Introduction to Biomedical Imaging Systems
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Lecture - 05
Important Signals

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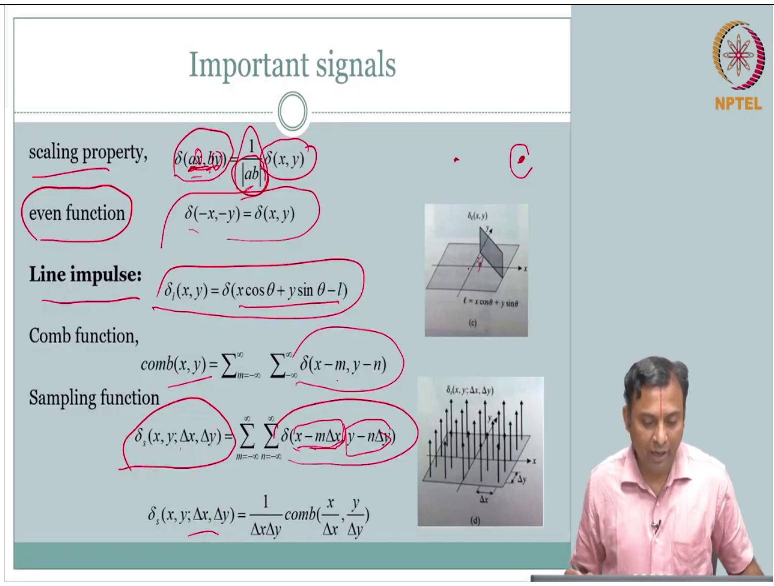

So, we have few more Important Signals that, we want to cover before wrapping up the review of the signals part of it. So, we did point impulse, line impulse and then we talked about a sampling function and the combination of comb function. How it can be written for the sampling function, right.

So, now if you really think about it, there are whole different sets of signals, several of them were very interesting. But we will stick to those most common ones, which would be repeatedly used in this particular course, ok.

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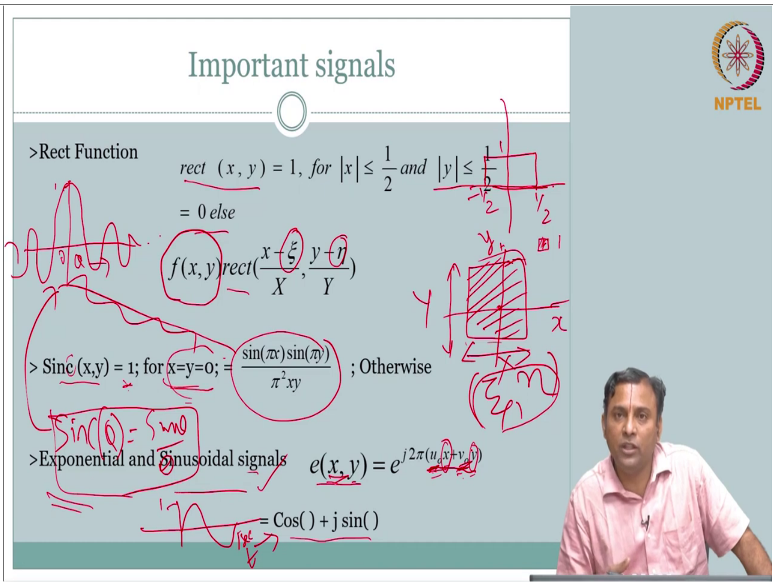

So, having said that let us move on; you are going to see rectangular function, rect function, right. So, I am sure this is the same name it goes when we did 1 D right or rather when you did 1 D or you should have seen the rect function before. How does it look? Well, you have one axis right; you have a time axis, right.

So, how will the rectangular function look? That is my independent variable and then so, that was my t. So, it is 1 right, over a range, rest of the time it is 0; this is how a rectangular function in 1 D looks like.

So, rect function in 2 D is the direct extension; in which case how will it be? Instead of t, now we are going to use or we have started using the variables like x and y, the spatial variables, right ok.

So, that means you are going to have, rect function is going to have x comma y two dimensions and it is going to be having 1 over what range; if in 1 D it is minus half to half right, in 2 D it has to be spanning an area, right.

So, it has to be minus half to half in x direction, minus half to half y direction. So, if you take the plane, you have a rectangular region, right. So, you need to x, y; third dimension right, if you have to visualize, you need to have the third dimension.

So, I am not having any colour sketch here. So, I am just going to shade this and say, this shade is value 1, ok. So, your rect of x comma y is just to be complete, the legend is this represents 1 ok, rest of the places it is 0, clear.

So, this is your rectangular function. So, it is direct extension of 1 D. So, it is a unit area here right, this is the unit area. Again imagine what it can be used for? I think that is the important, just definition of this is fine. But can you see the powerfulness of this function? Well, we did point right, we said delta function point impulse. And what was it used for or at least we said that it is very powerful to do what? In fact, we extended it and said sampling, right.

So, it is very useful to pick by itself definition is there; but it is most useful to pick values of a function f of x comma y, you can pick the value of f of x comma y at any location, any point in that space using the delta function. Likewise here instead of points, you can pick any area, unit area from any location.

So, you can use this unit area function rect function to pick any area from function f of x comma y; in the plane of f of x comma y, you can pick wherever you want. So, you can move the rect function right, you can center it and you can have a f of x comma y, which is the 2 D space. So, there is a function which is there in that plane; I can pick a region of interest right or area of interest by using the rect function. What does this mean? This is offset.

So, instead of 0 comma 0, I can move this rectangle to different locations and center it around some arbitrary epsilon and eta right. I can center it there; if you center it there, then I can have Y cross X.

So, I can have a rectangle of width and height X and Y centered around epsilon and eta, clear. So, see the powerfulness. So, you can pick areas. So, that is where this becomes very useful right, mathematically to use this, ok. So, that is one important function.

And then another powerful function is this guy. So, if you are looking at this and you are thinking, he made a typing error right; it is y is I know sine, what is this sin c, it is not a typing error, you should immediately recognize this is a sinc function. You must have heard this in first you know of even signals and systems or first level signal processing wherever you deal with, you would have heard about this.

Sinc function, what is a sinc function? Clearly by the form you say sinc is equal to 1, when x is equal to y equal to 0 kind of tells you already that, this is defined; otherwise x and y if it comes in denominator and when they are 0, you cannot determine it.

So, essentially you can already sense that, you have a value of 1 at x is equal to y equal to 0; means this x and y are going to be in the denominator. So, what is it going to be otherwise, other locations, right? So, it is going to be, if you have recognized your sinc of theta to be sin theta by theta, right.

And if you know this form right, which you should probably know; then you are going to say do the same thing, extend it to two dimensions. So, sin pi x sin pi y by pi square x y, so this is your sinc function. How does the sinc function look?

So, in one dimension, I mean it is easier to draw in one dimension; like if you take 1 D signal, so I am going to draw that, but you can you know start to visualize this in as a 2 D function as well.

So, I am going to just, actually not that sorry; this I am going to do just 1 D, ok. So, how does the 1 D look like? I have some variable right, I have to have at origin, I have to have maximum value; other locations it is moving like this, how does that look like, go right so on and so forth, so on and so forth.

So, clear. So, this is your sinc function; it has a maximum value at the origin and then it asymptotically reduces, but it oscillates, it goes both positive and negative, ok. So, that is your sinc function and this can be extended to two directions, that is what here sinc of x comma y. So, if you actually start to imagine this, easy way to visualize is perhaps; what happens if you take this sinc function, this is in one direction I drew.

So, let us say if this is x direction, you pretend draw the other direction also it is going to be like this; but it is going to be perpendicular to this. So, you can always see it is kind of a bell right, having ripples on those direction. So, you can start to imagine that, ok. Is there anything else that you need to brush up, yeah sinc you are saying sinc is you know sine form.

So, that means it is important that we need to understand basic sinusoidal signals. What is the big deal about sinusoidal signals? Wow, it is very powerful; because it is a template signal, that it is a basis. So, you could essentially use sinusoids right and relate it to a more general exponential, so that you could get your we will. So, you could basically you know under certain circumstance, you could essentially capture any signal or write any other signal any in in the form of purely sinusoids, a combination of sinusoids, right.

So, we need to understand what is sinusoids and what is exponentials; you know this, right. So, when I say you know this, formulation you may know; what is the exponential and what is sinusoid you might know the formulation. And what is the relationship between exponential and sinusoid that also you probably know, right. It is a direct extension from 1 D to 2 D; e of x comma y is e power j 2 pi times u naught x plus v naught y, ok.

So, mathematically it is fine, because one dimension you would have already again, instead of x and y you would have been more familiar with t right time. So, you would have already seen exponential function to be e power j 2 pi f t right or e power j omega, you know angular frequency and your linear, so omega is equal to 2 pi f.

So, you would have already seen that, right. So, if you extend that to here analogously; instead of t, you have x and y, instead of 2 pi f t, t is replaced by the spatial variables. So, what should u naught and v naught correspond to? It better correspond to, it is an analog of whatever you had in your one dimension, what you had f t f is a frequency.

So, what is the relationship between frequency and u naught that you see here or v naught that you see here? That is a frequency where you are talking about number of oscillations per time. So, the units there in 1 D that you would have learnt is cycles per second or whatever time unit's right, that is your frequency; whereas here this is also supposed to be frequency due to the analogous nature.

But this what is this frequency? This is a oscillation, but there is an oscillation in x direction, there is a oscillation in y direction; u naught is in x direction, v naught in is y direction.

So, that means, this is oscillations per length dimension right; because x is length direction, y is length direction. So, if x and y are in millimeters, then you are going to have number of cycles u naught is going to be number of cycles in the per millimeter in the x direction; likewise v naught is going to be number of cycles, number of oscillations in the y direction.

So, the meaning of frequency is same right, from 1 D we extended to 2 D; 1 D the independent variable typically you would have been exposed to time and therefore, you take it for granted. The frequency there is so many cycles per second; whereas here that mathematically it is a same representation, but the context and the meaning is slightly different, because the units are different, ok.

This is fine, is there anything else, is it absolutely clear; if it is absolutely clear if you at this point time, you think it is absolutely clear to you, then let us do one small exercise, right.

When you read this, take pen, paper; draw, I am going to do the easy one, you are going to try out the difficult one. So, I am going do the easy one, which is one dimension; what happens if I have a sinusoid of 1 hertz or one cycle per second right, how do I visualize it, right? How do I visualize it? How do I plot it, right? How do I visualize it? I plot it. So, I know how to.

So, I have a cycle per second, I said 1 hertz. So, let. So, I am making my life easy here, right. So, I have one cycle per second. So, this is t axis 1 second. So, I have drawn sinusoid of 1 hertz, I have drawn here some amplitude, right. So, one does not matter. So, now, I am going to tell you have a paper pen ready; how do I draw right, how do I draw a sinusoid in 2 dimensions, ok.

Think about it, that is what we will do, I will not rob you of the surprise. So, you think about it, before that let us just make it little more explicit; you can write exponentials and right. You have the you know the relationship between exponential and how do you write it in terms of cosine and sine right, it is a e power js cos plus j sin, right.

So, you can write that and therefore, you see the relationship between exponential and sinusoid. So, you can by combination, you can get sinusoid in terms of exponentials, cosinusoid in terms of exponential right; exponential e power i x in one dimension probably you would have even you know memorize these formulae's; e power i x plus e power minus i x e power i x minus e power minus i x right you will get cosine or sine right.

So, you can get that. So, similar things you can do here. So, the objective here is exponential is very powerful, because you can represent one or the other right; you do not worry about the face, you can have sine or cosine both are. So, getting back to what we wanted to do; if you are very comfortable, you know all these relationship between sinusoid exponential and I made my life easy by plotting.

I know how to visualize the sinusoid of 1 hertz. Can you plot right, I should not say a plot, can you visualize; because I am talking about two dimension, you can plot, but we saw the difficulty right. So, can you visualize a or represent a two dimensional sinusoid, ok? I think you have enough time, hopefully you started thinking.

The first thing is after this you should start to question me and say look how do I, you are telling me plot a two dimensional sinusoid, but what is the frequency you are. There are two variables, in one dimension there is only one variable and so, you said 1 Hertz it is taken for granted, it is the time axis, there is a cycles per second.

But when you have two variables right, when you have two variables x comma y; what frequency are you talking about? Is it the oscillations in x direction, oscillation in y direction; how do what do you want me to do, that question should come to you. If that is clear, then rest of it is straightforward.

So, before revealing the combinations that you can you know represent I really wish you try it once and then see what is going to come, ok. So, what I am going to show next is typical example of sinusoids with different frequencies, how do you represent them. So, a image of two dimensional sinusoids is what I am going to show, ok.

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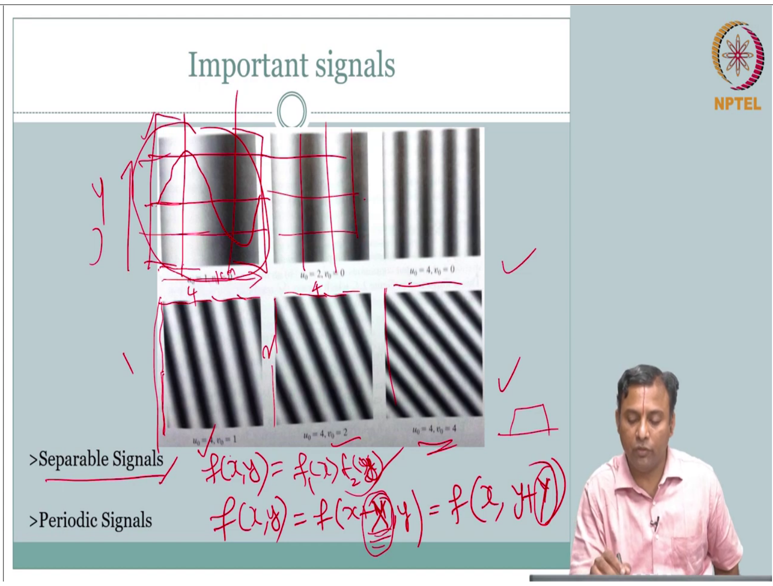

So, this is an image of two dimensional sinusoids. So, at the face of it when you look at, it looks like we understand; but let us spend some more time to really see if we can understand or can we read out this in English what this is representing.

So, recall the definition. What is the definition? frequency I mean the you know when I have cycles oscillations per x, that is my u naught or frequency along the x dimension. If I have oscillations in y direction, that is my v naught or the spatial frequency, spatial frequency in x direction, spatial frequency in your y direction.

So, if you look at the first image for example, what is varying, which direction it is varying and how do you calculate the frequency; can you comment what frequency components are there in this signal? A very rudimentary way to look at it is, I know the definition let me brute force; let us say for example. I take one line right, I break the problem from two dimension to one dimension.

So, if I plot this, if I take white to be positive and black to be negative right; if I do that, if I take along this line, the values will be like this, it is one cycle, clear. So, that means if this direction is x direction spatially; that means there is an oscillation of one cycle in whatever unit say, say let us for example, let us take this as you know 1 centimeter, just to be realistic 1 centimeter.

So, that means I have one cycle in 1 centimeter, clear. So, the frequency is there is a frequency in x direction, which is u naught and that is one cycle per centimeter correct, that is the brute force interpretation, right clear.

So, I could see that from here also; so that means if I take the profile here, it is still going to be like this; if I take the profile here, it is still going to be like this. So, what does this say; irrespective of where I am in my y direction, irrespective of where I am in my y direction right, I am going to have only oscillation in x direction, y direction there is no oscillation, right, y direction if I take, in y direction. If I plot, it is going to be whatever this white value; the same value is going to be there.

If I take this line in y direction, that is going to be some negative value whatever that, right. There is no variation, what does that mean? That means, there is no oscillation means zero frequency. So, clearly you can see that, what do we mean by oscillations per unit length that is your spatial frequency. So, you have a spatial frequency of one in the x direction, you have a spatial frequency of 0 in your y direction.

So, quickly can you run through the next two, what do you think it will be? Now, it is straight all of them are vertical bands, so same logic; if I take profile here there will be oscillation. So, there is oscillation in the x direction; but if I take profile like that, I do not have any oscillation.

So, it is very similar to the first block that we covered; there is no oscillation in the y direction right, in both these images. Whereas in the x direction, here I saw one cycle, here I can see two cycles, here I can see two, three, four cycles clear.

So, clearly you have u naught and v naught are the spatial frequencies, now I hope you get the feel for what do we mean by frequency spatial frequency; I mean this is actually straightforward extension from 1 D, but to visualize this and get a appreciation is already not trivial, ok.

But mathematically it may be trivial, but to get a feel for it, you should have that joy right; I mean that I hope you will see, you enjoy if you get that you experience that, good luck, ok. But this is the thing. So, if you do this, immediately if you understand this, you should natural inquisitiveness, you should look at the bottom and you say wow this is not vertical right, but it is at some angle.

So, what could be, how do I comment on this? Maybe you can go from here to here you can do another thought exercise. What happens if I have that is left to you to work out, what happens if you have, no oscillations in the x direction, but some oscillations in the y direction, ok.

So, please do try yourself, u naught is equal to 0, v naught is going to be 1, 2 and 4, how does that look clear. So, you try that on your own. So, what does this mean? This looks slanting right and the angles change. So, a smart way to do is, I will interpret it as basic as possible right; if I take y axis just now we covered, how many cycles are there, there is a black, there is a white. So, there is full cycle that goes over the black and you know comes positive and goes over white.

So, there is one cycle in y direction. So, your v naught is 1. So, if I take my x direction I see one, two, three, four I see four right four cycles; so that means this has both oscillations in both directions u naught and v naught both are nonzero.

So, depending on which is faster which is slow right, which frequency is high or which frequency is low the slant can change. So, for example, here we talk it is 1 and 4 right, here if you look it is 1 and 2 and 1, 2, 3, 4. So, it is 2 and 4; likewise here what it is 1, 2, 3, 4; 1, 2, 3, 4, so it is 4 and 4, right. So, clear.

So, 4 and 1 x direction, this 4 y direction is 1. So, your u naught is 4, v naught is 1; x direction is 4, u naught v direction is 2. So, v naught is 2 and then you have 4 and 4, clear.

So, much for a simple sinusoidal oscillation. So, now, you can start to see when you look at a image, when we talk about spatial frequency right; you can already see how do I look at an image and kind of eye ball it and expect what kind of frequencies will be there, ok.

So, that is something that you should train yourself, ok. Before we complete the signals part of it, we need to just touch upon two characteristics of the signals certain signals, which can be very helpful to work with. So, separable signals; what is separable signals? So, so far what we have is f of x comma y, that is what we are interested, two dimensional signal.

But then you know we are lazy right, I mean we are always lazy; if we know something, we do not want to and if we know how to use something right, we do not want to learn something new, if I can make use of what I know to solve a problem, we will do only that, right.

So, f of x comma y is a two dimensional signal, but I am telling you know what; I have done lot of work in one dimensional. I have taken courses, I have done so many problems, you know I have all the solutions ready for one dimension. This two dimension how do I now you know look at it?

So, if it turns out the signal f of x, y can be broken into say f 1 of x and f 2 of x sorry f 2 of y right; that means you can separate this 2 dimensional signal into two 1 dimensional signal. If you can do that, this kind of signals are called as separable signal. So, we like to work with separable signals for the reason I mentioned right; because we can split it into two 1 dimensional signal and I know I can solve, I know how to work with it, ok.

So, quickly if you can recall, can you gather some examples from what we have done so far; can you see if you can recognize some of the functions we covered earlier, which of them are say separable signals, right?

Student: Rectangles.

We covered rectangle, do you think rectangle could be a separable signal; yes area right 2 dimension it is area, 1 dimension we saw the rect. What is the area length into breadth? So, I could have two 1 dimensional signal; one with length side, the other say the breadth side. So, rectangular function is you can separate. What is the other function that, sinc function, right.

So, some of the signals that we already saw right; if I know how to work with sinc function in 1 dimension, I would rather break the 2 dimensional sinc function into two 1 dimensional sinc function and work with it, ok.

So, that is separable functions. Periodic signals I should not even, you know spend time you know with you know by this time in 1 dimension, you would have learnt about periodic signals. So, how do we extend that to 2 dimension? In 1 dimension recall what was the idea of periodic signal; if you have a function say for example, f of t right, the value remains same after every period, if the time period is capital T, then you have f of t is same as f of t plus capital T.

So, every or n capital T; that means every you know fundamental period is capital T, so it repeats every time. So, if you know the if you know the function or the signal within this time period, one time period; then you know it for infinity right, you know how it repeats.

So, you have to extend that to 2 dimension. How do we extend it to 2 dimension? Only catch is f of x comma y, this needs to be same question; periodic where right, I have 2 dimension. So, it has to be periodic in both direction. So, I can say f of x comma y is same whether you take f of x plus some period right, you can call this capital X or let us put capital X, just because we used this for the rect function right comma y, this better be same as f of x comma y plus capital Y.

That means, it is periodic both in x direction and y direction with the; here we call it what spatial period if you want to call it right, being capital X in x direction, capital Y in y direction, clear.

So, quickly if you recall, we have any examples that we have covered so far? Well, we had sampling function right, comb function; you already saw there it was at delta we actually used this x before, right.

So, all of those are your periodic function; of course just now whatever we covered sinusoid. Only difference is the here the frequency is 1 over u naught is right your frequency 1 over capital X and sorry 1 over capital X and 1 over capital Y that will be your v naught, the frequency should be inverse of the time period or sorry your spatial period, ok.

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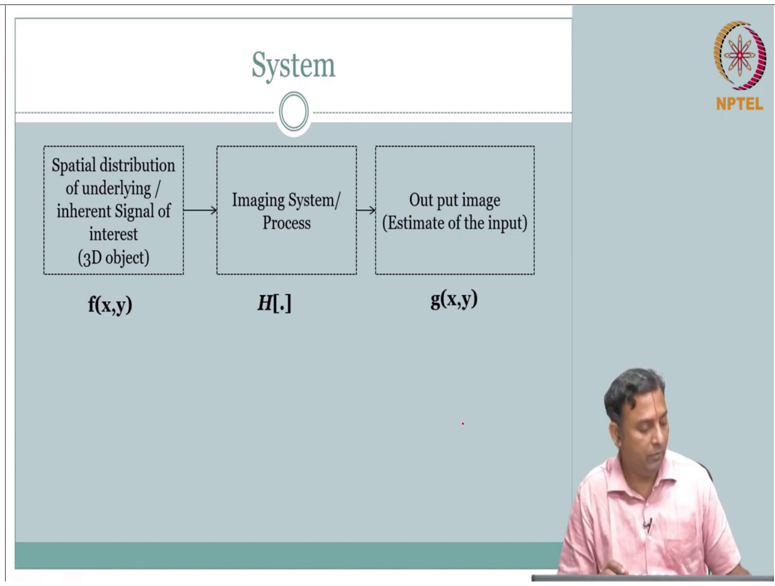

So, I think we will stop here; the next review is going to be on 2 D systems. And we are good to go to individual systems just after that or may be few lectures after that, ok. We will stop here.