Thermodynamics (Classical) for Biological Systems

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Module No. #05

Phase Equilibria

Lecture No. #32

Liquid/Liquid and Solid/Liquid Equilibria

Welcome!

In the last class, we had looked at vapour-liquid equilibrium as a special case of equilibrium; and we had worked out the special condition for vapour-liquid equilibrium based on our original conditions of equilibrium – the five equations that we had seen very early in this module. And we had also worked out an example problem to better understand how the equilibrium comes about, and how we can use the various parameters that we have developed or defined to come about with useful relationships at equilibrium, in the case of vapour-liquid equilibrium.

What we are going to do is in this class, essentially, is to develop the governing equations for liquid-liquid and solid-liquid equilibria. There could be various different kinds of equilibria. We are just going to take these two, and develop the governing equations, because these occur quite often in the case of biological systems, especially bio-process related activities as well as some activities that deal with analysis of biological molecules and so on.

And what we are going to do, how we are going to do develop these governing equations is as exercises, which means you are going to develop it. That way, it builds up your confidence of addressing and developing equations in hitherto unknown situations. And, a complete appreciation of the principles that are involved also comes about as a part of doing such exercises. So, we will do these as exercises. I am going to start you out, then I will give time, and of course, I will present the equations, the governing equations, the way I have done it. We can cross check after sometime.

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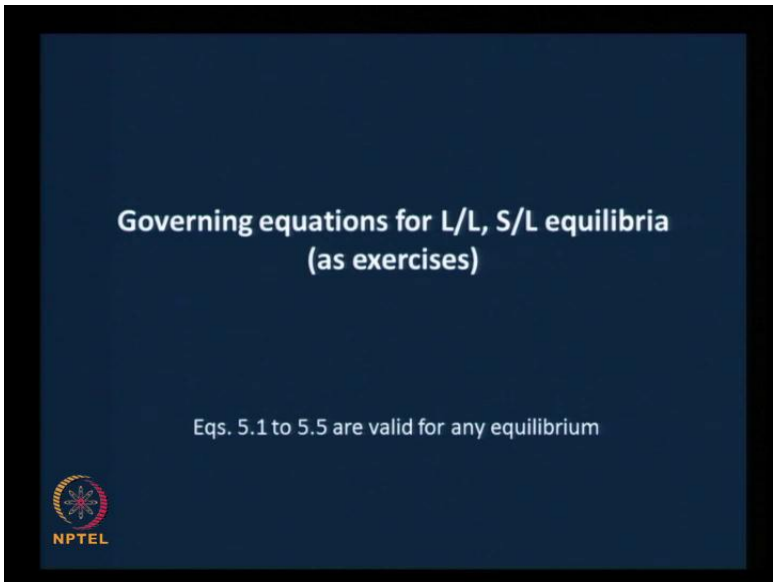

What I would like you to remember, when you are developing these governing equations for liquid-liquid and solid-liquid equilibria … is the fact that equations 5.1 to 5.5 and further – you know we had stopped at 5.5. Essentially equation 5.1 is T alpha equals T beta equals T gamma, the sameness of temperature or the equality of temperature across the various phases at equilibrium. The equation 5.2 was the equation of pressures or equality of pressures across the various phases, P alpha, P beta, P gamma, and so on.

Then equations 5.3 to 5.5, we had written for three components; one, two and three. What we had written there was that the chemical potential of each component was equal across the various phases, in which it is present, of course. We are assuming that all the components are present in all the phases. Therefore mu 1 alpha equals mu 1 beta equals mu 1 gamma, and so on was equation 5.3. mu 2 alpha equals mu 2 beta equals mu 2 gamma, equation 5.4. mu 3 alpha equals mu 3 beta equals mu 3 gamma, and so on was equation 5.5. These are the 5 fundamental equations that are necessary for the thermodynamic equilibrium in any system. Need to keep this in mind, and we need to develop it.

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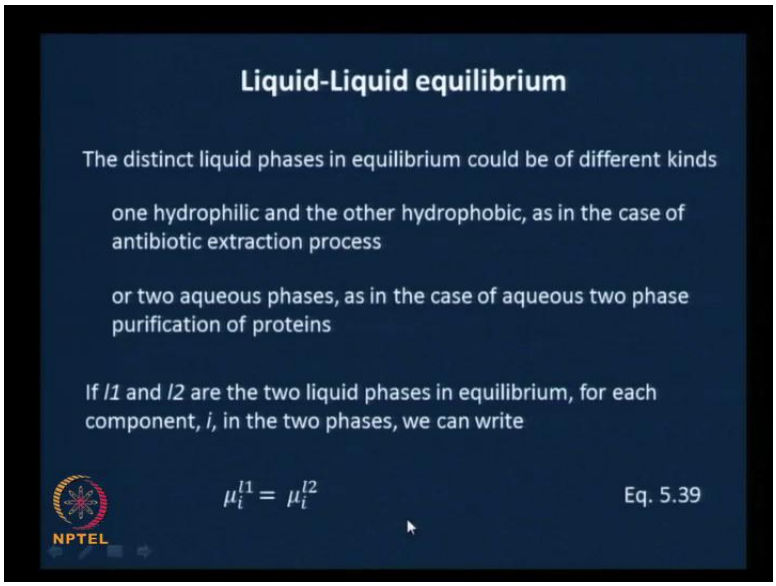

Let me start you out for the case of liquid-liquid equilibrium, which means there are two liquid phases … distinctly separate. … They could be of many different kinds. One could be a hydrophilic phase, the other could be hydrophobic phase. It is easy to imagine a hydrophilic phase – hydrophobic phase; let us say, a mixture of oil and water. They are not going to mix with each other. You can clearly demarcate these two phases. One such system may be toluene-water system and so on, so forth, is actually used in the anti-biotic extraction process. One hydrophilic phase, which is the water phase, in which the anti-biotic is present after it is produced by the organisms, after the bio-reactor phase. And, what is added may be hydrophobic liquid is added to extract most of the anti-biotic into the hydrophobic phase; because, at equilibrium, the concentration of the anti-biotic in the hydrophobic phase is many fold the concentration of the anti-biotic in the hydrophilic or the water phase. That is the basic reason for using this two liquid process for extraction and purification of certain bio- molecules such as anti-biotics. So, this is one example of a liquid-liquid … or of the occurrence of two liquid phases. … Equilibrium conditions are the limiting conditions. More interestingly, they could be even two aqueous phases, two water based phases, but which are distinct phases … one sits on the top of the other.

An example is different mixtures of let us say polyethylene glycol and ammonium sulphate salt. If we mix them together then depending on the composition they could form two different, distinct phases. In fact such two aqueous phase system, or aqueous

two phase systems as they are called, are actually used in the industry for extraction and purification of proteins. Why do we need to do that?

You know that proteins get their activity because of the way the amino acid chain is folded. And the way the amino acid chain is folded … has relevance only in an aqueous system, because, if you recall from your early classes, may be an initial life science course or molecular biology course, biochemistry course for that matter … that a protein folds itself according to the forces between them as well as the forces between the various constituents of the proteins, their amino acids, their side chains and so on, and the environment that it is present in.

If the environment that it is present in is water, it forms a certain conformation which is active. You put the same protein molecule in a non-aqueous phase then the forces are completely different. And the way it gets folded may result in a molecule that is not active at all; because, it is only the three dimensional folding of the protein molecule that gives that its activity. Therefore, what is done is, two aqueous phases are used. And that is based on the fact that the solubility of protein in one of the aqueous phases is higher may not be 10 or 100 fold higher but may be about a few fold … 3 to 5 fold higher than the other phase. And this was used to concentrate and purify the protein molecules.

Therefore, liquid-liquid systems are used quite extensively in biological systems, and we are going to look at … liquid-liquid systems, in general. What I would like you to do is take about 20 minutes from now; because you need to think about various things and so on. And the question is this, if l 1 and l 2 are the two liquid phases in equilibrium, for each component i in the two phases, develop the expression that can be written. It is as simple as that. To give you a hint, the first five equations, equations 5.1 to 5.5 are valid. Also take a look at how we went about doing pretty much the same thing, but for a component i, that is distributing between the vapour and liquid phases in the case of vapour-liquid equilibrium. Those are the hints that I am going to leave you with. Take about 20 minutes, and then we will look at what the solution is.

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So, you would have found it easy to write the condition for equilibrium which is nothing but the equality of chemical potentials. We are of course, going to take the conditions of temperature equality and pressure equality as a given. So we do not have to write them

explicitly here as mentioned some time earlier; that is considered as a given thing. Here mu i in the l 1 phase must equal mu i in the l 2 phase; i is the component which is distributing between the two liquid phases l 1 and l 2. As mentioned earlier l 1 could be a hydrophilic phase and l 2 could be a hydrophobic phase. Or l 1 could be one aqueous phase and l 2 could be another aqueous phase, as in the case of aqueous two phase extraction. Let us call this equation 5.39.

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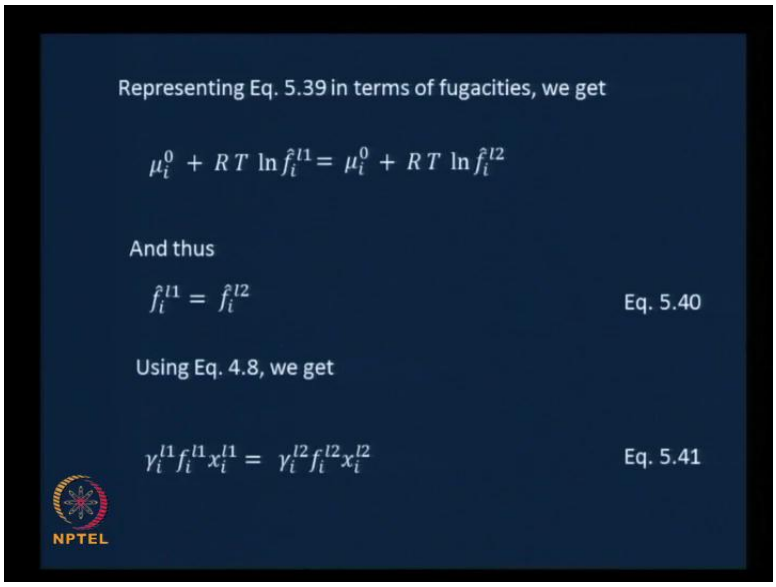

Now, if we expand equation 5 39 in terms of fugacities, we can write mu i naught plus R T ln f i in l 1, hat. Hope you recall this. f i hat equals mu i naught plus R T ln f i in l 2, hat. This is the fugacity of component i in l 1 in a multi-component system. And similarly, this f i l 2 hat is the fugacity of component I, in l 2, in a multi-component system. Therefore, it is quite easy to see … mu i naught is going to cancel with mu i naught; R T cancels with this side … this side. And then ln f i l 1 hat equals ln f i l 2 hat. And therefore, f i l 1 hat equals f i l 2 hat.

Now if you go back and look at expressions for this, you could write … which is essentially equation 4.8 in the previous module. You could express f i l 1 hat in terms of the pure component fugacity, which is easier to estimate. This is written as mu i l 1 f i l 1 x i l 1; where x i is the mole fraction and gamma … this is not mu this is gamma … gamma i l 1 f i l 1 x i l 1; gamma i is the activity coefficient, x i is the mole fraction and f

i is the pure component fugacity in the l 1 phase must equal gamma i l 2 f i l 2 x i l 2. This becomes the condition for equilibrium. Let us call this equation 5.41.

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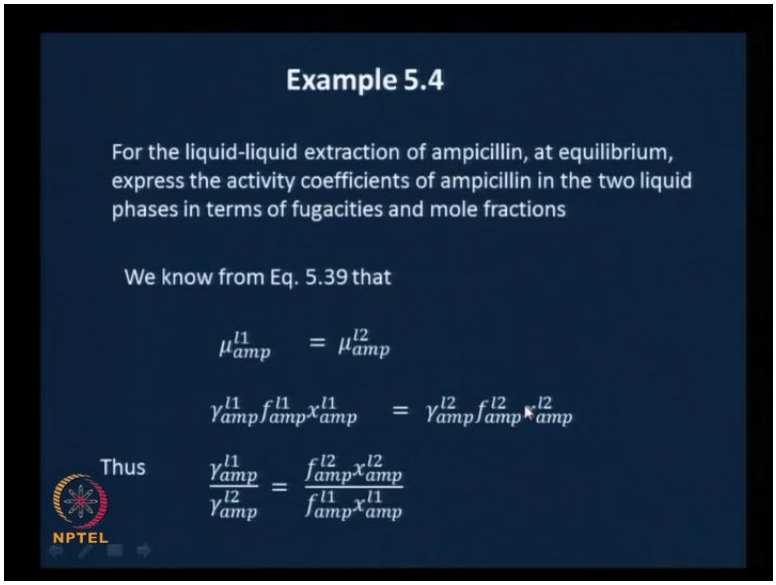

To get a little more comfortable with this, let us work out an example. For the liquid-liquid extraction of ampicillin, at equilibrium, express the ratio of the activity coefficients of ampicillin – that ratio is missing here – the ratio of activity coefficients of the ampicillin in the two liquid phases in terms of fugacities and mole fractions. Take about 15 minutes. Go ahead and do this please.

This is on the same lines as the derivation. We could directly use the final relationship that we got just a while ago. We know, if we employ equation 5.39 directly, … see, this is the case of liquid-liquid equilibrium and ampicillin is the component i that is distributing between the two liquid phases. We know that from equation 5.39 that mu of ampicillin in the l 1 phase must equal mu of the ampicillin in the l 2 phase. And writing this in terms of gamma i or bringing this down to fugacities, and then writing it; either way it is ok. gamma of ampicillin in the l 1 phase times fugacity of pure component ampicillin and the mole fraction of ampicillin in the l 1 phase must equal gamma of ampicillin in the l 2 phase fugacity of ampicillin in the l 2 phase and the mole fraction of ampicillin in the l 2 phase. And therefore, what is required is the ratio of the activity coefficients of ampicillin, gamma ampicillin in the l 1 phase divided by gamma of ampicillin in the l 2 phase, is nothing but the product of fugacity and mole fraction in the

l 2 phase divided by the product of fugacity and mole fraction of ampicillin in the l 1 phase. This is the equation that we were looking for, or the relationship that we are looking for in this example.

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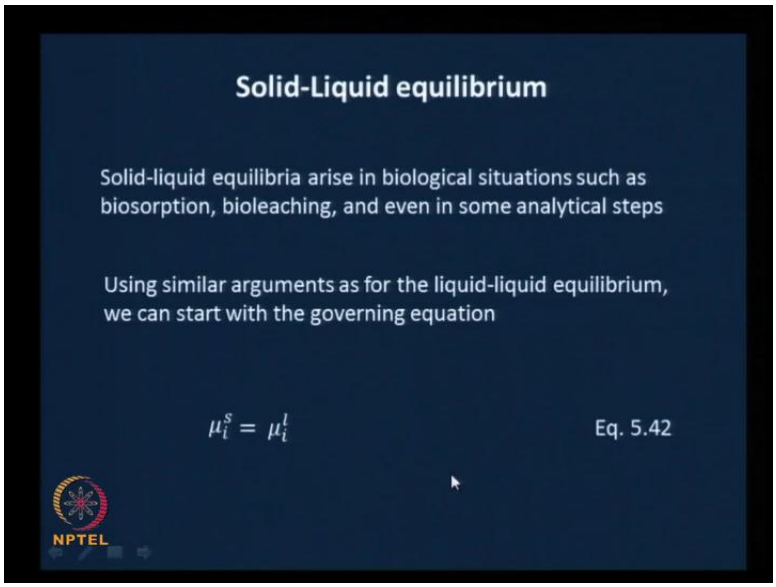

For completeness, I would … let me actually present this since we are almost out of time in this particular lecture. Let me just present this. It is quite straight forward. It is an example of solid-liquid equilibrium. There are many different examples of solid-liquid equilibrium in biological systems. They arise in situations such as biosorption which is the removal of, let us say, toxic trace metals using a biological agents such as cells and so on. Bioleaching which is taking the metals or useful components out by using biological means, and even in some analytical steps you come across solid-liquid equilibrium in biological systems. … We know that equations 5.1 to 5.5 are valid. Therefore, using similar arguments as for the liquid-liquid equilibrium, we can start with the governing equation that mu i in the solid phase must equal mu i in the liquid phase; equation 5.42.

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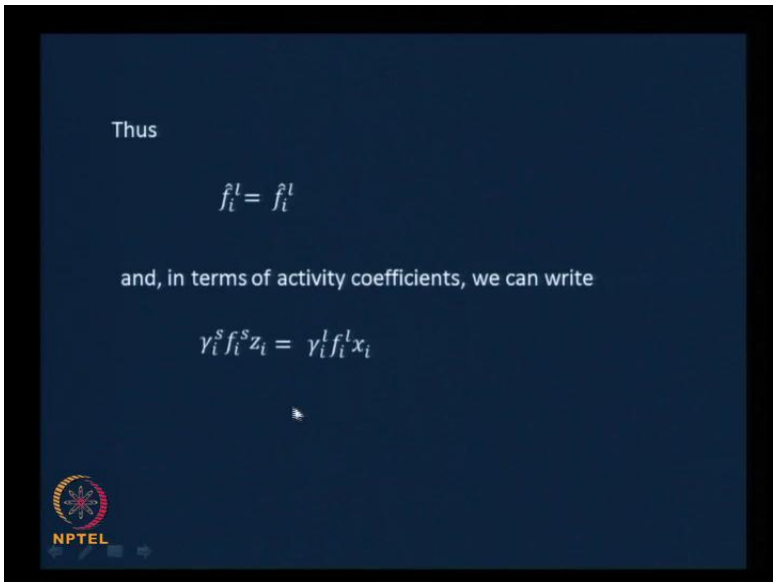

And therefore, the fugacity of i in the liquid phase … this is solid phase … must equal the fugacity of i in the liquid phase. … In terms of activity coefficient, the gamma i in the solid phase times f i in the solid phase times the mole fraction in the solid phase, given as z i, equals gamma i in the liquid phase times fugacity coefficient in the liquid phase times mole fraction in the liquid phase. This is the governing equation for solid-liquid equilibrium. When we come back in the next class, we will review whatever we have done in this particular module on phase equilibrium. See you then.