

**Material and Energy Balances**  
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**Module No # 01**

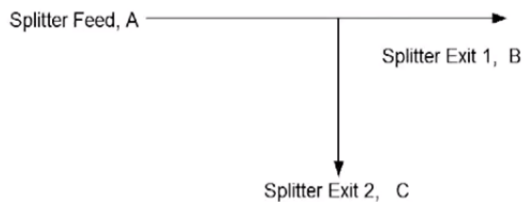
**Lecture No # 04**

**Material Balance Calculations for Single Units Without Reactions - Part 1**

Hello everybody we have now looked at all the fundamental that are required for performing material balances. So now in today's class we will talk about performing these material balance calculations for single unit system which do not have the reactions. These are the simplest possible system there are many types for processes which are actually just single units and non-reactors. We will go through some examples of these processes and try and solve problem related to these processes.

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## Divider/splitter



- Total balance :  $A = B + C$
- **Important:** Composition of streams A, B and C are same
- Only one independent balance equation (total)

The first and simplest of system is a divider or a splitter the flow chart shows what a divider or splitter would be you have one splitter feed which is then divided into multiple splits you can write a total mass balance which is A which is splitter feed equals B + C which are the splitter exits you need to understand that composition of stream A, B and C are the same. So a simple example would be consider a 10 grams per liter solution of NaCl flowing through a pipe.

If you were to divide this one stream into two or more streams the concentration of sodium chloride is not going to change it is only going to be the flow rates that are going to be divided.

So the concentration of all these three streams will remain the same because of this only one independent balance equation can be written which is the total balance equation. If you were to write the component balance equations they would be dependent on the total balance as all the concentration will be the same and get cancelled off.

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## Material Balance – Splitter

- A stream of growth medium containing 60% glucose and 40% growth factor A flowing at 500.0 kg per hour was divided into two feed streams for different cell cultures. One fourth of the growth medium was sent to fermenter 1 and the rest was sent to fermenter 2. What is the flow rates of glucose and growth factor A in the feed streams for the two fermenters?

$$\begin{aligned}
 60\% \text{ Glucose} &\Rightarrow \text{Stream 1: } 0.6 \times F_1 = 75 \text{ kg/h} \\
 &\text{Stream 2: } 0.6 \times F_2 = 225 \text{ kg/h} \\
 40\% \text{ GFA} &\Rightarrow \text{Stream 1: } 0.4 \times F_1 = 50 \text{ kg/h} \\
 &\text{Stream 2: } 0.4 \times F_2 = 150 \text{ kg/h}
 \end{aligned}$$

Now let us try and solve the simple example problem which illustrates this here is the example problem which illustrates what we just learnt about the splitter. A stream of growth medium containing 60% glucose and 40% growth factor A is flowing at 500 grams per hour and is being divided into two feeds streams for different cell cultures. One fourth of the growth medium was sent to fermenter 1 and the rest was sent to fermenter 2.

You have been asked to calculate the flow rates of glucose and growth factor A in feed streams for the two fermenter. As I was mentioned earlier the first step was solving any material balance problem is to identify a convenient basis. When we read the problem statement we find that 500 kilo grams per hour of feed is entering into the system this information has been provided to you. So you would use this as a basis so we start with basis this is 500 kilograms per hour of feed.

Now the next is to draw of flow chart which would clearly represent the process so you have one feed stream which is flowing at a rate of 500 kilogram per hour and contains 60% glucose and 40% growth factor A. So this is divided into two streams which we call as the splitter exists so

the exist we will say there has a flow rate of  $F_1$  kilograms per hour and second one can have a flow rate of  $F_2$  kilogram per hour.

The problem statements says that the feed entering into the splitter has been dividing into two streams and one fourth of growth medium goes to the first stream and the rest goes to the second stream which means three fourth of the feed entering would go into the second stream indicating that the value for  $F_2$  would be equal to  $3 F_1$ . So instead of writing  $F_2$  we would be better of writing  $3F_1$  which incorporates the correlation between  $F_1$  and  $F_2$ .

As we already know the splitter make use that the flow is split into multiple streams but it does not affect the concentration of substances present in the stream. This mean these two exit stream would also contain 60% glucose and 40% growth factor so the concentrations are the composition for the two streams are 60% glucose and 40% growth factor A the same goes for the second stream also.

Now to solve for  $F_1$  and  $F_2$  or we would have to write the total mass balance equation so the total balance equation would be you go back to the fundamental equation which is accumulation equals input – output + generation + consumption in this equation the system is at steady state which means accumulation term goes to 0 and you have a input term you have a output term there are no reactions happening which means the generations consumptions terms also go to 0.

So the equation can be simplified as input equals output when we write this equation using the flow rates given we would simplify it as feed which is 500 kilograms per hour is equal to  $F_1 + 3F_1$ . So this gives you a flow rates as  $F_1 = 125$  kilograms per hour and  $F_2 = 3F_1 = 375$  kilograms per hour. Now that you have calculated the total flow rates of two exit stream you have to go about and look whether that is all that was asked in question.

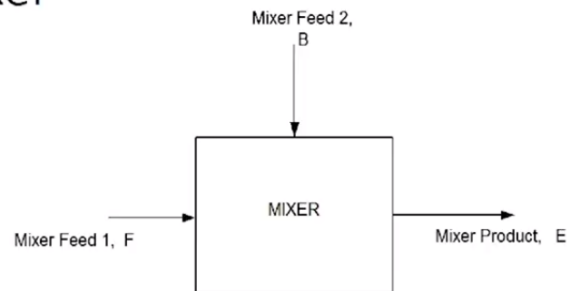
When you check the question you find that the question asked for the flow rates of glucose and growth factor in streams to the fermenters which means our calculation have not been complete. So we need to now calculate the amount of glucose and growth factor A present in the two streams. So we already know that 60% of the stream would contain glucose so this implies for stream 1 and stream 2 the amount glucose would be 0.6 times  $F_1$  and 0.6 times  $F_2$  respectively.

So in stream 1 mass of glucose leaving would be 75 kilograms per hour and in stream it would be 225 kilograms per hour. So 40% of these streams contains growth factor A implies stream 2 so stream 1 would contain 0.4 times  $F_1$  which is 50 kilograms per hour and stream 2 would contains 0.4 times  $F_2$  which is 150 kilograms per hour with this our calculations are completed. We know that the mass flow rate for steam 1 is equal to 125 kilograms per hour out of which 75 kilograms is glucose and 50 kilograms per hour is growth factor A.

For stream 2 the total mass flow rate is 375 kilograms per hour and the glucose flow rate is 225 kilograms per hour and growth factor A flows at 150 kilograms per hour.

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## Mixer



- More than one input stream
- One 'mixed' output stream
- How many material balances can you write for such a system?
  - Equal to the number of components present in the system

With this we can try to move on to the next step is mixture is another common system which is used for any process here what happens is you have more than one input and there is one final output. So multiple stream containing different components are mixed together to form one final output which is the mixer product so here you have multiple balance equations you can write can you identify how many independent material balances you would be able to write for such a system.

Obviously you can write the total balance equation and you would also be write component balance equation so would all of these independent or would one of them dependent on each other. What you would find is the summation of all the component balances will be equal to the

total balance. So the number of equation which are independent that can be written for mixing point would be equal to the number of components that are present in the system.

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## Material Balance – Mixer

- Primary antibody deficiencies are treated with subcutaneous immunoglobulin (SCIG) replacement therapy. This SCIG product containing 10% IgG, 5% sorbitol, the rest water can be prepared by mixing a concentrated IgG solution (A), containing 50% IgG and the rest water, a concentrated sorbitol solution (B), containing 40% sorbitol and the rest water, and pure water (C) in the right ratio. How much A, B, and C should be mixed to produce 10 L of SCIG product? Assume that densities for all the solutions is equal to 1 g/cm<sup>3</sup>.

Basis - 10 L =  $10 \text{ L} \times \frac{1 \text{ g}}{\text{cm}^3} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{1000 \text{ cm}^3}{\text{L}} = 10 \text{ kg}$

Total:

$$I - 0 + x_1 - x_2 = x_3$$

$$E = 0$$

$$A + B + C = 10$$

Here is an example which we will try and solve to understand the principles behind material balance in a mixture. Primary antibody deficiencies are treated with subcutaneous immunoglobulin replacement therapy this SC IG products contains 10% IG G which is a type of immunoglobulin and 5% sorbitol and the rest is water.

This mixture can be prepared by mixing a concentrated IG solution which contains 50% IGG and rest water or concentrated sorbitol solution containing 40% sorbitol and rest water and pure water in the correct ratio. How much A, B and C which are the three different solution that needs to be mixed to produce 10 liter of SC IG products. You can assume that the density for all the solutions = 1 grams per centimeters cube.

Now let us try and solve this problem where do we start the first step for any material balance problem as I said is identifying the basis here the basis would be 10 liters of SCIG products is formed had also informed you earlier in the previous lecture that we should avoid volumetric flows and volumes as for balances. So we should convert this volume into mass before we proceed and perform balance equations.

So basis which we use for this system would be 10 liters which can then be converted to mass using the density. So this 10 liters using the density of 1 gram per centimeter cube can be converted to mass would be 10 liters times 1 gram per centimeter cube times 1 kilogram per 1000 grams times 1000 centimeters cube per liter.

So now your liters get cancelled off your grams gets cancelled your centimeter cube gets cancelled off and your left if mass in terms of kilograms which is 10 kilograms all thought this conversion is simple I have still followed a meticulous pattern of written down all the units so that I ensure that there is minimal chance of error. So this kind of a systematic problem solving will ensure you do make mistakes like dividing the conversion factor instead of multiplying or vice versa.

So now that we have the mass we can write total balance equations so we know that there are three streams which are coming in so that flow chart would look something like this we have three streams which are entering into the mixer to find to finally form one product. So if this is the mixture we have 1, 2 and 3 input streams and one final output stream.

Let us call these as A, B and C we can label these with the information on the concentration which have been given so A has been told to be 50% IGG and the other 50% is water. B is 40% is sorbitol and the other 60% is water C is 100% water. Your final product is formed which we will call as P is 10% IGG 5% sorbitol and rest which is 85% is water. Now that we have all the information in the flow chart.

Let us try and calculate actual flow rates for each of these streams the total balance equation can be written as input – output – generation + consumption = accumulation again being in study state accumulation will go to 0 no reaction and generation and consumption go to 0 and you end up with input = output. So the input here are A , B and C and your output is your product which is 10 kilograms per hour now after writing the total balance.

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## Material Balance – Mixer

- Primary antibody deficiencies are treated with subcutaneous immunoglobulin (SCIG) replacement therapy. This SCIG product containing 10% IgG, 5% sorbitol, the rest water can be prepared by mixing a concentrated IgG solution (A), containing 50% IgG and the rest water, a concentrated sorbitol solution (B), containing 40% sorbitol and the rest water, and pure water (C) in the right ratio. How much A, B, and C should be mixed to produce 10 L of SCIG product? Assume that densities for all the solutions is equal to 1 g/cm<sup>3</sup>.

Comp. balances:

$$\text{IgG: } 0.5A = 0.1 \times 10$$

$$A = \frac{1}{0.5} = 2 \text{ kg}$$

$$S: 0.4B = 0.05 \times 10$$

$$B = 1.25 \text{ kg}$$

$$C = 6.75 \text{ kg}$$

$$W: I = 0$$

$$0.5A + 0.6B + C = 0.85 \times 10$$

$$\text{LHS: } 0.5 \times 2 + 0.6 \times 1.25 + 6.75 = 8.5 \text{ kg}$$

$$\text{RHS: } 0.85 \times 10 = 8.5 \text{ kg}$$

$$\text{LHS} = \text{RHS}$$

We can now write the component balances so let us start with let us first start with IGG. When we write the IGG balance you would have 0.5 times A = 0.1 times the product which is 10 so again we have used the same equation which is input – output + generation – consumption = accumulation and considering steady state and no reactions we got input equals output and from this we calculate the value as 1 / 0.5 which equals 2 kilograms.

And writing a balance equations for sorbitol we would be able to get 0.4 times B = 0.05 times 10 so based on this we can calculate the B as 1.25 kilograms. So substituting these two values in your total mass balance you would be able to calculate C as 6.75 kilograms. So these are the masses which has to be mixed to get your final product which is 10 kilogram of SCIG product.

So you can also convert them back into volume so that you can get the values as liters instead of kilograms. So the value would be 2 liters 1.25 liters and 6.75 liters of A, B and C finally giving you a total volume of 10 liters of SEIG product. So you can actually verify whether the answers you are got are correct. So the last slide which we looked at when we saw performing the fundamental or engineering calculations was validating the answers we are arrived at.

One possible way to do it would be to use different equations and perform the calculations using different approach. So here what we can do is we can write a component balance for water and try to solve and see whether we get the same values. So we can plug in these values to the water

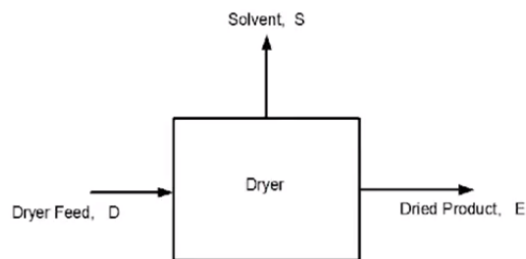
balance equation to confirm whether our answers are correct. So if we were to write the water balance equation it would look this you again have input equals output.

So input is  $0.5 \text{ times } A + 0.6 \text{ times } B + C$  which will be equal to  $0.85 \text{ times } 10$  now these values of a B and C can be plugged to see whether these this equation is valid. So your left hand side would now become  $0.5 \text{ times } 2 + 0.6 \text{ times } 1.25 + 6.75$  this adds up to 8.5 kilograms your right hand side is  $0.85 \text{ times } 10$  giving you a value of 8.5 kilograms.

So your left hand side is equal to your there by showing that the calculations you had performed and values for the values of A, B and C that you obtained are actually correct.

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## Dryer



- Wet feed is dried by supplying heat
  - can be solid, solution or slurry
- Solvent stream is vapor and solid-free
- Product stream is solid, may contain solvent

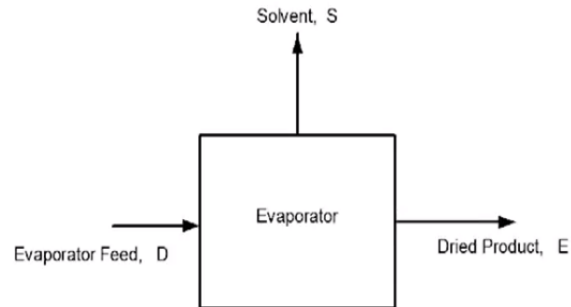
Now let us consider a different process drying drying is a process where you have one feed which enters into a dryer and this system gets heat supplied to it there by some of the solvent gets evaporated and you end up with dry product and the wet feed which enters in to the system can be a solid a solution or a slurry the solvent stream is a vapor which is solid free only the solvent get evaporated and leaves the system as the vapor.

The solids do not accompany this solvent stream would leaves the dryer the product is the solid however it does not have to be only in the solid some of the solvent may still be present in the final stream because not always are the feed fully dried and you end up with the bone dry material you might want to reduce the concentrations.



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## Evaporator



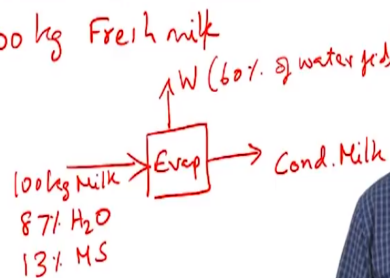
- Similar to drying, except feed is always a liquid

Another process is an evaporated which is similar to drying except here the feed is always a liquid. So you have an evaporated feed which is a liquid sent to an evaporator where heat is supplied to remove some of the solvent and you end up with the dried product.

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## Material Balance – Evaporator

- Unsweetened condensed milk is a shelf-stable canned milk product. If 100 kg of fresh milk, containing 87% water and the rest milk solids, is condensed using an evaporator to remove 60% of the water from the fresh milk, calculate
  - The mass of water evaporated
  - The final mass and composition of condensed milk produced



So let us look at an example for a evaporator which would help us understand the fundamentals of performing material balances for the system unsweetened condensed milk is the shelf-stable canned milk product. If 100 kilogram of fresh milk containing 87% water and the rest milk solids is condensed using an evaporator to remove 60% water from the fresh milk you are asked to

calculate the mass of water which is evaporated and the final mass and composition of condensed milk product.

Please read the milk one more time and make sure that you fully understand what information has been provided to you and what you need to calculate. We will now proceed to try and solve this problem but I would strongly recommend that you pause the video and try solving it yourself and see to whether you are having the understanding of how to go about it and then see whether my calculations match with yours.

So let us see how I would solve the problem systematically to get the correct answer so the problem statement says that 100 kilograms of fresh milk is entering into the evaporator so we will use 100 kilo grams fresh milk as the basis as 100 kilograms of fresh milk. Now let us try and draw the flow chart which present the information that has been provided to us in this problem.

So we have evaporator for which is fed with 100 kilograms of milk and it is told that this milk contains 87% water and the rest is milk solids which is 13% would be milk solids and this evaporator gives you an final product where 60% of the water has been removed. So these would be your condensed milk and water has been removed from the system as a vapor and this is basically 60% of water fed.

Using this information let us see if we can calculate the amount of water which is evaporated and the mass and composition of the condensed milk.

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## Material Balance – Evaporator

- Unsweetened condensed milk is a shelf-stable canned milk product. If 100 kg of fresh milk, containing 87% water and the rest milk solids, is condensed using an evaporator to remove 60% of the water from the fresh milk, calculate
  - The mass of water evaporated
  - The final mass and composition of condensed milk produced

$$\text{Water entering} = 0.87 \times 100 \text{ kg} = 87 \text{ kg}$$

$$\text{Water evap} = 0.6 \times 87 \text{ kg} = 52.2 \text{ kg}$$

$$\text{Total} : 100 = W + P$$

$$P = 100 - W = 100 - 52.2 = 47.8 \text{ kg}$$

$$W : 0.87 \times 100 = x \times 47.8 + 52.2$$

$$x = \frac{87 - 52.2}{47.8} = 0.728$$

$$\begin{aligned} \text{H}_2\text{O evap} &= 52.2 \text{ kg} \\ P &= 47.8 \text{ kg} \\ x_{\text{H}_2\text{O}} &= 0.728 \text{ kg/kg} \end{aligned}$$

So the problem state tells you that 60% of water which is entering in to the system through the fresh milk is evaporated in the form of water vapor so let us first calculate how much of water is actually entering into the system that would be water entering = 0.87 times 100 kilograms which is 87 kilograms. So water evaporated would be 0.6 times 87 kilograms which will give you a final value of 52.2 kilograms.

Now we can write a total mass balance for the system we looked at which would be 100 which is the fresh milk entering = W + P which is masses of water leaving the system and the condensed milk which is leaving the system. So the final product which is condensed milk would be 100 – the amount of water which is evaporated which is 52.2. So you end up with 100 – 52.2 which is 47.8 kilograms.

So the final milk condensed milk is 47.8 kilogram we still need to calculate the composition of the milk. So writing a component balance for the water we would be able to get 0.87 times 100 which is entering through the fresh milk = X times 47.8 where X is the concentration of water in the condensed milk + 52.2 which is the mass of water which is evaporated from the system. Solving this equation we can calculate X as 87-52.2 divided by 47.8 which is 0.728.

So this would give you the mass fraction of water in the condensed milk so now final answers are water evaporated = 52.2 kilograms final product produced which is the condensed milk is 47.8 kilogram and this contains water at 0.728 kilograms per kilogram so this gives you an final

answers. Let me walk you through my balanced equation one more time so that there is some clarity.

So the total mass balance was reduced to input = output because there are no reactions and there is no accumulation for steady state process. So we would have input which is 100 = the summation of two output which are W and P and the component balance for water is written as mass fraction of water in the input stream times the flow rate of water in the input stream giving you the total mass of water entering the system and this is equated to the mass of water leaving the system which is two component.

First one is water leaving as water evaporated form the evaporator and the other is water leaving the system as a component of the condensed so these two had been written down as 52.2 and X times 47.8 substituting all the values we have calculated the value for X and finally we understand the amount of water that has been evaporated and the amount of product which has been formed and the concentration of water in the condensed product.

I hope you are able to follow this problem without much difficulty so we will move on to the next concept we will conclude the lectures for today's class and we will continue with more systems which are single units without reactions thank you and see you in the next lecture.