


Introduction to Biomedical Imaging Systems
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Lecture - 42
MRI_Phys_S17-S20

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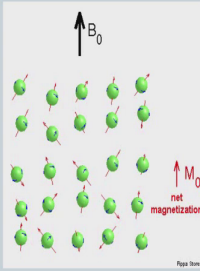


Nett Magnetization


Slight preference for "up" direction

Sample becomes magnetized

Magnetization vector:


$$M = \sum_{n=1}^{N_s} \mu_n$$


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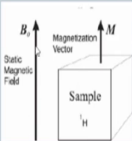


Now, we need to build our material from here as to what is the signal, right. It would be you see this is the net magnetization is z-direction and that the strength of it we need to relate it to some of the variables that we have been talking about.


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Macroscopic Magnetization at Equilibrium




The diagram shows a 3D rectangular prism labeled 'Sample' with a width 'W'. To its left, a vertical arrow labeled B_0 is labeled 'Static Magnetic Field'. To its right, a vertical arrow labeled M is labeled 'Magnetization Vector'. Both arrows point upwards, indicating that the magnetization vector M is aligned with the static magnetic field B_0 .



So, let us take a small sample volume for explanation. It is a nice cuboid that we have taken which is not that different I mean your images if you actually recall in the images we call about pixel it is always some you know square or rectangle when you go in a volume you are going to have volume element.


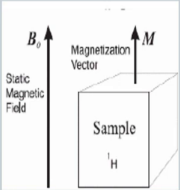
So, we were taking a sample volume, which look which is a cuboid here and when there is. So, when you subject that volume to the external magnetic field B_0 the static magnetic field, then there is a net magnetization M that is along the direction of B_0 .

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Macroscopic Magnetization at Equilibrium

- Equilibrium value: M_0
 - Same direction as B_0
 - Depends on $x=(x,y,z)$ only
- Magnitude of M_0 $M_0 = \frac{B_0^2 \hbar^2}{4kT} P_D$
 - k is the Boltzmann's constant
 - T is temperature
 - P_D is the proton density, **which depends on the tissue type**



So, at equilibrium what do we mean by equilibrium to take the sample volume put it in the external magnetic field. Initially each of the nuclei are all in different random directions. So, give it some time, right. After you give it some time every nuclei is trying to align to if it is a hydrogen it is trying to align to it is up preference at 54 degrees there are more of them or it is going to be an 180 minus 54 for the other direction, but then there are more on the up direction.

So, you have a net magnetization after you leave. This is called as M_0 which is the equilibrium value. So, that is the net magnetization at equilibrium from that sample ok. Of course that depends on x, y, z many that depends on the volume itself there you are picking, how many nuclei are there inside that volume, right. So, if we take a different volume then I might get a different M_0 because that may have a different number of hydrogen nuclei.

So, depending on the volume which you pick from the x, y, z the amount of signal that is there or amount of M or amount of equilibrium M_0 that you are going to get is going to be different. So, the magnitude of this magnetization vector is given by this formula right here and you will notice that there are several constants known constants so, which we do not have to worry about. So, Boltzmann constant temperature T is ok it is all body temperature.

So, we assume that the material that we are taking the sample that we are taking is in body temperature. So, T is not a problem, k that Boltzmann constant is not a problem; these are universal constant right, k. So, what are we left and then there is a h which is Planck's constant. So, again that we do not have to worry about we do not have much control it is universal constants.

So, what do you now have? You have the magnetization vector M_0 which is at equilibrium it is actually dependent on B_0 which is the field strength of the static magnetic field and then proportional to gyromagnetic ratio square of the gyromagnetic magnetic ratio. What is that gyromagnetic ratio? It is the material property. So, if I choose my material, if I say I am interested in hydrogen then the gamma is same, right. How do I know it is hydrogen if it is gamma, then I know the signature frequency.

So, if I get signal at that frequency then which was 42 point whatever for 1 tesla right. So, depending on B_0 that you are using you may have gamma the megahertz. So, your gamma is for hydrogen. So, that also is not a problem for me. So, your net signal that you are getting is directly proportional to P D. What is P D? proton density; what do we mean by protons? So, positively charged protons.


So, in case of hydrogen atom you have only one proton. So, in that sense it is technically called as proton density, but in our case we are doing hydrogen. So, they loosely use they say proton density and it is given that for biomedical we are mostly interested in hydrogen. So, we have to be careful in the jargons that people use. So, total density is strictly proton density, but that is positively charged how many protons are there in that volume.

But, in our case or in biomedical MRI mostly first importance is hydrogen, right. It is somewhat not said, but it is essentially amount of hydrogen nuclei that is there the density. So, that is your M_{naught} . So, essentially what we are now getting towards is you take a sample volume, if you can measure M_{naught} you can see that if you choose a magnet, so, B_{naught} is chosen, gamma frequency you are tailoring it to hydrogen.

So, the signal strength that you are getting is corresponding to the hydrogen frequency, then that M_{naught} is directly proportional to only the number of protons. So, if I choose my sample same from different locations. If a sample has more P D then it will have more signal you know signal if location has less water content less hydrogen therefore, that will have less signal. So, I will have more signal or less signal dependent on your P D.

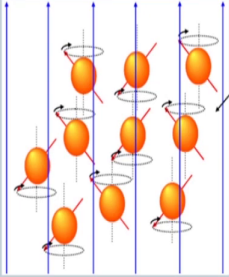

This is the this is the fundamental signal that we are talking about. I would not say signal as yet because this is the physics this is the quantity that we are going after how do we record and what is the signal we already spoke initially, right. We have to record this magnetization where we have magnetic vector that is looping, then I can actually correct get a current put a coil get a current that will be proportional to this. So, we get to the detail, but essentially this is a signal origin that we are talking about.


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Larmor Frequency

- Spins Precess at a single frequency (ω_0), but incoherently- i.e., they are not in phase, so that the sum of x-y components is '0' with net magnetization vector in Z-direction.


$$\omega_0 = \gamma B_0$$




So, just to complete the idea about definition of this Larmor frequency, the Spins Precess at a single frequency ω_0 , but notice I mean this is the another very easy to state concept, but difficult to comprehend. They are not in phase. So, that the sum of x-y component is 0 with the net magnetization vector in Z-direction. I mean that is very complicated to understand. Imagine this is what we are doing. What do we mean by the statement that is made thereof not in phase?

So, let us imagine that each of you are sitting in a class each of you are have this is your micro spin, right. So, I am saying now all of you to spin. So, each of you are going to do this. Of course, they do not say anything further. Each of you some will be doing it here some will be doing it there, some will be doing it here most people who are lazy will have hand down they

will do it here in whichever direction they were actually sitting a handle stick they will do it, right.

So, essentially initially it is random nuclear spins direction that is how it is; each one is a microscopic magnetization vector in different directions, but pretend now I am subjecting this to external magnetic field, right and let us say that is my voice command saying, everybody align the spin in the direction going from floor to roof. So, the moment I say that each of you are going to. So, each one are initially at a different direction so, net vector is 0.

But, now I have instructed you to all align along the direction going from floor to roof this is akin to applying external magnetic field that is in the plus Z-direction. So, what is going to happen? Each of you are trying to feature starting position you had you have to come try to make a attempt to come here, but when you come here you are not you are not going to align, you are not going to precess around.

So, now, notice of course, some of you will go down, right and then do it, but the nature of things is most of you will obey the command; that means, the nature says that most people will go to the upstate than the down state. So, there is going to be net positive, right.

So, now notice carefully what has happened. Because of the external magnetic field that is applied or I said in you know in this analogy I said, everybody align yourself in the vertical direction you are try to come and you are spinning and there is going to be net magnetization in this direction, but when you are spinning you are spinning around this direction which is called as at some frequency which is called as Larmor frequency given that you all are supposed to behave same you are all hydrogen, right.

So, that means, no matter what happens after some time is given right after some time is given after the external magnetic field is applied, after some time is given after I asked you to come here eventually right, everybody would have come doing this. Although you will start so, now, I am saying ok you are doing this you all are same nuclei hydrogen. So, you are all doing the same frequency right you are all spinning at the same speed angular velocity, right.

You are doing it very, but now I say stop that is I apply I switch off the external magnetic field or in some sense I say stop. What happens? Instantaneously each of you if you stop you will notice even though all of you are at the same angle you know in this up direction you are stopping, you will notice with respect to the vertical angle some would have stopped here, some would have stopped here, some would have stopped here, some would have stopped there.

So, the angle of inclination is same because your all hydrogen and you have the resting state that you have achieved at equilibrium, but then when I say stop even though you are doing this when you stop you maybe at different positions, right. The angle could be same, but you might have stopped when in the other directions you are at different locations some will be, but some will be horizontal some can be in the you know the other direction the x direction some could be in x-y direction.

So, what is happening? That means, you are all supposed to be not in phase this is what is meant by not in phase. So, I mean I am going to get into that little more detail, but from here if you actually position yourself and use all the knowledge that you have accumulated from mathematics and physics, you could image immediately guess oh I have a net magnetization vector he is talking about 3D Z-direction, X-direction, Y-direction.

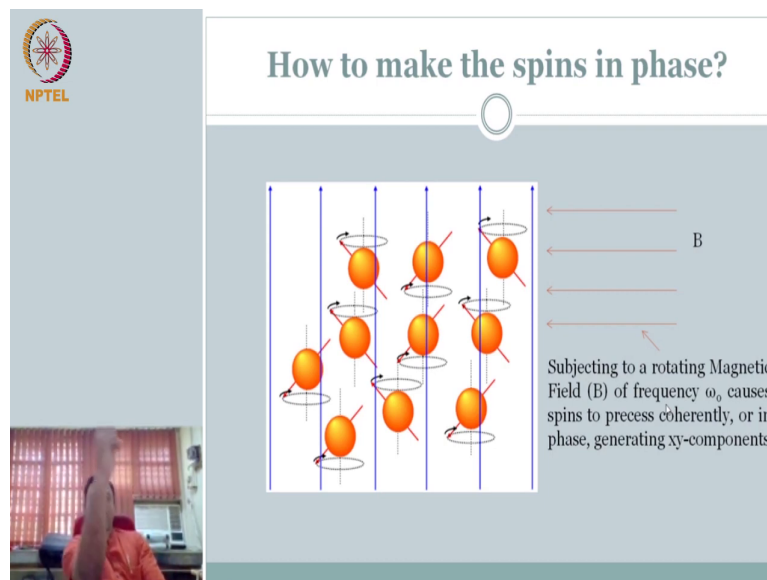
If net magnetization is there in Z-direction because all of you are trying to come or most of you are here in this direction and therefore, there is going to be a net magnetization in the Z-direction, no problem with that. But, if you are in different phase what does that mean; that means, if I project each of you on the x-y plane, the x-y plane the directions can be every all through 0 to 360, right whichever wherever you stop.

So, that means, in the floor if you sum all the vector components in the x-y direction the magnetization components in the x-y direction I do not have any signal because it will again be in all directions. So, it will all sum to 0. So, now, you understand that when we talk about magnetization vector, there is a net magnetization vector in the Z-direction, but if you are in

all different phases that is if you do not make an extra effort then there is still randomness in the x-y direction and therefore, the component along the x-y plane will go to 0, right.

So, let us put some images to explain. So, this is what it is the blue color arrow mark that you see is the external magnetic field, right. So, each one is spinning at different orientations when you apply the external magnetic field they are all tried to align and these are the you know relate this to the different hands where each would have stopped. So, these are the spinning around and there is a M Z.

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So, now this might make sense how to make the spins in phase. Now, that you understood what do we mean by in phase each one is there is a net magnetization in this direction, but each one of you are stopping right each one depending on when, where you started obeying the command you would have joined the precession at different locations.

So, there is going to be you know different orientations that you are going to if you start a little later than the neighbor you will continue to be that same amount late every time, right. So, how do we make this in phase, right? That is a next level of engineering that we need to talk about.

So, we can make to we can make this happen if I am able to communicate and say that means, if this says if I can communicate create another magnetic field that can actually talk with or have influence with this spinning at the same frequency, right. So, relate to your regular forces that we know the gravitational field what we can do.

You have something like this if I am moving, right? How do I push this fellow to if it is doing this how will I push this to down? I can I have to push to ground, but then if I just have a force like this then I only hit one time this will go here. This will go then next time it comes it will go here then it will come.

So, if I have to push him down, what I need to do is I have to move with this guy push him down and move with this guy, I have to push him down move with this guy, push him down move with this guy, push him down move with; that means, I have to move with this guy.


If I have to move with this guy I have to have the same frequency, right. Here that means, if I have to apply external magnetic field I have to apply the external magnetic field at the same frequency of the Larmor frequency of this guy in the other direction. So, now, not only you have the external magnetic field that was static where your net magnetization in the Z-direction, now I am applying another magnetic field which is B which is rotating ok subjecting to rotating magnetic field.

And, that means, it has a frequency of ω naught which has to be you know it has to this analogy, right. It has to move at the same velocity. So, your angular frequency ω naught should be exactly same as your Larmor frequency of what you are doing, if you do that then given some time all of you are going to have hand. So, now, wherever you started if I give

another command and say no, you go slow you go fast, but next 2 seconds all of you make sure that you your magnetic spin is in front of your nose, right just to make it interesting.


So, wherever you go started to make that extra effort to bring it back and from there on all of your hands will spin with not only the same angular frequency, but also same phase. So, if I take one instance, then I will not see several vectors I will see only one hand because everything else will be synchronized, ok.

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Process Involved in MRI

- Put patient in a static field B_0 (much stronger than the earth's field)
- **(step 1)** Wait until the nuclear magnetization reaches an equilibrium (align with B_0)
- Applying a rotating magnetic field B_1 (much weaker than B_0) to bring M to an initial angle, α , with B_0 (rotating freq=Larmor freq.)
- $M(t)$ precess around B_0 at Larmor frequency around B_0 axis (z dir.) with angle, α
- The component in z increases in time (longitudinal relaxation) with time constant T_1
- The component in x-y plane reduces in time (transverse relaxation) with time constant T_2
- Measure the transverse component at a certain time after the excitation (NMR signal)
- Go back to step 1
- By using different excitation pulse sequences, the signal amplitude can reflect mainly the proton density, T_1 or T_2 at a given voxel



So, just go back to our previous process involved that we have covered this step put patient in a static field. So, you know what happens now. Put in a static field all your microscopic magnetization start to align given a net magnetization in the Z-direction. So, wait until nuclear magnetization reaches an equilibrium, right that we talked about. Now, there is a net

magnetization in this direction, but how do I measure it that is the next level of engineering that we need to do how do I measure this.

So, in order to measure this the first step I am trying to make is I will apply another rotating magnetic field, so that I can bring this component in the x-y direction ok. So, M of t precesses around B_0 Larmor frequency the Z-direction. Now, you have a component in the Z-direction which is referred to as longitudinal relaxation. So, of course, just to complete the point if I now say remove the external magnetic field, right, now I say, ok you all relax what will you do you will go to your resting state wherever it was.

So, that means, if I remove the external magnetic field after I make you all obey the command, they remove the command how fast you go to your native state is a characteristics of you, right. Some may go immediately, some may go low, some may go slow, some may go immediately, some may take their own sweet time, some may not go well let us not take that case here. The point is you are trying to relax, right. So, you are going to take some time to relax.

And, given that we are we have made our components in the XYZ direction the component in the Z-direction. So, you are all forced to go up, right, what will happen if you have to relax? If you have to have to relax you are going to go to 0, right. So, when I start I have coherent signal the component in Z-direction increases in time with longitudinal relaxation. Why? Because your B_0 is still there the static field is there.

The component in x-y reduces with time that is you are becoming out of phase right each of you are trying to go back. So, each of you will have out of phase. So, measure the transverse component at certain time after excitation of NMR you get the NMR signal, that is, you have in the static magnetic field is always there. So, you have always the B_0 right, but then I have moved the B_0 push to this B_0 into the floor and I have given another command so that they are in phase.

So, I have made this magnetization vector initially it is 0 in the x-y plane. I have given another rotating magnetic field made everything coherent and I have pushed the value to the

x-y plane. Now, if I remove that external rotating magnetic field it will start to relax; that means, it will start to de phase. So, your component in the transverse plane is going to go to 0, but it is going to try align itself to the external magnetic field.

So, your z component will go up that is what they mean by relaxation can you go into this again and again and this is the whole crux of this solution. So, now, you do this again and again you get multiple time that is a signal. So, now, we have come until understanding what is the magnetic vectors signal magnetization vector? And we have come to the level of understanding that we could manipulate this M, so that you could get longitudinal signal.

I mean and you have the magnetization vector in the x-y direction or in the z-direction that much we have understood. And of course, the speed with which it relaxes right when you remove the excitation has to be a material property. So, that is one is proton density we saw how fast it de phases is one time signal which is T 2 they call and how fast it comes back to equilibrium after the excitation is removed is another time constant which is T 1. I am sure you would heard of T 1, T 2 kind of MRI images, we this is our real understanding exactly what is mean by that.