Introduction to Biomedical Imaging Systems

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Lecture - 06
System

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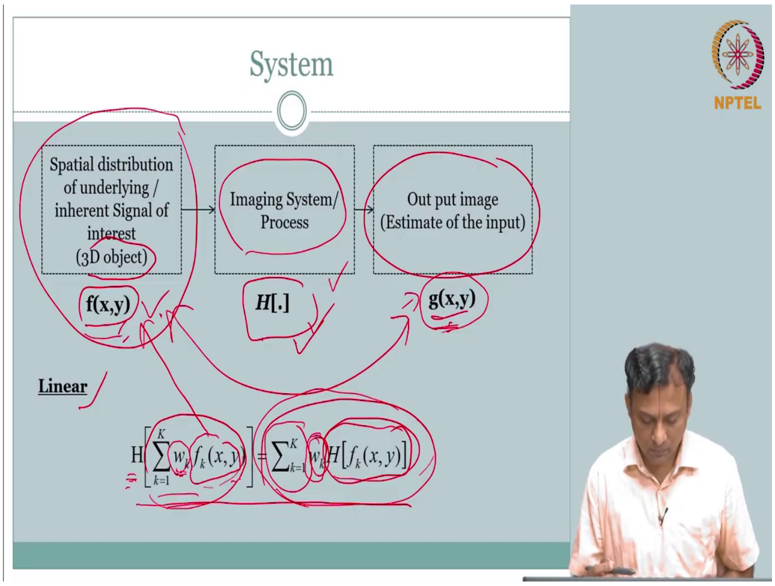

So, after a brief review of signals more specifically 2 D signals and some of the interesting signals that will be routinely used its time now to review systems. Recall remember we are going to take a signals and systems approach. So, with the imaging system we have some input right we have an input which is our 3 D spatial distribution.

So, 3 D object, but its essentially human body, but you have some inherent signals of interest. So, we have a distribution of whatever property as seen by the principle of the imaging system. So, we will denote f of x, y as the underlying input spatial distribution function.

And your system is going to be represented by H which is very typical you would have seen this in you know typical 1 D signals and systems and then output image is going to be represented by g of x, y. I mean I will try to be as clear as possible as we go forward, but the general intention is to use f of x, y for input and g of x, y for output ok.

So, what is the system? I mean quickly to run through what is the system or signal we covered. So, this is signal, this is another signal right. So, what is the system? Well, as you can see here anything that operates on an input at the you know broadest level it is something that operates on the input to give you another signal.

So, it operates on a input signal to give an output signal right. So, in our case it is going to be a spatial distribution at the input and a spatial distribution at the output. The objective of the system is to make sure the output as close as possible to the input or the unknown underlying distribution ok this is fine.

So, essentially what this says is, you can use a system right you can use a system to capture the underlying take the input and transform it into an output which seems nice I mean it kind of connects everything, but it does not really tell you we cannot really mathematically write it and do analysis of the system in this all encompassive manner right.

So, it is too general. So, what we need to do is ok a system is something that operates on an input to generate an output is fine, but can we whittle down to some desirable conditions right or properties of such a system where we could write out the mathematical expression to convey the system operation right what the system does what the system behaves.

So, if you want to characterize analyze the system, how do we write it down mathematically? Is there ways or simple assumptions that we can encounter or sorry apply to make this happen. So, one of the first important property of a system is going to be linear nothing different from what you would have known from 1 D signals in systems.

So, when you say linear linearity what does it mean? Well, think about I mean immediately you may call about some theorems and say how to prove linearity, but what is the big picture what does it try to convey? What does linearity imply physically right? You have H of right this is your system you have system what is this? We really do not know i just written some formulation some representation.

But what do we know here? We know this is f of x, y what is this f of x, y? That is my input. So, if you have to read this what this physically means, it says this is some weight let us call w as a weight. So, you have a function f of x comma y. So, if I give f of x, y to this system H if this H operates on f of x comma y, I gets an output g of x, y. What happens if I give to the system this signal this input what is this input? This means, f of f k of x comma y f of x comma y I know what is this f k of x comma y? f 1 of x comma y f 2 of x comma y.

So, these are different input functions each one has their own weightage. So, weighted sum right this is a summation. So, when you give an input that is not one input, but it is a sum of weighted sum of several inputs. If you pass that through the system, the output right output is going to be what? Output is going to be I have another summation here I have the weights which is same as w k.

So, it is sum of correspondingly weighted this is the interesting deal H of f k of x comma y what is H? H of this guy H of f of x comma y is g of x comma y which was the output to that particular input; that means, H of f k of x comma y means if the input was f 1 of x comma y, the output would be H of f 1 of x comma y would be g 1 of x comma y.

So, essentially this represents the output to individual inputs f 1 of x comma y, f 2 of x comma y. So, physically what does this mean? This means if I provide to the system right if I present to the system a weighted sum of different functions right the output of the system right the output of the system will be sum of correspondingly weighted output to corresponding inputs ok.

So, make it life simple, I mean is very similar in fact, but for the variables use very similar to your 1 D right linearity. So, you would have used a t, but here you are using sum x and y.

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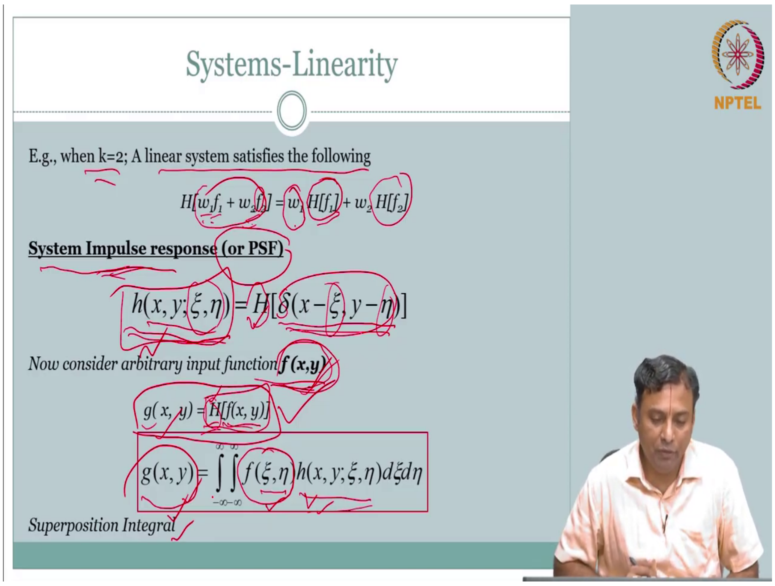

So, just to write it out. So, if k is equal to 2 just you know I have two signals then a linear system satisfies the following that is when I present to the system a combination weighted sum of weighted combination of individual inputs w 1 f 1, w 2 f 2. So, if I present f 1 I know what is the output. The output is H of f 1 if I present f 2 I know what is the output.

Output is H of f 2 if I present a combination weighted combination of this w 1 f 1 plus w 2 f 2 then the output is going to be corresponding outputs weighted by the same weights corresponding weights right w 1 and w 2 clear. So, this is very important property and why is this important? Then essentially we are getting to what happens if we can have a signal seen as a combination of signals.

So, we will treat everything with one input signal and then we can perhaps use this linear system theory and say I can if I can see any input given any signal, if I can break that signal as a weighted combination of multiple signals right then I know how to handle it ok. So, it will become clear as we go forward.

So, the idea that I said this ok if you why is this powerful is there a way we could talk about some arbitrary signal, but before we move into arbitrary signal, the whole idea of reviewing signal before this class is there are some interesting signals one of the first interesting signal that we reviewed was, it is not here it is point impulse ok.

So, we would like to know because we know everything about point impulse we talked about its importance what was the importance, what was the salient feature. Well point impulse you know at you know the delta function remember in 1 D we call it delta function here we call it as point impulse.

Essentially what it did is, we noted that by itself its definition is fine, but it becomes interesting because it applies right it cola cooperates it operates on a signal and then you have some interesting behavior I recall one property we covered right shifting property. So, we will come to that. So, the idea is ok that is an interesting signal.

Now, what happens if you present that signal to the linear system right. So, what happens to the output if my input is a interesting signal that interesting signal is nothing, but a point impulse. As you know from your 1 D when I put delta function as input the what is the output? The output that you get is called as your system impulse response right similar only thing is we are going to call here we call it as point spread function because impulse we talked about point impulse.

So, when it comes through the system the point is spread. So, the system how does it respond to a point? It spreads the point. So, it is the point spread function ok also called as the point spread function. So, how do we write? So, your H of x comma y right this is a arbitrary function right epsilon minus epsilon and eta we will talk about that.

So, what happens this is your H system. So, if I present my system right if I present my system with an input what is this input? Input is a point impulse delta right delta of x comma y is how we defined, but then we made it little more general by saying it can also be moved around right.

So, we present with the arbitrary delta function which has an offset epsilon and eta then the. So, this is the input into the system the output is called your point spread function. So, clearly this is good right. So, if I know about the system if I can talk about the system if I want to analyze the system.

Now, all I need to do is analyze this H of the behavior of H of x comma y comma zeta and eta that will tell me about the system, but not so, desirable thing is it is a variable there are four variables there. So, it becomes common it is a four dimensional signal, H becomes a four dimensional signal four dimensional signal is good mathematically it might be good I mean.

So, its good to work with it, but when you talk about practicality, it might be important for us to understand what the physical meaning of this is. Can we reduce it to a more tractable form yet not lose the generality not lose the underlying see mathematics always tries to capture the underlying physical meaning right.

So, we should not lose that part, but at the same time we have to make some conditions or look for certain assumptions, look for certain ways to reduce the mathematical intractability ok. I mean, but it is doable right, but its not practical. So, we will try to see if there is any simpler way that we can invoke certain conditions first thing that comes to.

So, before we do that let us find let us just connect the two concepts. So, now, if you have an arbitrary input function f of x comma y right. So, if you have a arbitrary input function this we know right. The first slide that we started then system review we said if you present it with f of x comma y present what the system.

So, it operates H operates on the f of x comma y to give an output g of x comma y. So, for an arbitrary input I know my output ok. So, now, the question is can I write an output if this is my input right if I am presented with the input that we see right. So, how do you the interesting thing is this. If you present a arbitrary signal f of x comma y I get g of x comma y operating through the system.

So, that is good right, but I cannot simplify that without knowing anything further about the H, but now I know something about the H what is that H something that about H I know? That is the system impulse response. So, I know if I present an impulse to the system right if I present an impulse to the system I know my impulse response.

So, now can we think about presenting the system with impulse multiply? So, the arbitrary signal right f of x comma y. I can actually see that arbitrary function as a collection of impulse function at different locations this is what we saw through shifting property right. When you look at go back and when you review the shifting property you will recognize that any arbitrary signal f of x comma y can be seen as a collection of right.

You are using this delta function go look we can use that to pick points on the surface right from f of x comma y. So, that is at if you are picking it from epsilon comma eta then the value of f is picked at that location right that is what your shifting property we covered that is what we covered for your point impulse.

So, here essentially what we do is, I know something about this H which is the system impulse response how does it behave to the impulse? So, now, you see how cleverly an arbitrary signal f of x comma y can be seen as nothing more than a collection of right different locations you have the impulse you are picking of that value and you have the double integral your summation.

So, your g of x, y this is your output to a arbitrary input f of x, y can be connected through the behavior of system impulse response ok. So, let us see. So, this is good, but not so, good. The good news is now it seems like there is a mathematical form to connect the input and output. In fact, any arbitrary f of x, y can be operated upon if I know my H of x comma y comma zeta comma i.

If I know this that is in other words if I know the system impulse, then I could generate or predict any output ok. So, that is good, but what is not so, good is in order to evaluate this double integral you have to know everything about this H which may or may not be possible all the time. At least knowing H all the four dimensions may not be possible all the time.

So, what do we do? Of course, if you should have probably recognized this is a equivalent. So, this is your super position integral ok. Super position integral connects your output to input and system function in the form of right integral. So, this super position theorem you would have studied right for linearity.

So, this is just a extension of it, but it gets little complicated because the dimensionality is increasing otherwise mathematically is a straightforward extension. So, now, what we will do?

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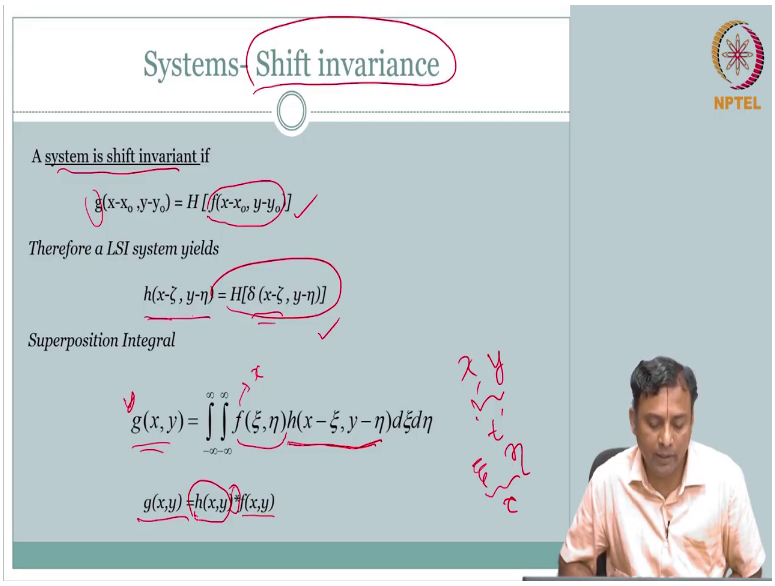

We will essentially proceed forward and try to see if we can address this issue. We have four dimensions for H can we reduce the dimension is there any way to make it little more mathematically tractable? So, as we can see here we will invoke this idea of shift invariance. So, if you are wondering you know I have done linear the in one dimension I have done it.

So, I know linearity is similar what is the shift invariance? Probably, I have not heard shift invariance access, but you probably would recognize time invariant because most of like I said most of the time in one dimension we talk with the time as your independent variable. So, you would have heard something called linear time invariant system right that makes like life lot easier for you in 1 D.

Similar thing here we have a equivalent which is shift invariance so; that means, what would it try to do? I have my impulse instead of time invariance means you are moving along the time the offsets in time right. Here there should be shift in position spatial shift whether your H is sensitive to where you shift or not is the idea here. So, it turns out if the system is shift invariant if it is shift in variant, then this is what happens H of. So, you have f of x comma y. If you have f of x comma y the output is g of x comma y.

If I present f of x minus x naught y minus y naught what does that mean? I have an input, but now I am moving the input right I am shifting it to the input space and then I have a system. So, what comes out of the system? The output. So, the output shift by the same amount ok. So, if this is my input right and I am moving.

So, if you shift my input, I present it through the system my output moves exactly the same way right exactly the same way that is what is your shift invariance in other words it does not matter how you arrived there. If you presented with that input f of x comma y you get the output g of x comma y.

If you shift the input that is you shift it in x naught and y naught your output of your f of x comma y we will also move by x naught and y naught. So, why is this important? Now suddenly you see we had four dimensions right the H can be no brought down to only two dimension. Now, I am not worried about shifts I am not worried about the offsets I am only worried about x and y right where it is and how it integrates with the signal that is what is going to determine my output ok.

So, therefore, LSI system right yields H of x minus eta y of sorry y minus eta as H of delta of the shift. So, if I present a shifted input right delta functions output is shifted version of the system function this is very powerful right. So, using the shift invariance if it is a LSI system, then the shifting of your delta function or in point impulse just shifts your system impulse response ok.

So, now looking back to a super position integral right we just saw that and it there was a H with 4 dimensions. So, let us see what happens. So, your g of x comma y which is your output is becoming f of zeta comma eta times this guy. So, now you notice suddenly it becomes little more tractable right. So, I can get my g of x comma y similar concept g of x comma y I have you know f of zeta comma eta in terms of my H right.

So, system connects input and output are related by the system function only thing is it is now more compliant system, it is not only linear it is also shift invariant ok. So, this you can you will I mean this is this looks complicated because of the variables are kind of new to you, but if I tell you just quick example if you replace this x and y right or imagine this x and y is your t right and your zeta and eta to be your tau.

Now you kind of course, you can even simplify it little further because of the denotions you can start call this y as g as y and input as x right then you get and of course, you will not have 2 integrals you will have d tau right.

x of t H of t minus tau, d tau integral you get y of t this is your convolution only thing is its variables are different and you have one more dimension. So, it looks new, but otherwise this is straightforward extension ok. So, what is this? That means, I expect that you know what; that means, that is your convolution.

So, output is nothing, but your input system response convolved with your input that is your output clear. Exactly same or equivalent to your one dimension that you probably are very familiar its an extension ok. So, nothing more to (Refer Time: 25:02) there.

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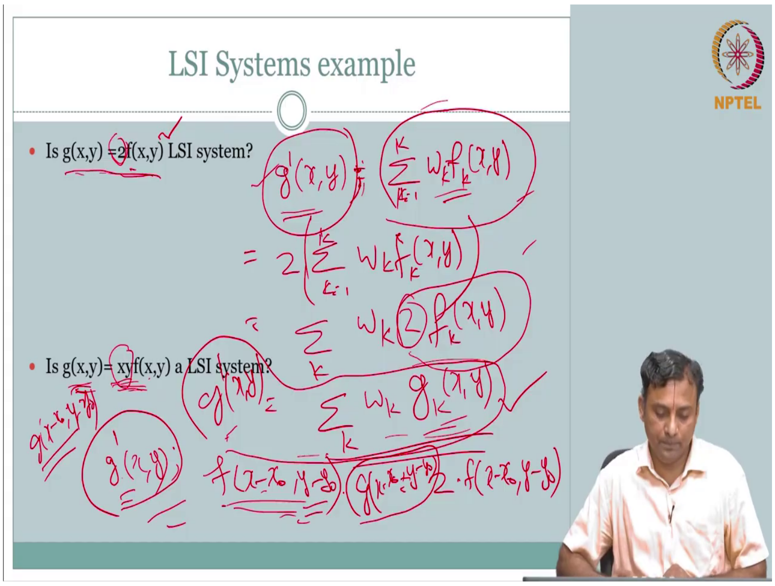

Let us move on let us get some examples done right. There will be homework or assignments where you will be able to work on some of this again level here is going to be just a review. So, I expect that you go through the textbooks and you know signals and system textbooks, you may have lot of 1 D brush up your familiarity with 1 D so, that this becomes easier.

So, now, the question is this linear shift in variant system? The deliberate reason I picked this is because most of you when you look at this you may probably pounce and say I know the answer I mean you know eye eyeball I know what it is. But the challenge in signals and systems especially now that it is now 2 D is going to be when you try to solve something, then you will get lot of confusion unless you are systematic in writing right.

If you had while you write you are able to see the physical connection only then there is a then it is a real material that you really have learnt it. If your eyeball and you feel looking at it I know right what will happen.

Then probably if it gets to a difficult see I am deliberately taking some easy examples here, but in real world you know its going to be lot more challenging. So, if you have to bother with real signals little higher signals that are little more complicated its better you develop good habits. So, write it out right.

So, when you have like this what do you do? First thing is what is what is linear system linear shift invariant system. So, what is linear system? Linear system says if I am presented with arbitrary input right when not arbitrary sum of different individual inputs. So, if I have f 1 i get g 1, if i get f 2 i get g 2 if I am submit presented with summation weighted sum of right.

So, let me I think it will be good if you can write as we speak right. So, if I say for example, just for clarity, I do not want to call it g of x, y because we want the check whether g of x, y is linear system or not. So, if I say g dashed of x comma y is the output that you get when you present it with a combination weighted combination of input signals right.

So, let us say the let us say you have a weighted combination let us say use the same variable k 1 to some capital K. So, this you want to do with f of x k f this thing right it is exactly from your previous slide. So, I am calling this right the output to this is this guy rather what should I say, let us use the same variable I mean I do not have H explicitly.

So, let us just that is not equal to. So, if this is presented to the system right this is the output ok you see the carefulness here because I did not have H here. So, I have given the system. So, H is hidden there ok. So, this is the case. So, let us see what is this a linear system or not what happens if I do this? Let us see what happens if I present this as the input I am given the system here right.

So, it is going to be this is my input. So, the output is going to be 2 times this guy k w k f k of x comma y of course, if this is the case how do I; I mean I could write it little more clearly as 2 times the input just to recognize that this is f this is x the 2 is probably to do with the system. So, let us just put a bracket right.

So, this immediately you can recognize ok I can do this k w k right I can get the two inside 2 f k of x comma y. So, you can eyeball now of course, let us write the next step and say if your eyeball now what happens? This appears to be very recognizable what is that? 2 times f of x, y is g x y.

So, 2 times f k of x, y will be or can be recognized as g k of x comma y clear. So, now, what does this say? This is my g dashed of x comma y. So, when you read this out what in what comes through? My new output right when presented with a when presented with the weighted combination of inputs right. My new output is nothing, but correspondingly weighted sum of individual outputs to the outputs to the individual inputs.

Wow is not that what we defined as linear so; that means, it says g of x equal to 2 of 2 times f of x comma y is a linear system because it we just read it out exact same physical interpretation of this of this relationship, you know amounts to the definition for linearity.

So, next is. So, is this ok linear is fine is this shift invariant what do you do for shift invariant? I need to shift the input. So, let us say if I present the input instead of f of x comma y right I present my g of x comma y you get the output when I present instead of f comma y x minus x naught comma y minus y naught.

If I present to the system right shifted version of the input what is the output? That is your g dashed of x y right. So, then what happens? If I present this then my right you look at this look at here if I present this as an input my output is going to be 2 times if I 2 times f of x minus x naught comma y minus y naught going to be g of x minus x naught comma y minus y naught.

That is g dashed of x comma y is nothing, but shifted version right both are same. So, essentially this is shift invariant as well clear. So, let us do one more just we will not do it one more time again now we can eyeball, but then we can eyeball with some now steps written already.

So, g of x comma y is equal to x, y of f of x comma y is this a linear shift invariant system? So, looking at it you can quickly see the similarity between the this problem and the previous problem. If you recognize x, y to be 2 right pretty much you can follow the same steps.

You can say let us call g dashed of x comma y as the new output when presented with the system when the system is presented with a collection of inputs sum of collection of inputs right weighted sum of inputs. So, you can pretty much see that you can carry forward this only thing is instead of this 2 here you will have when you pass it through the system you will have output as x comma y right.

But you will recognize that if this is x comma y it does not alter this equation right. So, you will pretty much find g dashed of x comma y will be summation of w k g k of x comma y which says that it is linear system. Is this a shift invariant what do you do for shift invariance? Same thing right.

I have to shift the input what happens to the output? Is that same as if I shift the output of the individual output. So, for example, if I do shift of input right f of x comma x minus x naught f of y minus y naught if that is my input, let us say I get g dashed of x comma y right this is my output.

But if I look at here what is my shifted output? g of x minus x naught comma y minus y naught what is that? That is going to be x minus x naught into y minus y naught times f of x minus x naught y minus y naught. Clearly the x minus x naught and y minus y naught makes it unequal. So, the if you shift to the output that is not same as because you get x minus x naught into y minus y naught term which was only x into y if you consider presenting delayed input.

So, clearly it does not shift invariant. So, what the message is? Linearity shift invariance two or two concepts if you have linearity that does not mean it implies shift invariance or vice versa clear. So, that is something that is very powerful. So, we will end up using linear shift in variant system for our imaging system.

I mean I think the message should be very clear that it is not just because we can solve all this and get some easy mathematical traction, it turns out that the simplification does not really take away the ability to model right a general imaging system.

At least most of the imaging systems that we are going to cover here for the most common usage I think linear systems theory is very very much you know it physically also very representative right. We are not just doing things for mathematical traction, but it does not you know hold good in practice.

I think this is powerful yet powerful for the reason that it makes life simple yet it does not really it captures vast majority of the imaging systems I mean in in our case almost all of the imaging system that we are going to cover here introduce at least the most common imaging modes that are available you will be able to get through this ok.

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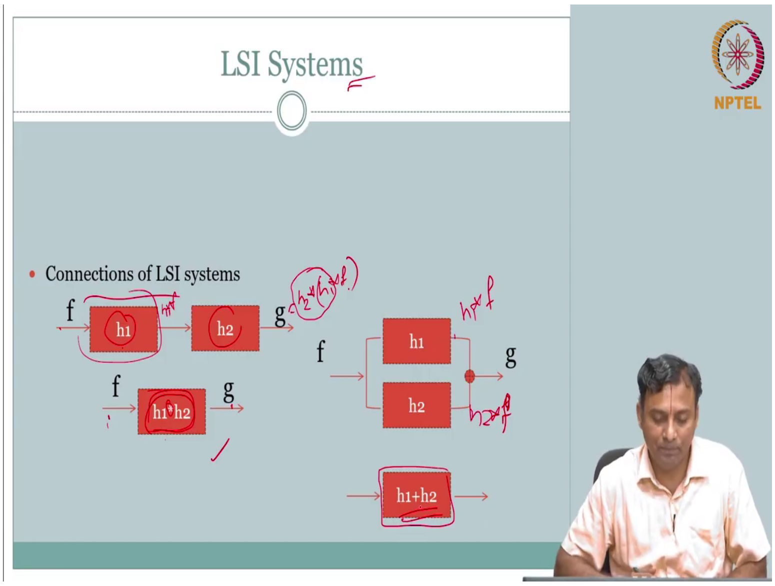

So, let us just quickly run through few additional important aspect. So, we covered else linear shift invariant system. So, now, the question is do we just have one system what happens if you have more than one system? And usually this happens right you have subsystems.

So, it depends on how you define input what is output and what is the system that is. So, for example, in this case if I am the input right I am say let us take a optical image the camera that we have. So, I am the source right and then there is a camera system that is recording, then there is a audio system that is recording. All of this is you know compiled together and that multimedia is streamed.

So, that is one system right, but then that multimedia is streamed using some internet and then you have you are going to log in and probably download the video and using your say for example, mobile phone or a laptop or PC or whatever that is going to read the data right and it is going to display it and you are going to have some speaker or ear phone or right.

So, you are going to listen and see. So, if you really look at it input is this 3 D distribution, output which you are looking right is a close up or better be a close representation of the input and then what consider a system here? Everything that went in between the input and the output the two representation and the image that you see of me right that is your output, this is 3 D is the input everything in between is actually one system as per what we have done right.

But then you see the problem it is good I mean its it its very general we are able to capture that. I know what the system is system consists of the camera and the microphone that is there in the recording room and then the PC that is used to process the data and then the internet and then your PC or your mobile phone that sub system.

In fact, you then is the speaker output or you are connecting it to some other home theatre probably for this you will not connect to home theatre, but I am just saying the idea is you have another. So, the all these are even though we said that it is one block system you can actually see this as a series of sub system.

So, you can have multiple systems right its about how you want to analyze right. So, you are going to encounter if I know how to characterize one system that is why we talked about linear in invariant shift invariant system, can we look at combinations of system because there is going to be subsystems right.

So, what are the ways we can look at combinations of system? So, what we call as connections of linear shift invariant systems right how can you connect? So, let us take for example, two systems just so, that we can eyeball and make pat our self and say I understand it right.

But its basically connection of multiple systems, but what are the ways you think you can connect two systems either you can connect through series right two systems in series or we can connect two systems in parallel right. So; that means, when I say system since this is shift invariant linear shift invariant systems each one all I need to know is if I know the H 1 I say that is a system I know if I know H 2 that is my system right.

So, now what is the if you connect them in series as you can see here, it is equivalent to having only one system whose system function is nothing, but that combines H 1 and H 2 what how does it combine? This operator what is that operator? Convolution.

So, if I have right if I have an input f the output of this LSI is going to be H 1 convolved with f and that output right H 1 convolved with f goes into the input what will be the output g? g is going to be H 2 convolved with whatever went in what went in? H 1 convolved with f that is the 1 that went in right.

So, you can then see; that means, my output and input output and input are connected through this system which is H 1 convolved with H 2 right. All these properties of commutative associative you know right I can do H 1 convolution the convolution properties H 1 convolved with H 2 or H 2 convolved with H 1 right.

All this you have to review, but otherwise you are getting one system equivalent that equivalent the system response is nothing, but convolution of the two subsystems this is in series similar logic you can extend if you have H 1 and H 2 right. So, what will be the output here? This will be H 1 convolved with f this will be H 2 convolved with f. So, g is going to be nothing, but sum of these two right.

So, if you have in parallel; that means, you can reduce that to a single equivalent system whose impulse response or the system function is nothing, but sum of the individual ones clear. So, and these things will come in very handy like I said physically when I talked about the example of you know recording and you seeing the video similar thing happens in the imaging system you will have several modules of the imaging system data acquisition, signal processing, display right all of this is a imaging system we say.

So, there are several sub modules and you will benefit from ability to recognize that you can start to look at a complex system as a connection of subsystems and therefore, you know how to analyze it.

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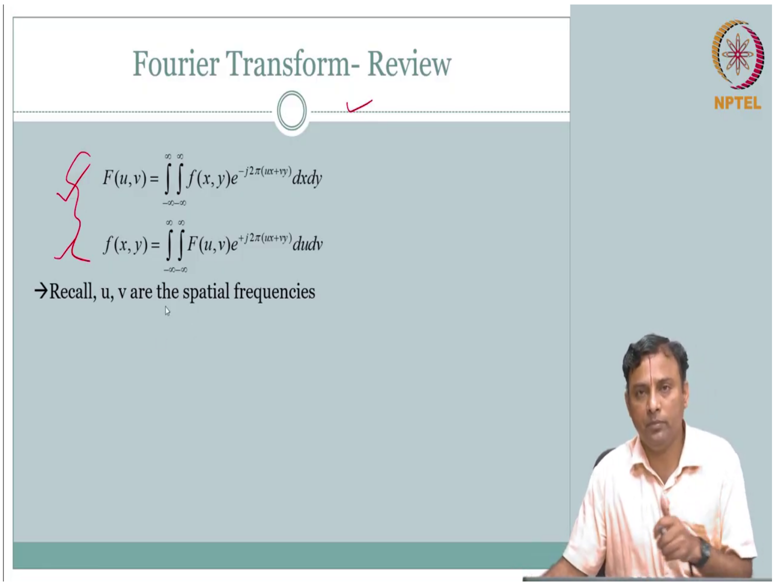

So, we will stop right here. We will start with the Fourier transform. Fourier transform is a very powerful concept whatever we did so, far we did in spatial domain right. So, we started to look at signal 2 D signal as nothing, but a spatial representation physical meaning was spatial representation of a pattern now can we analyze do the system analysis instead of convolution can we have another aspect another tool set that can be invoked right which brings us to Fourier transform ok.

So, looking at this you probably recognize it immediately, this is your Fourier transform and this is your inverse Fourier transform right. So, let us stop here let us stop here I will actually start from Fourier transform in the next lecture.