse what is the input? Think about I said 3D object let us be little more courteous so, it is not 3D object, we are talking about human beings.

So, what do we mean by spatial distribution of underlying or inherent signal? That means, right I am sitting here, I am human being so, if you take your camera as a imaging system, the output of which is what probably you are seeing on the screen right. So, the idea is I am a source, I am a 3D distribution of how light is the ability to reflect certain color of visible spectrum right.

So, may be here the color of the shirt that is sending out some color which comes out as senders of wavelength that is corresponding to pink right or my hair right that is you have black and no brown skin so, you have the wavelengths that come out that capture. But notice all of this its surface so, you are perceiving it, your vision is a imaging system right for interpretation purpose.

So, you are seeing me on the screen and that goes into your eyes and you perceive me right, but we are interested in imaging system right not the human imaging system, not the eye vision and stuff, we are interested in the imaging system how close it captures so, this is output is nothing but a estimate of the input.

So, if you are seeing me on screen, that is an estimate of how I would look in live so, this camera system is capturing me so, I am the input, output is what you see so, the output should be as close as possible to input right. So, in the context of medical imaging system right, what is going to be your input?

It depends on we have multi parameters right so, I am not what you perceive right now who I am on the screen so, if it is a ultrasound imaging system because you are perceiving me through your vision system which can take inputs in the visible range whereas, the if it is you know CT, if it is a X-ray based imaging system, it is going to look at me as what is it going to look at me? I am just a 3D distribution of the materials attenuation coefficient, how much can it attenuate the X-ray energy that is coming in that ability, that is the property that is what I am a distribution of that.

So, if X-ray system right X-ray imaging system, imaging system that uses X-ray to probe the tissue that is going to look at me as a 3D distribution of a material whose attenuation coefficient is going to determine how the imaging system is going to perceive me right. If it is going to be ultrasound imaging system, it is going to look at me as a distribution of acoustic interferences right, acoustic scatterers like we talked about mountain right giving echo's.

So, that is going to just see me as a water body with several interfaces that can reflect the sound back. If it is going to be a MRI system, it is going to look at me as of the other things, it is going to just look at me as a water body, I am I have hydrogen H_2O so, it is going to just look at how, what is the density of hydrogen right that is distributed across my body.

So, it is not going to see me as a skin or the color that you are seeing me right that is optical property whereas, it is going to see me as a the MRI is going to see me as a 3D of hydrogen atoms right proton so, what does the density is going to determine? How the imaging system MRI is going to capture me?

Of course, the output there should be or better be related right, closely resemble the underlying thing because you are going to make a diagnosis or whatever based on the output temperature so, you are going to say, there is lot of density here that is abnormal so, it better correlate with actual 3D distribution at that location there if there is some abnormality, the output estimate should correspond to the input right.

So, it is very intricate so, you cannot comment right, if you I should not say you cannot comment. If you have to make a very good use of the output image or you want doctors to be really making very reliable diagnosis or whatever based on the output image, this output image should be as close as possible because this is an estimate possible to the underlying ground truth. So, object of our imaging system is to faithfully do whatever faithfully get an estimate that is corresponding to the underlying ground to.

So, what we will do now? This is the big picture. So, we need to quickly run through what is signal, what is system, are there some signals interesting signals of interest that irrespective of the modality we need to you know brush up like mathematical concepts. So, we will start doing that so, first we will start with the signals review and then, we will go to systems ok.

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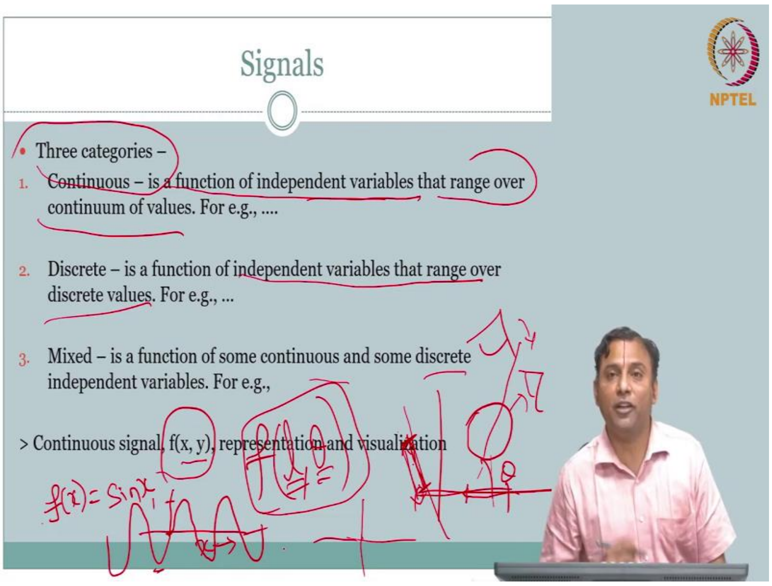

So, in that case, let us move on signals. So, what comes to your mind when you say signal? So, this is something that a given that I told this is going to be a kind of review, I would like for you to think loud when you watch the lecture just do not wait for listening to the lecture right, I want you to spend the time think about what you know already.

So, when we say signals, what comes to your mind right? What is it that what rings a bell what is a signal? It is nothing but a mathematical function of one or more independent variables and somehow that signal is going to can be used as a you know it is a mathematical form right so, it is kind of a model that better have some its better represent or model or correspond or capture something physical right that is when it becomes interesting.

So, signal is a mathematical function of one or more independent variables having said that if that is a broad concept that comes to mind so, what are the different categories that we can broadly classify these three signals right. So, again you probably know it so, I just encourage you to think before I say.

So, three categories what could it be think we have seen maybe something is continuous what is continuous? Signal is continuous. What do what does he mean by continuous? Why is he asking this question, what is continuous ok? Ok, I think you got the point idea is if you have continuous that means it is a function, we defined already the signal right it is nothing but a function of a independent variables at range over a continuum of values that is the key.

So, you have the independent variable right it is a f(t) or see typically I am using one-dimensional if you are done most of the physical variable that you would have used this time right so, you would have seen about s(t) or x(t) or that is something that the t is what usually you start when we do 1D especially in a course like signals and systems so, I think you are familiar with that. So, the t's are continuum right that is what it means by continuous function.

However, in our case, what are we interested? In 1D, I mean even though mathematically it has it can be any one independent variable, you are probably more comfortable with time as your variable right so, now, we are interested in 2D signals or 3D signals right so, let us take 2D signals what do we mean by that? That means you have a function that is having two independent variables ok and those are continuous or they range over a continuum.

So, in our case, what is the thing that comes to your mind? So, quickly recall some of the introduction we did on different modalities right; what you will notice is say for example, if you take X-ray right just a radiography so, what you, what are the axis the one that easily comes to your mind? I am a distribution of mu where attenuation coefficient to X-ray and I have a length, height, width so, my variables are in length dimension, and it is continuous right.

So, I could say that, the chest X-ray that we did and there you see that we have the f(x,y) right we typically right function in terms of its variable so, in one-dimension, we typically read x (t) or y (t) saying t is your time, but here, what we covered already x and y axis which are spatial right, which are in length scales so, f(x,y) this x and y right or the length scale so, that is a continuous variable.

So, you will start to see that inherently most things right which is having physical, which is capturing the physical world is going to be continuous so, like how time we use for one-dimension so, space you know is also going to be continuous so, that much is straightforward.

So, next is discrete. So, then you will quickly wonder, now that you told what is continuous, we know what is discrete right we have it ranges over a discrete value so, immediately what we think of is from probably are one-dimensional back ground, I know he is talking about instead of s(t)or x(t), we know it is s of you know discretized n delta t so, at different locations it is there so, he is talking about analogue to digital conversion after that we get discretized so, the t is now discrete a delta t time right.

Fine, that is artificially done because in live, you have you know physically you have continuous variable if you say t is continuum, but you are making the measurements at discrete interval so, that is fine, but here, so, the concept is similar right.

So, instead of doing it to one axis, if you do it in two axis, then that is discrete that much is find, but what we want to relate to this within the context of imaging system can you think of something from what we have covered so far something where you expect the underlying signal also to be at discrete or at you know discretized or take a range in of discrete values.

Probably, we mention this, know the concept of radio activities I mean right we said in nuclear medicine so, I said you take some radio tracer and it is decaying right so, the radio tracer is when it is decaying, you count how many photons, gamma photons come out and then you say that is the radioactivity and that is dependent on the wherever the physiology right, wherever activity takes place, you can have the radio tracer go there so, the concentration will tell you that is high activity or a low activity I think we I did really mention it.

So, in that context if you see, if you recall, we maybe at high school physics or something, you would have when you have radioactivity you are talking about some concept called the half-life right so, you see half-life is this every half-life is once the radioactivity takes place.

So, that is something that is inherently discrete in nature ok. So, with respect to 2D yes, your concept of extending 1D to 2D of discretizing the axis is fine I mean that is fine, but there are signals of interest which are inherently discrete in nature ok. So, third category could be well, you have continuous, you have discrete, what could be the 3rd one? A combination of both right.

So, from a explanation point of view, it is just merely completing right I have continuous, I can I have discrete so, I can have a combination of these two right, a functions so, you have some variables that are continuous, some that are discrete. Ok, is it just completing for the sake of completeness or can you think about a scenario or the signals that you are going to deal with the imaging systems where you would probably encounter this?

So, again, I am not asking you to jump into the detail because that is what we will do later, but just from introduction because you have seen different images, I have introduced certain terminologies right just based on that you should try to really challenge yourself to see can I find some examples from what he has thought earlier, what he has discussed earlier, what he has shown earlier.

So, yes, for mixed means definition wise its fine some is continuous, some is discrete, but a vivid example that you will end up dealing right is going to be for example, X-ray CT right we use that. In fact, I remember where we covered projection, we covered tomography right go look back those areas, you will realize that what is the idea in tomography, I take projections from different views right.

Remember I was using the cell phone saying one is width, one is height depends on the orientation and in tomography, you get collection of different projections from different views and then, you re-compute and get the 3D slice right that is what we talked about.

So, now, did you get the clue? One is projection is going to be length scale. Remember we said ok, I drew a circle and I said if you project, you are going to get a length, length scale right so, you are going to do this I think I drew some ice like this.

So, what happens? If a this is one view right, this is a projection of the circle here, what happens if I view from here right? Whatever angle that is so there also I am going to get a line right. A circular disc if you see, you project you are going to get a line, you are not going to see the change of line from front to back ok.

So, now, you get the idea, the length scale is going to be it is a continuous, you may choose to pick values wherever, but this variable what are we are going to measure if I am this is the signal, if I am going to measure the signal and mathematically write it, I am going to call this say some f(l) right, I have to have some coordinates defined so that if this is 0 degree, this is some other degree so, I can have some theta so, I can have a reference coordinate.

So, I could still do a regular coordinate system and say, this is 0 degree when I view from here, when I view from there, maybe I am moving 30 degree clockwise right so, I could have some theta. So, I can represent the collection of these projections right that is my signal as a combination of a continuous variable and discrete variable.

Because I am picking right, I am picking locations, I am getting views, I can have find stepping or not but the point is you are collecting from different views, finite number of different views inherently. So, your signal that you obtain is going to be a combination of continuous and discrete of course, then you use this signal, process this signal, create a image ok. So, that is something of interest.

So, now, of course, I just was excited, and I wrote on top of this never mind you should be able to see what we want to do is ok, having talked about signals and having talked about f(x,y) right we are just going to call it as f(x,y), a function that has two independent variables, x and y represents the two spatial directions.

So, now, the question is if I have this, how do I represent right? How do I visualize this ok? So, simple things again 1D you probably know right I mean you should know. For example, let us take any simple function one-dimension and see if you plot it right, how do you represent it or visualize it?

So, just for sake of continuity right before we get into the 2D f(x,y), if I tell that you have f(x) = sin(x) right this is a function, this is a signal so, how do you represent it? How do you visualize it? You will be able to quickly do, this is no brainer right, I will have the axis, I will plot x, I will have no magnitude there so, I will do small arc right you are going to so, sign 0 is 0 so, you are going to have maximum of 1 and then, it is going to go to minus 1 so, you can you will be able to visualize right.

But then, notice I mean this is kind of a I know it is a over b, but then this is one-dimensional, but in order to visualize this, what did I do? I have used the plane of the screen right, but plane of the screen is two-dimensions why? Because when we defined one-dimension, we said it is one independent variable and that forms one axis, that forms one axis but then, the value forms the other axis.

So, when you have one independent variable, I am using another dimension so, for visualization or representation. So, this becomes I need a plane 2D to represent or visualize so, that means, going forward if I have f(x,y), what would you expect? How do you represent? How do you visualize? What do you think ok? If we paid attention to this part, then quickly we will recall, that means, f(x,y) better have x and y as two axis right and then, you have to have a third dimension for the values clear.

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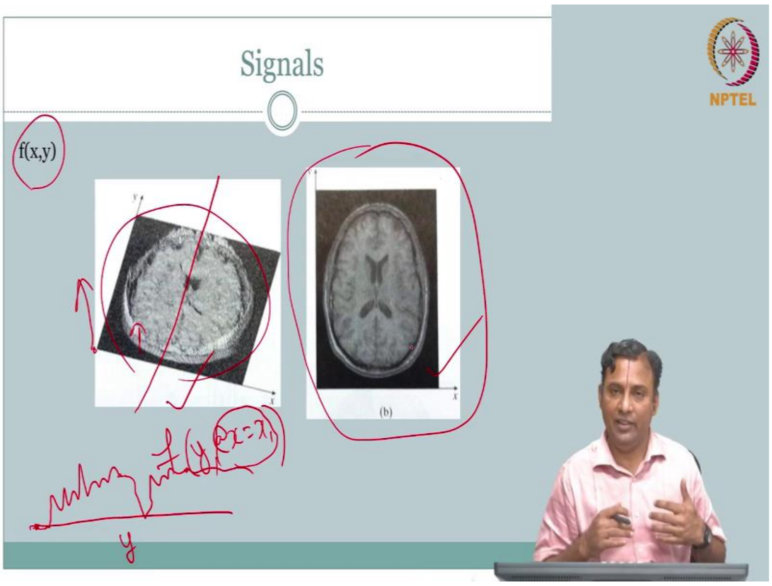

So, now f(x,y) if I want to visualize or represent, straight forward extinction of or understanding of 1D representation leads us to something like this where you have x axis, y axis clear x axis, y axis and a third axis where you can see the valleys and peaks here. So, if you were to take some cut here for example, now this cut is going to be a 1D signal right, it is going to be f (y, x) at x=x1 so, this is not varying. So, this is going to be a 1D variable.

So, if I take that then, I know how to plot it right this will be my y; this will be a y so, you are going to have value so, this is 0 for example, just for right this is if that is 0 right, this is 0 so, if I start its going to have some black is 0 for example, then I am going to go up, then the lot of fluctuation, then there is when I get to there I see black goes to 0 and there is lot of fluctuations so, this is a signal.

So, its 1D signal, I am able to plot it this is fine we know. So, like this, you have done right what you visualize f(x,y) you get this. So, this is actually very important to get this insight because if we are going to get the rod you get the data, then you have to do the image reconstruction what if you want to do image processing right.

Basically, if you go into your workspace right if you are reading the variable f (x,y) this is what you will see this is how it is so, you can get a feel for how the signal is varying. However, if you show this for the end user say you show this to a radiologist, he will just say what is this you know this is not a typical image I see.

So, he is going to use the term image so, you have been very correct in your representation, you have done f(x,y) you have actually plotted it, but in imaging systems, output we are expecting to be a image so, how do we do that? Because looking at this you really you cannot I mean you when I reveal the image, you will see y this what you see here as a signal looks weird to the doctor ok.

So, what do so, then how do you represent? How do you visualize? We do not worry about plotting, we use what is called as pixel element that is your pixel right. So, we convert this into an image what we do is the third dimension we color code let the maximum value be white, minimum value be black right, you can have different different schemes of mapping this.

But so, you essentially encode the third dimension encode as it color code if you do that you still use x and y, the third dimension that was coming out right when you plotted instead of that we converted that to a color representation. So, if you do that, this is what you see, the same f(x,y) the same signal, it is just that you are visualizing this by color coding the third axis.

So, now you get the point. So, if it is one-dimensional signal, you needed two-dimensions to show the plot how the signal is varying. So, if it is going to be two-dimensions, then you need three-dimension. So, you can imagine if you have a three-dimensional variable, three-dimensional signal, how would you know visualize that ok? That is a thought exercise I mean you know you can do that.

So, now you get that image is not a big deal, x,y and the third color that much is followable right, but this is something that you will appreciate, we have seen these are you know brain scan, if you and I can appreciate that, clearly the clinical practitioner is going to be very comfortable only with this, he is not going to look at this and say, your mathematically correct, it is same f(x,y) you know, but he is not going to make a diagnosis based on that clear.

So, this is fairly straightforward, but I wanted to spend this time because based on my experience, I noticed that the small stretch of imagination if you do not appreciate, then all the reconstruction and all the things that when use whenever you use f(x,y), there is no feel, there is no intuition, there is no appreciation for how this is happening so, it becomes just a mathematical exercise, but if you to spend this extra 5-10 minutes understanding this which is probably not complicated, but I found this to be helpful from the student feedback ok.

So, so much for the signal, then what do we have? So, we call what is called as picture elements. So, if this is a picture, you have picture element that is your pixels ok so, after that I won't say anything, you all know what is pixel and you know how many pixels is good, how many pixels is not good for your application you know you know that. So, it has relationship to the image quality so, that we will cover little later ok fine.

So, f(x,y) so, for 1D we know 2D, just for extension we need to work also for 3D ok and the reason for that is some of the modalities that we will cover inherently you will have you know I am going to cover the whole volume so, can I just do slice so, even this slice if you look at it, this is saying that this is infinitously thin right, everything is on plane of paper, you do not have the z direction x and y, z is there is no delta z, there is no width that is what it is mapped to.

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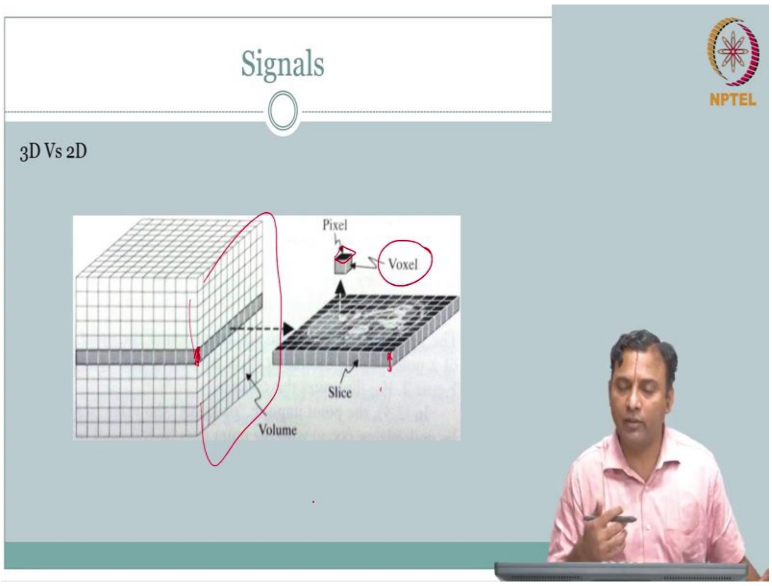

But in reality, we are 3D so, that is going to be width. So, what we will encounter is not picture element, we will encounter volume element. So, when I say volume element, you will probably not recognize, but if I say volume element is abbreviated as voxel, then you might probably start to relate right seen voxel when they MRI for example, right where you start to talk about voxels because it is all volume elements that you are going to play with.

So, that means, this is a 3D object, I have a 2D slice which is going to be my image so, this is your pixel. So, when you have a height right or slice thickness to it, then it becomes a volume, a surface area times height or depth is going to be a volume ok. So, we are going to call this now as voxel and several of these slices if you stack, then you can cover the 3D.

So, we will use volume elements when we are doing 3D and when you are doing imaging, typically even though it is the same value within this thickness, you are going to project this on plane of paper that means, you are saying you are collapsing that depth information, you are projecting on plane of paper so, it becomes a 2D so, we call it image and it is called picture element pixel ok.

So, let us quickly move on. So, we now got hang of what signal means and how we represent or visualize two-dimensional signals and of course, three-dimensional signals as well right here. So, now, it is in order for us to proceed quickly to revise, refresh our self on some typical signals of interest that you will encounter several times ok.

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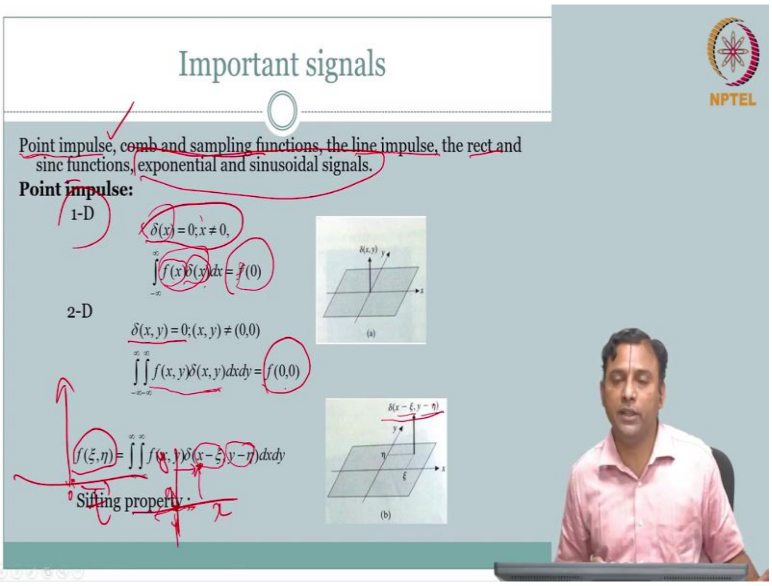

So, what are the specific signals that we are going to encounter here? We are going to talk about point impulse, comb and sampling function, line impulse and few other functions like rect and sinc of course, relationship between exponential and sinusoidal signals. I mean this is just scratching the surface, the ideas these are all unique this by covering these, we should be able to address 99 percent of the introductory material that we have.

However, if you are going to work on any particular signal processing aspect more perhaps you are advised to explore this further right, but that is not intended for this course, for this audience. So, similarly we will run through you have 1-D right one-dimensional signal, I already plotted a sinusoidal so, you know what it looks.

So, if you have point impulse right, what is 1-D point impulse? Does anything have you seen point impulse in when you covered 1-D? No, what do we mean by that? I mean you know we probably have heard impulse right so, you at the origin, you would have seen something like this a delta function, direct delta, impulse function you would have heard that what is point impulse?

Same concept instead of 1-D here right so, for example, this was 1-D at t equal to 0, you have everything packed so, that is your impulse, direct delta whatever you want right. So, point is going to be when I have two-dimensions at the origin right at the origin that is a point.

So, point impulse, impulse is same right, it has all the energy at that location 0 width, your point impulse should mean similar just that it does not have any thing on x or y say x and y. So, it does not have all the energies planked at the origin of x,y. So, point impulse so, we can 1-D you are familiar, you would heard about delta function so, delta of x is 0 and x is not equal to 0 so, that means, everything is packed only at 0, outside 0 you do not have any value, it is all 0 that is its definition.

So, the uniqueness of this is hardly this function is used by itself right. Mostly, it gets its meaning, or the way it is do a way it gets utilized is delta x combines with some other function, it kind of combines so, here, it is multiplied with some arbitrarily function f of x so, delta x multiplied with f (x).

$$\delta(x) = 0; x \neq 0,$$

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0)$$

And then, the energy that is integrated over that sum right that comes out to be f of 0 that means, the functions value, this is a any function f(x) when it is combined with a delta of x right, then you can essentially pick this f of 0 that means, the function f(x) whatever value it has at 0 can be picked out right because of this delta function.

So, delta function is unique in that sense that it is not by itself it is defined, but it is interesting because it can be used with other signals to extract, pull out the value of that arbitrarily function wherever delta function exists ok so, that is your 1-D. So, you can extend the same concept to 2-D as well so, that means, it is defined only at origin right at (0,0) by itself does not, you know it is not; it is not utilized, but then it combines here right two-dimensional signal instead of f of x here, it is f(x,y).

$$\delta(x, y) = 0; (x, y) \neq (0,0)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\delta(x, y)dxdy = f(0,0)$$

And you will notice quickly this can be used f(x,y) means it is pulling out f(x,y)'s value at (0,0) so, because delta function is defined, point impulse is defined at (0,0), the function f(x,y), any function f(x,y), its value at (0,0) can be pulled out. Wow, this is fantastic right why? Because f(x,y) is a two-dimensional in space, it can have any value anywhere.

What if I want to know that f of evaluate that f(x,y) at particular location right? So, I want to locate a f(x,y), I want to know at this location, this location is some offsets from here, I know this value now right f(0,0)using the delta function an integrity. What if I want to know a value here? I can use my delta function to pick how do I do that? I have delta function at (x,y) right it is like this in the space.

Now, if I can shift it right so, I can shift it by some epsilon, eta, then your delta function is shifted right (x-epsilon, y-eta) so, recall the definition of the point impulse, you will quickly see this operation f(x,y) is f(x,y) is existent over the space, but my delta function is existing only when this argument is 0.

$$f(\xi,\eta)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x-\xi,y-\eta)dxdy$$

That means, when x minus epsilon equal to 0 or x is equal to epsilon y is equal to eta only there you have delta non-zero that means, only there I can get my function right. So, that is your shifting property. So, you can basically use shifted version of your delta functions to pull out values from f(x,y) at those locations right this is shifting property ok.

So, as we go further, I think this is all 1-D equivalent so, you know if you had already appreciated 1-D powerfulness of your delta functions del a and whatever you are pulling especially in the linear systems concept, this is a straightforward extension ok. So, you can move your delta function; move your delta function and then, pick at f(x,y) so, the value of f(x,y) at this location can be pulled out using the shifted delta function point impulse ok.

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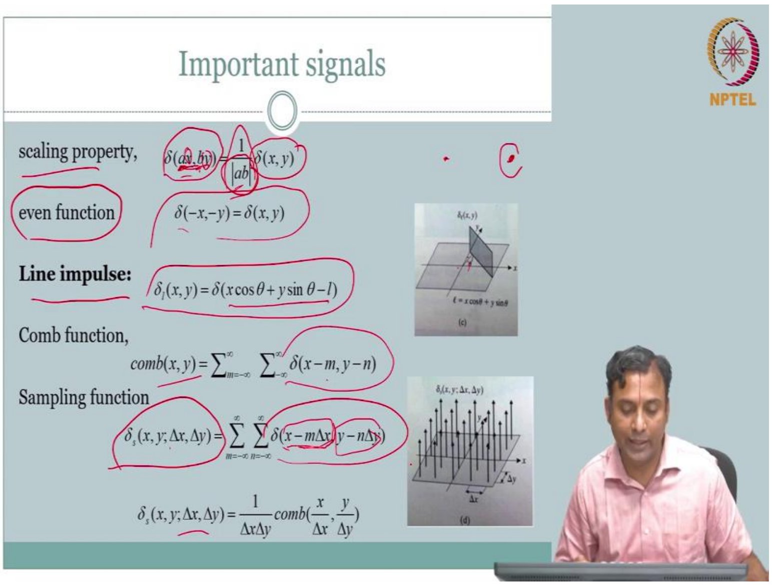

So, other others properties like scaling or even function or straightforward extension. So, if you have you know we had only x and y before, but if you scale the axis right, if you do ax and by, then you will get delta(x,y) you will get this here so, it is a scaling property. So, if you scale this axis here, the value gets shift the scale ok. So, this is your axis scaling, this is your amplitude. So, if you change here, if you scale here, it gets divided by this value right so, as to keep the energy constant.

$$\delta(ax,by)=\frac{1}{|ab|}\delta(x,y)$$

So, the idea is this is your scaling property, this is what is called as even function again this is you can even though it is listed separately, this is nothing but what happens if you are a and b are is equal to minus 1 right? a equal to b equal to minus 1 would essentially be delta(-x, -y) which will be 1 by 1 of delta(x) so, it is just a derivative right so, but it is specific it is called as even function. So, those things are straight carry forward from your 1-D signals that you probably know nothing much.

$$\delta(-x,-y)=\delta(x,y)$$

So, let us quickly move to the another important idea of line impulse. Line impulse ok, you now have the value of point impulse right, point impulse is very immensely useful, you will look at it from a image quality perspective eventually right. If you have a ideal point at 0 and if your imaging system is going to take so, I have a ideal point ok, this is your ideal and then the imaging system is going to take this point and is going to show this point right, the objective is your output image is an estimate of ideal input.

But because of the imaging systems point right this is a point impulse, systems response to this so, your point spread function you will see that this is bigger one so, there will be blurring ok. So, it is related to resolution, blurring, image quality so, it is because it is very important point impulse is an important concept it is a important mathematical operator. So, why then line impulse similar?

So, now instead of just using point, if I want to characterize a imaging system, I could use calibrated way to do it and it might be beneficial some say instead of making a point, if you have to make a 3D object that means, I am going to talk about a sphere instead of making a very small sphere like infinitesimally small sphere I am placing it and trying to image which may be challenging, it may be easier to have line targets, a wire with different dimension.

So, in calibrations line targets become very useful and therefore, to mathematically represent that, we need to have line impulses. So, first we write right you can have line impulse what does it mean? That means I have a delta function that exists only along the line that is non-zero only along the line. So, I have delta of here l is equal to x cos theta plus y sin theta so that means, where is this line? This line is in space (x,y) space right so, it has some angle, the norm to this line from the origin makes an angle theta and the length along the direction of the norm right is l.

$$\delta_l(x, y) = \delta(x\cos\theta + y\sin\theta - l)$$

So, now, you can look at write a line in this space (x,y) with different orientation and at different locations from the origin ok. So, this is your line impulse. So, immediately when you do that, you can think about different functions so, comb function. What is a comb function? You know the comb that we use, how does it look? It as you know it is usually a comb you have a line and then, you have teeth’s right that is your comb.

So, comb function should similarly have that means, it is mimicking that. So, you have a line, but it has values right, the spikes only at spatial locations. So, comb of (x,y) is nothing but a collection of this at different offsets from m and n, different locations m and n right (x,y) plane at locations of m and n, you are moving this delta only there it is there so, if you visualize that it look like a comb.

$$comb(x,y) = \sum_{m=-\infty}^{\infty} \sum_{-\infty}^{\infty} \delta(x-m, y-n)$$

And again, why is this important? We already talked about using these delta functions to thick values of the continuous function so, naturally that means, you can wish envision if there are going to be a spikes right, delta function here like that (m, n), then maybe I could you and I pick off from f(x,y) if I pick up values only at discrete location (m, n) that means, it is equivalent to sampling also.

So, you can view this as a sampling as well delta of (x,y) and you are picking samples space delta x in the x direction, delta y in y direction which is nothing but a collection of delta functions where x right it is defined only x is defined that x minus m delta x meaning whenever this argument goes to 0, whenever this argument goes to 0 so, that mean x is equal to m delta x and y is equal to n delta y that is when you have the values that you pick, those are the discrete location so, that is your sampling ok.

$$\delta_s(x,y;\Delta x,\Delta y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x-m\Delta x, y-n\Delta y)$$

So, that is why you will kind of use this more and more again you could use the same argument, if you could get these relationship between sampling function and comb function. So, you can use this comb function to do sampling. So, I can take the space out and I can get discrete locations, valued at discrete locations.

$$\delta_s(x,y;\Delta x,\Delta y) = \frac{1}{\Delta x \Delta y} comb(\frac{x}{\Delta x}, \frac{y}{\Delta y})$$

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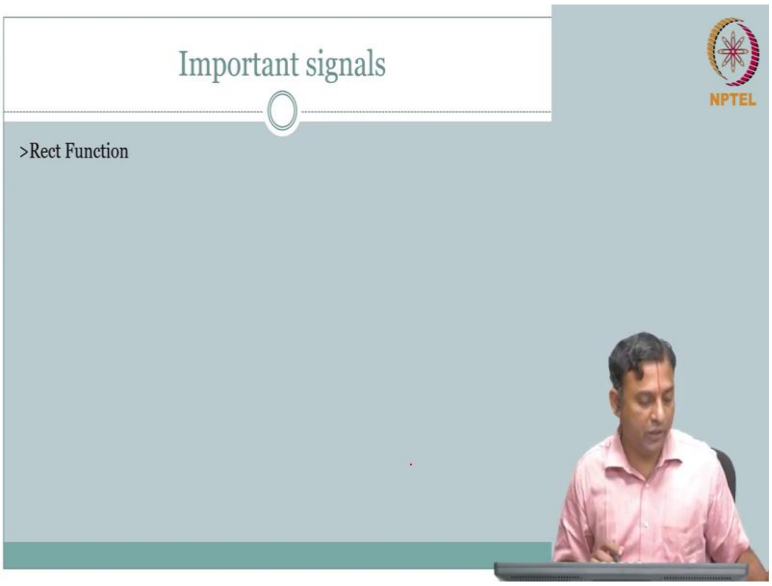

So, we will stop here and we will continue the next lecture from this location.

Thank you.