**Introduction to Biomedical Imaging Systems**
**Dr. Arun K. Thittai**
**Department of Applied Mechanics**
**Indian Institute of Technology, Madras**

**Lecture - 43**
**MRI_Phys_S21-S28**

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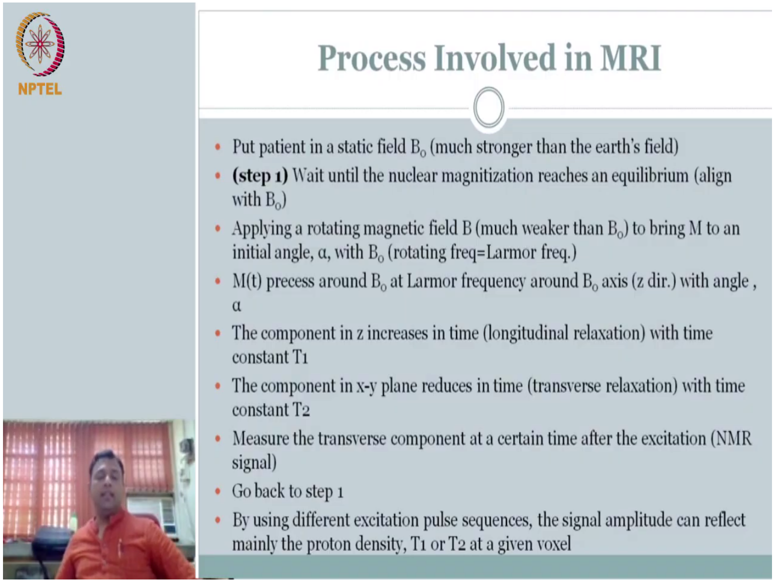


Let us move further to really understand where we get the signal in MRI ok. So, far what we have built is the overall process involved we talked about how you put it in a you know static magnetic field you allow time it will all start to align itself in the z direction the convention that we are using. And of course, when you have it is not just going to align it is going to have a Larmor frequency it is going to precess around right.

So, your magnetization vector has a net magnetization in the z direction with no further action the components in the other x y direction is going to be random. So, you are going to have net magnetization only in the z direction, in the x y direction it will all sum to 0 just because of the effect of only static magnetic field. Now, in the next step that you see if we can introduce another a rotating magnetic field then we will be able to manipulate this magnetization vector.

So, that in the x y also you will start to make them in phase right therefore, you will have magnetization vector component in the x y direction as well. And then here the steps talk about what happens if you switch off the your static magnetic field is still there, you switch off the additional excitation rotating magnetic field. Once you do that what will happen? In the x y direction it will start to become a random right. So, it will start to relax in the x y plane and also in the z direction it will start to align get back to its equilibrium value right.

So, there will be a increase in the z direction, so that it gets to the maximum possible equilibrium value in the x y direction the signal relaxes to 0. So, the rate at which the time constant involved we refer to T 1, T 2 we will talk about this again understand this further. So, now, what we need to do is all this intuitive understanding of magnetization vector changing with the time right in the presence of static magnetic field and the manipulation with the rotational magnetic field.

We need to put them in form of equation the motion of this magnetization vector we know this motion qualitatively we have described this, but we need to mathematically describe the equation that governs this motion ok. So, that is what we will attempt to do in the next 10-15 minutes. Of course, that motion has to be somehow related to the signal that comes out.

So, we talked about magnetization vector capturing the physics of the signal, but what do we measure that we have not really talked about which will be the discussion point after we develop the mathematical framework the equation, then we will conclude with the signal that is measured.

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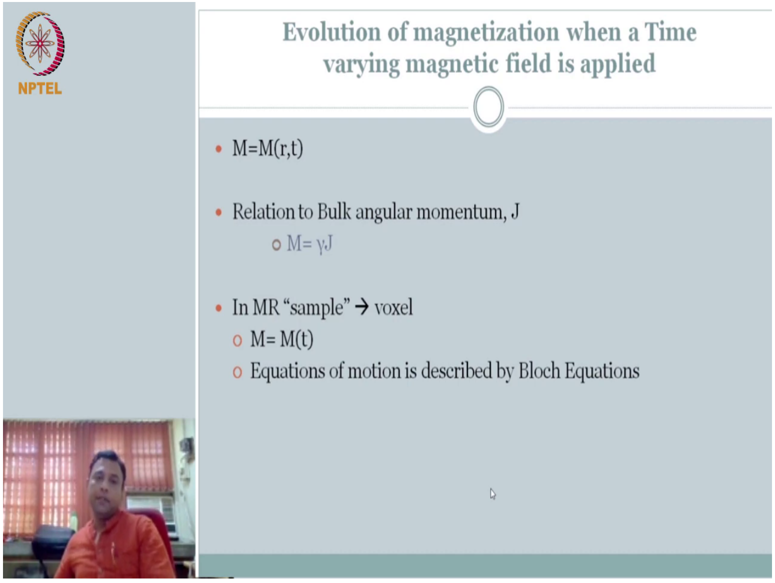


So, to start with we have the magnetization field that you are putting in the static magnetic field. So, you have microscopic you put them in. So, first thing that we know is this magnetization vector is a function of r and t; that is where is it where is this volume coming from and it is varying with the time. So, it is a function of r and t. We talked about the relationship between this microscopic level, we talked about the spin angular momentum and the magnetization vector remember we used a mu the microscopic spin. So, we had a relationship there.

Therefore, naturally if you take a volume right then you have a net magnetization vector capital M that has to be related to the bulk angular momentum you have angular momentum due to individual nucleus. So, if you take a volume you have many of them. So, there is a bulk angular momentum capital J is related to your net magnetization vector capital M using M is equal to gamma J the same you know analogy that we had with the micro spin, mu is equal to gamma angular momentum of individual spin. Same thing is hold, here we call capital J as the bulk angular momentum.

So, now what we are interested is if you take a sample right we take a sample volume we will call that as the volume element right voxel. So, when you have this small sample volume then we focus our attention on that then we can reduce this M as a function of space and t we can reduce it to M of t because we will pretend that now we are intrusted a small volume and there is no much variation in that.

So, you take a small volume we call that as M of t I mean, it will just worry about how does that small volume the magnetization vector from the small volume is going to behave in that can we describe the motion equation of the magnetization vector using a small sample volume right. And this equation was derived by Bloch which was called as Bloch equations which we will try to put together.

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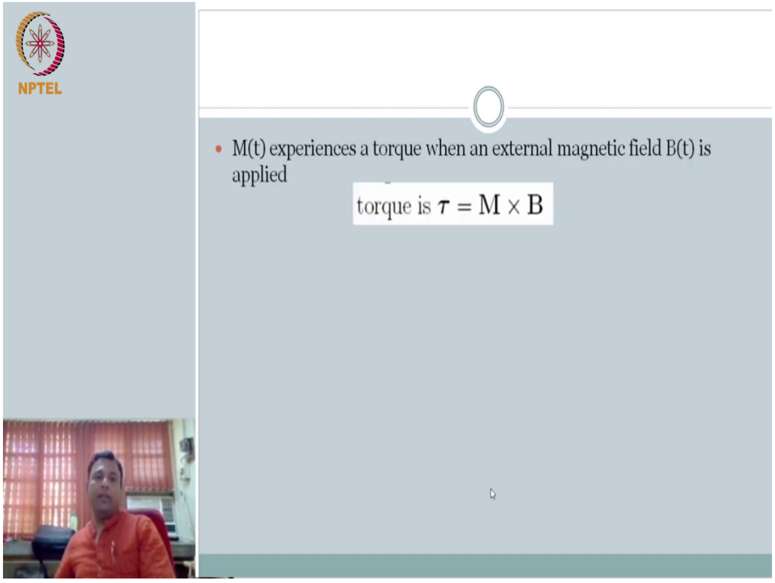


So, starting with what we know right when you have M of t it experiences a torque. So, we have the magnetization vector net magnetization vector. If it is experiencing if you have a external magnetic field then this magnetization vector is going to experience a torque which is given by tau is equal to M cross B. Very similar to what you have with the gravitational field I will show example of the top and maybe that will make you remember, recall the material that would have covered in your Engineering Mechanics when you talk about gyroscopes ok.

So, you have a torque which is M cross B.

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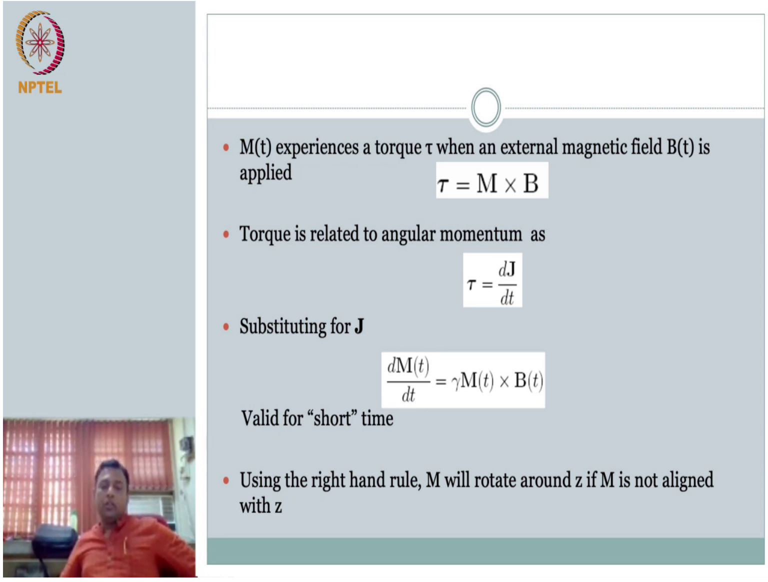


So, now, we know torque is related to the angular momentum again this you should have you will know inside out maybe if I tease you and talk about the linear counterpart this all will make sense. Now, what is linear counterpart? f is a force is equal to mass into acceleration what is acceleration is rate of change of velocity, so mass into B v by d t. So, you can have d by d t of M v is the linear force linear motion right. So, not the angular motion that we have here, so linear motion, so d by d t of M v is the linear force.

So, force governing the linear motion. So, what is M v? M v is mass into velocity which is your linear momentum. So, rate of change of linear momentum gives you your force governing the linear motion. Analogously you will have angular momentum rate of change of angular momentum will give you the rotational force which is your torque right. So, it is all make sense though the pressure basic, so tau is equal to d J by d t.

So, we can actually fuse the previous slide we had M is equal to gamma J. Now, we are having tau is equal to M cross B. So, we can eliminate J here by combining these two. So, we can have d by d t of M of t equal to gamma M of t cross B of t ok. So, we have just combined M is equal to gamma J and tau is equal to M cross B and tau is the related to J by the dou by dou t. So, we have fused these three and eliminated J.

So, now what you have is this is valid for short time, again this is a very troublesome notation short time we really do not know. I mean for now we will say short time means the not you know in relation to the time constant remember we talked about T 1, T 2. So, somewhere we will have to define the short time which will clear which will become clear when we when we progress further. But for a moment we will assume that this is valid provided the time instances that we are talking about is short

So, using the right hand thumb rule we can say M is rotating around z if it is not aligned with the z direction. So, it does this is the z direction it is rotating around z direction right this is what we have so far.

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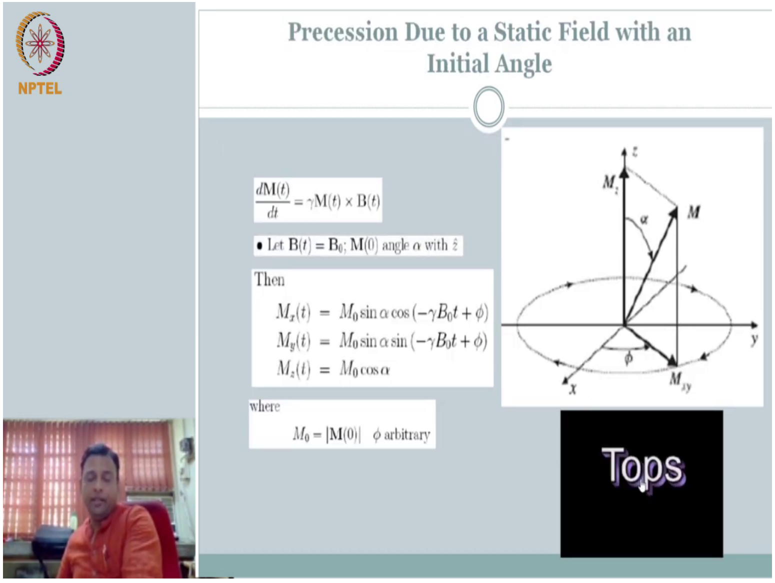


So, now let us essentially describe this with little more detail. So, you have now what we are talking about is describing this precision due to static field right. So, we will have to now reduce the general equation to the context that we have. So, we will say the static the B of t is nothing, but we will pretend is your static field and let us say that you are starting at some angle alpha arbitrary angle alpha.

So, now what happens to your motion equation d by d t d of M of t is gamma M of t cross B of t is what we had. Now, I am saying B of t is actually nothing, but your static magnetic field which is B naught. So, B of t becomes B naught and let say that your magnetization vector when you started right at 0 time equal to 0 was aligned at some arbitrary angle alpha with respect to your z.

So, when you put this together and we can actually tease the magnetization vector into different components. So, you can have M x of t, M y of t and M z of t right in terms of the variables that you see here. Of course, your phi is your phase in the x y direction. So, this is your magnetization vector of each of the components fit into each of the component. So, magnitude of your M naught here is nothing, but the magnitude of M vector at time equal to 0, phi is arbitrary.

So, this governs this is the motion equation that tells that the magnetization vector is precessing around in the z direction ok. This is very similar to in the gravitational field counterpart your top experiment right.

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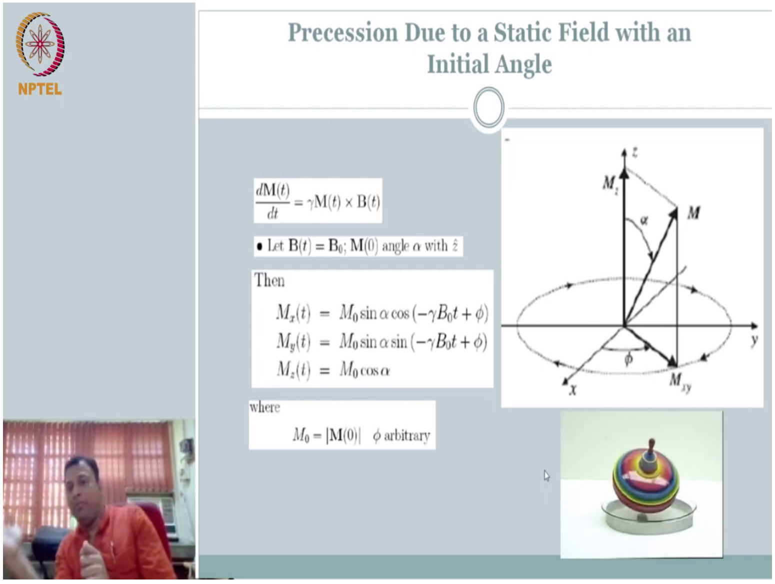


So, when you have the top it is rotating. So, you have axis of rotation, but then there is a gravitation that is trying to bring it to the floor. So, this is your top if you keep the top without angular momentum what is going to happen, if I place my top here without any angular momentum no spin what will happen it will fall down. So, gravity is trying to pull it this way.

So, therefore, what you have is you have a angular momentum it is rotating about this axis or rotating about this point ok.

If you want to really read the material go read about rotation about a point here in engineering mechanics syllabus. So, it is rotating about this point when it do not have angular momentum gravity is pulling it to this direction. So, your rotation moment is going to be in this direction, but it is going to spin about the axis there is going to be angular momentum in the direction of spin. So, essentially what is happening because of that it is moving around in this direction right that is exactly what you have here the precessing around the z direction.

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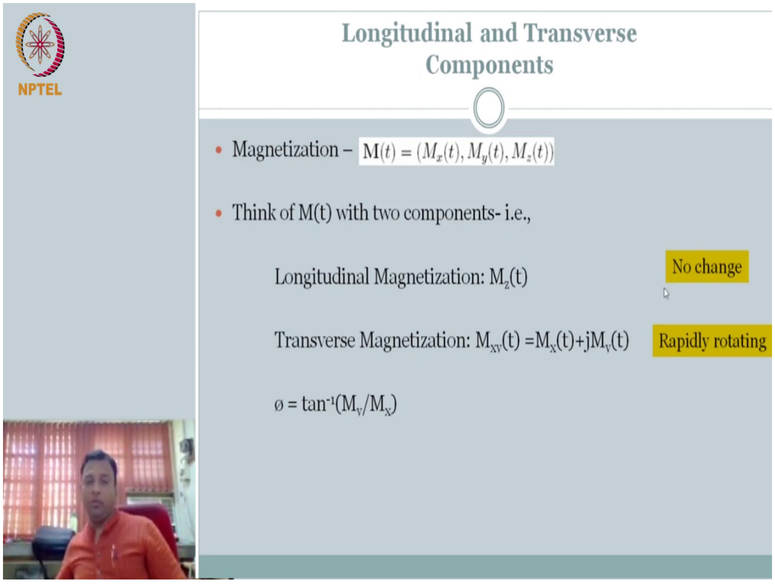


So, now, what we will; now what we will do is we go further we want to understand the signal. So, we now understood the magnetization moment how it is changing with the time we are able to describe it, we need to now proceed further to measure this quantity ok that is what we are going to build. So, magnetization vector we have split it into M x, M y, M z, but you will notice that it is profitable that there are two instead of working with three variables we could work with two variables by making this intuition that in you know we have already talked about M z of t M z of t.

So, that is your z direction, so you should retain your M z of t because that is what we have built. But, then when it comes to the other M x and M y instead of having M x and M y separately as two variables we could actually talk about the component on the floor ok. That will in other words one we will call it as longitudinal magnetization this is the z direction, the other we will call as transverse magnetization which is M x of t that is the vector magnetization vector on the floor which of course, has M x and the M y j M y it is a complex variable.

But, and of course, the phase of that is given by tan inverse of M y by M x, but notice carefully what we have done we have split it into two components. One is the z direction which we will call as longitudinal magnetization, the other is the magnetization in the floor ok. You will notice that the z will not change rapidly no change longitudinal magnetization if you have your z direction it will come to the equilibrium value. For example, after all wait for several time in the static magnetic field the z is achieved the net magnetization in z direction.

Whereas here in the x y what is happening? It this is the frequency it is rotating rapidly right. So, your x y is changing the projection on the floor is spinning fast whereas, on the z direction it is not changing much. So, one component the transverse magnetization is rotating rapidly whereas, the z direction is not changing ok. So, that is a very key component that we want to understand.

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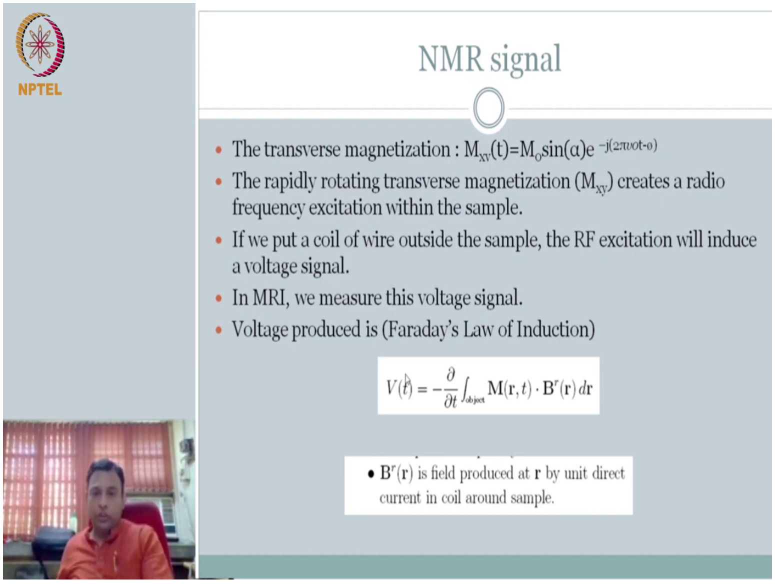


So, now let us get into the NMR signal. So, we have understood the magnetization vector it is spinning in the transverse direction it does not it is changing less rapidly or not changing in the z direction ok. So, now what we want to do is we will write the transverse magnetization which is M x of y of t as from the previous equations that we have right sin alpha e power. So, this is your rotating component it has a phase, it has some starting value sin alpha is the starting value; alpha is the some arbitrary wherever it started.

So, the rapidly rotating transverse magnetic field, so when you have see we did this in the when we started we said when you have current loop right when the charges rotate spin it is like a current going in circular loop which will give raise to your magnetic field. Now, we are talking about magnetic vector rotating right it is rotating. So, when you have a sample and the magnetic vector is rotating then it creates a RF excitation right it is a reciprocal electromagnetic.

So, when you have magnetization vector that is rotating rapidly then the sample is experiencing RF excitation. So, what will happen is if you put a coil of wire outside the sample, this RF excitation will induce a a voltage. So, you have initially when we talked about we were talking about the charged particles right charge spinning which is current across a loop giving raise to magnetic field the M vector. Now, we are saying if you have a M vector that is rotating fast and you put a coil that will induce a voltage right this is essentially the signal we measure.

So, we measure the voltage in the MRI or in the MRI NMR signal is essentially measured as a voltage using the Faraday's law of induction. So, we have magnetization vector we built our case understanding the meaning of this magnetization vector and what it physically signifies right. Now, we are saying in order to measure that if we put a coil then this magnetization vector right because it is cutting it; it will induce the voltage and this voltage will be proportional to the M vector ok.

So, how do we measure it? Simple we use reciprocity what do I mean by that, if I have a coil and I pass a unit current on this coil it is going to create a magnetic field right that is you apply a current you get magnetic field. Now, I am saying if you measure this if I have magnetic field and I keep a coil it is going to induce a current that is a reciprocal. So, if we can do the forward then I can say if I have a magnetic field and I put a coil this will be the voltage or the current then the voltage that is coming.

So, I can characterize this I can have this B r which is my received coil in some sense it is calibrated. So, receive coil a sensitivity ok. So, now my voltage that is induced is nothing, but you have the whole object in the whole object we have M r of t, we have this magnetization vector that is rapidly changing. So, the rapidly changing so, dou by dou t rapidly changing magnetization vector right. So, that when it is captured by B r of r that is the voltage that you are getting.

So, the whole voltage the sum of all the voltages that you get from the entire object due to this rotating magnetic field cutting the coil and inducing a voltage that is proportional is your signal.

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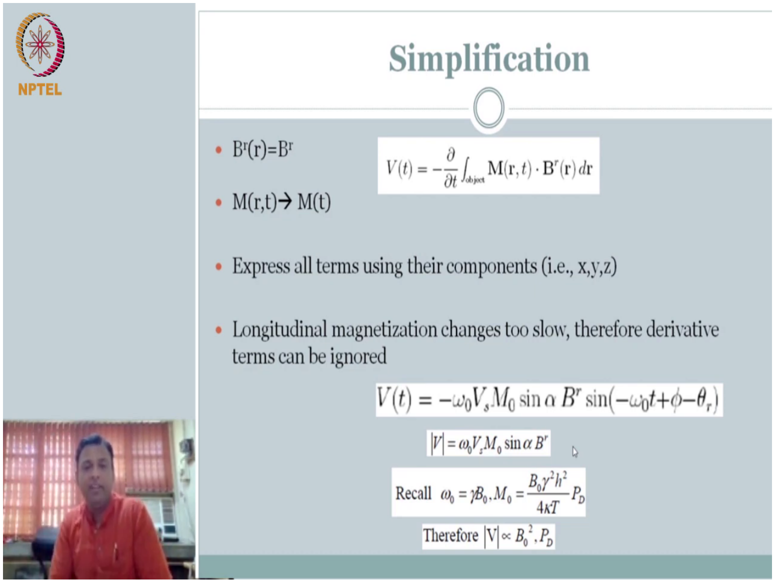


So, let us take a deeper look at this. So, V of t the voltage that you measure is going to depend on this M r comma t and your received sensitivity. So, you will have to simplify this to make some meaningful understanding of what is happening. So, this is the big picture. So, now, let us simplify it first simplification will be ok I see this B r of r.

That means, the received sensitivity is a function of spatial location. Why should it be? Let us assume that if I have a coil which is equally sensitive to all r that is if I apply a a current it provides it creates a magnetic field at location r that is how we got about this B r using the reciprocity. So, we will say that whatever magnetic field wherever it is created it will proportionately create the corresponding voltage.

Therefore, it is not dependent on where it is if it is homogeneous let or it is uniform right we assume that B r as this reciprocity that is it; it is uniform then we can have just B r instead of r it will just be B r right it is not it is sensitive to each location equally likely. Then you have this M r of t oh. So, your volume right M r is function of location t is function of time. So, why should we worry about that if you take a small sample volume.

So, your r is at some location you are taking a small sample volume then we could pretend that within that sample volume there is not much change that is you take a sample volume and say this is homogeneous sample volume right then I can get rid of this r ok. So, what I have done is let us assume B r of r is r that is it is uniform sensitivity of the coil, then I have M r of t reducing to M of t meaning I have taken a small volume.

So, it is a homogeneous volume there is no much changes within the given volume, if that is the case then I can write all these terms in terms of x y z coordinate. Now, you have these vectors right. So, I am going to split this into components and write it. So, I can write longitudinal magnetization changes to slope right we talked about this magnet longitudinal magnetization is your M along the z component. The z component is changing slow at least we can pretend it is changing slow compared to your the other direction. So, at least that much we can assume without going deeper.

So, essentially we substitute all these such simplification, we can write V of t is nothing, but omega naught V s; V s is now sample volume not voltage this is a very common mistake students have when you read the text V s is sample volume not your voltage ok so beware of that. Then you have your M naught a sin alpha B r and the sin quantity ok. So, notice now that this is your voltage that you are getting we can make some sense of this if you really look at it, what is this saying you have some magnitude which is changing sinusoidally.

So, this is a sinusoidal signal not only that you know what is the frequency of the sinusoid right you already see this omega naught. So, essentially what you are recording is a sinusoidal signal whose frequency is also not a big deal because you know it is omega naught. So, the only big interest to us now is to understand the amplitude part of it. So, your signal that you are measuring is a sinusoidal signal of a particular frequency which is more right there is a Larmor frequency that you are operating and it has an amplitude.

So, the signal amplitude is what we need to understand little more intuitively. So, what do you measure? You measure a voltage and the voltage magnitude of the voltage is omega naught V s M naught sin alpha B r. So, what does this say what do you know from before? You know something about omega naught that is the Larmor frequency. What is the Larmor frequency? It is related I mean you know it is related to the magnetic field you can increase the frequency by increasing the field strength we saw this before.

Likewise what is your M naught? Oh magnetization vector that also can be increased by increasing the field strength. So, we can already recall omega naught is gamma B naught and M naught is this quantity B naught gamma square h square the we remember we talked about this and then you have the proton density again it is coming here also proton more the proton density. So, you can see the signal that you are recording is proportional to square of your B naught and proton density this is very important.

So, now you understand if I go from a half a tesla machine to 1 tesla machine, 1.5 tesla machine, 3 tesla machine the signal inherently is same my volume is same, my proton density is same, but if I increase this the field strength my signal strength goes up therefore, if my signal strength goes up I can get better quality signals right. Of course, we have not talked about noise yet we will come back, but at least we are increasing the signal, but we are not increasing linearly we are increasing it non linearly.

So, if I go from 1 half 0.5 tesla to 1.5 tesla it is not like I am increasing the signal strength by three times, I am increasing the signal strength by nine times ok. So, that is a very very big advantage. So, now, you see the fundamental signal that you are recording in MRI which is in voltage is nothing, but a sinusoidal signal, the strength of the signal is proportional to square of the magnetic field that you are using.

Of course if you are measuring it from a volume if you increase the volume right you get more signal, but you know the challenge without talking much we know the challenge. If I increase the volume yes I can increase my signal, but then what is happening? The resolution is going to go down right I can say I get this net signal from the head which is fantastic I will have a lot more signal, but then I am interested in seeing the changes within the brim. So, I need some smaller volume, so that I can localize better. So, this is the inherent trade off that we will have right.

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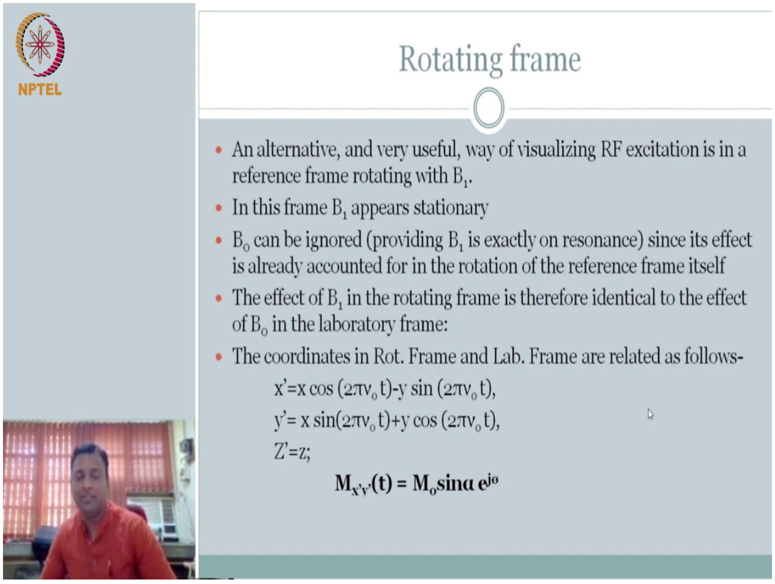


So, we will stop here. In fact, we will just do one more before we stop which is now what we need to do is we need to go to the next stage. Now, we know the signal in a static field I know how to define this, I know how to measure the signal. What did we want to do? We want to make it coherent we started with some alpha right some alpha value when will it be maximum this voltage maximum, the magnitude when your psi alpha is 1.

What does that mean? Alpha is 90 degree, what was alpha in a thing; oh if your magnetization vector was on the floor right then you get the maximum signal that is what it says. So, we started some alpha as arbitrary if you can make this alpha go to the floor that is 90 degree then you will get the best signal. So, the idea is how do we manipulate this alpha, we talked about apply another rotating magnetic field with the same frequency then you can manipulate this magnetization vector. So, now, you see that you get that insight. So, now, we can manipulate the transverse component.

So, we need to add one more signal one more rotating magnetic field to manipulate this alpha then we could get signal. So, we need to build our material to add to our signal equation the motion equation what happens we add additional rotating magnetic field right now we have simplified it for a static magnetic field and talked about motion equation for this thing. So, now, we need to add one more rotating magnetic field, so that we can manipulate the magnetization vector. So, it will become little complicated from visualization point of view if you use laboratory coordinate system.

And therefore, we could go for rotating frame of reference where essentially what we are talking about is, imagine you are applying another magnetic field right it is another magnetic field. So, this is my because of the static magnetic field this is what my magnetization vector is doing. Now, I want to apply another one, but that has to also rotate right only then I can push it to the so, that is also having a frequency which is same as the Larmor frequency.

To describe this it is going to be difficult it will be easy if I say oh if I sit on this right if I sit on this motion then I do not care about this one all I see is it is going to go out. So, that is reference rotating frame of reference if you are going to sit on the reference frame or in this case the rotating magnetic field that you are going to apply, if you are sitting on it then you are moving the magnetization vector how will it remember right.

So, in this frame B 1 appears stationary, your B naught can be you are already done right, B naught can be ignored because B 1 is already in the same Larmor frequency so, that effect is not an issue. Effect of B 1 is therefore, is identical to the effect of whatever we covered right now if you sit on it that will be the effect that you will see I mean it will be very mathematically you can do the transformation.

And come up with the rotating frame of reference M x dashed y dashed of t that is the motion of the magnetization vector described with respect to rotating frame of reference x dash y dash is equivalent to your this one ok. This is very easily I mean it mathematically it is very simple it is difficult to get a grasp of it.

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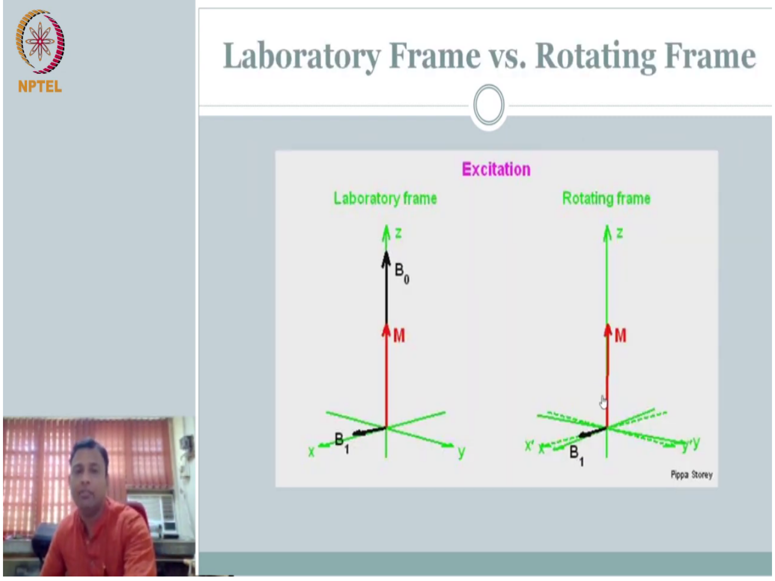


Let us I just have a animation that will may help you understand what we are talking.

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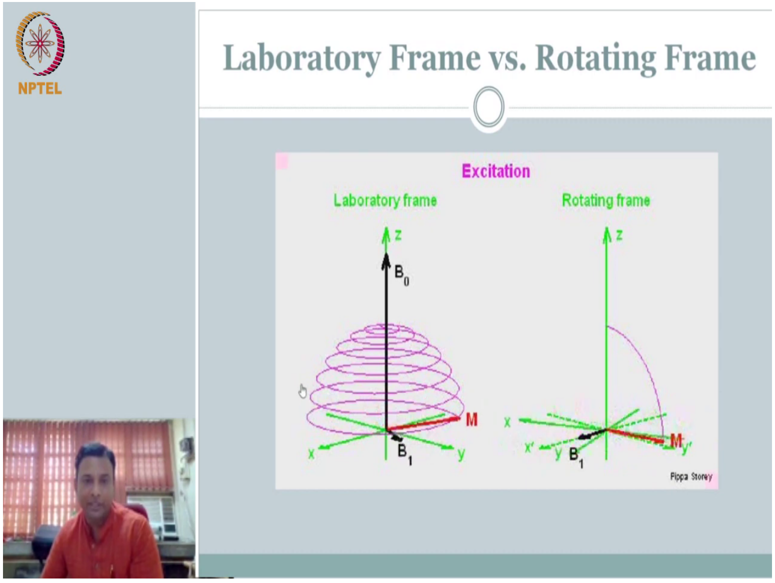


Right intuitively the equation that you saw is representing this in the laboratory frame of reference the magenta is the motion path right.

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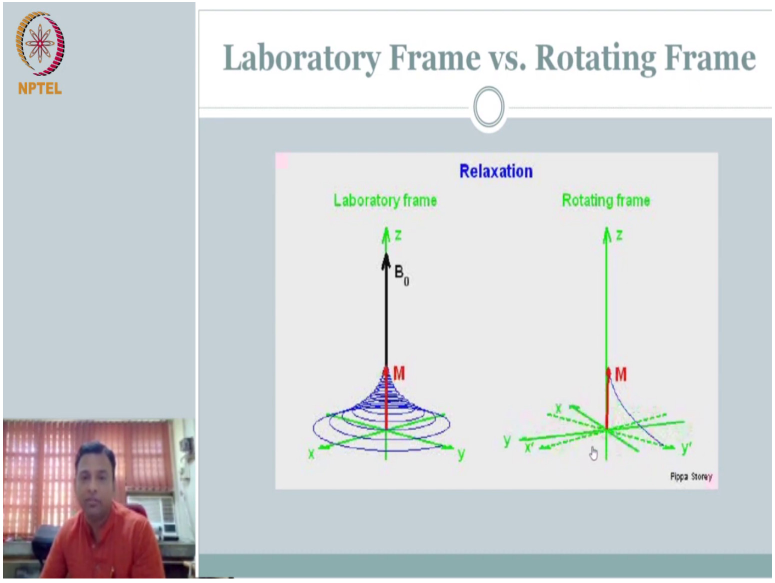


This is how you are representing M which is changing in time ok. If you do that this is what you are relaxation that you see. Whereas, in the rotating frame of reference the frame itself is right you are sitting on the rotating frame B 1 right the B 1 you are sitting on B 1 we are sitting on B 1 then you do not have to worry about that rotating. So, you are just looking at this.

This is a two different way of viewing it you become more familiar with it I mean you should you would have been introduced to this in several courses in different context, but now try to train to get a intuitive understanding of what happens when you have B 1 it may be easy to visualize the effect in rotating frame of reference ok. Instead of having this spiral each time I show my hand if you pretend it is rotating frame of reference I do not have to do this. I will just say this M is going down M is coming up right that will be the easiest way to do it. So, let us stop here.