Thermodynamics for Biological Systems: Classical and Statistical Aspects

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Lecture – 58
Defining Beta in Boltzmann Distribution Law

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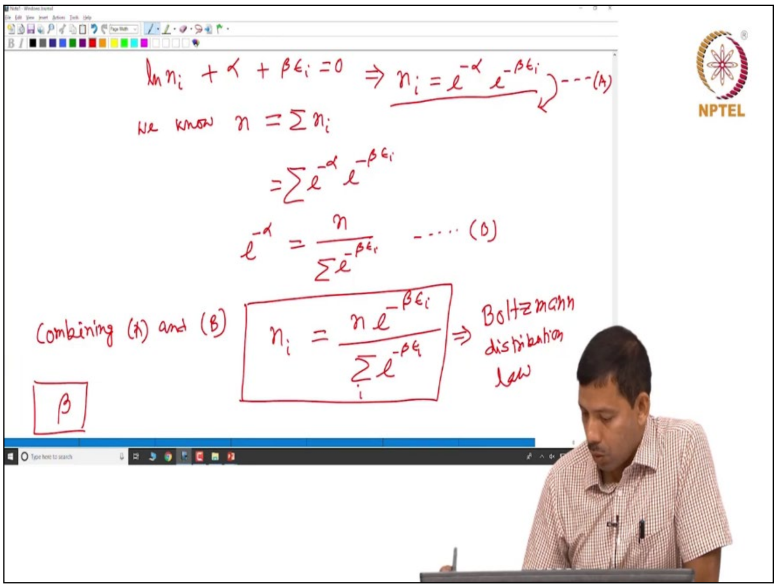

Something is still missing here, or something is still there which we do not know and that is beta. So, beta is an undetermined multipliers what we proposed to get the dn_i's independent so β is still a constant and which we do not know what it is. So, our next step would be to find out what this beta is but, before that let us try to make this Boltzmann distribution law a little more generalized.

$$n_i = \frac{n\, e^{-\beta\varepsilon_i}}{\sum_i e^{-\beta\varepsilon_i}}$$

This is Boltzmann distribution law

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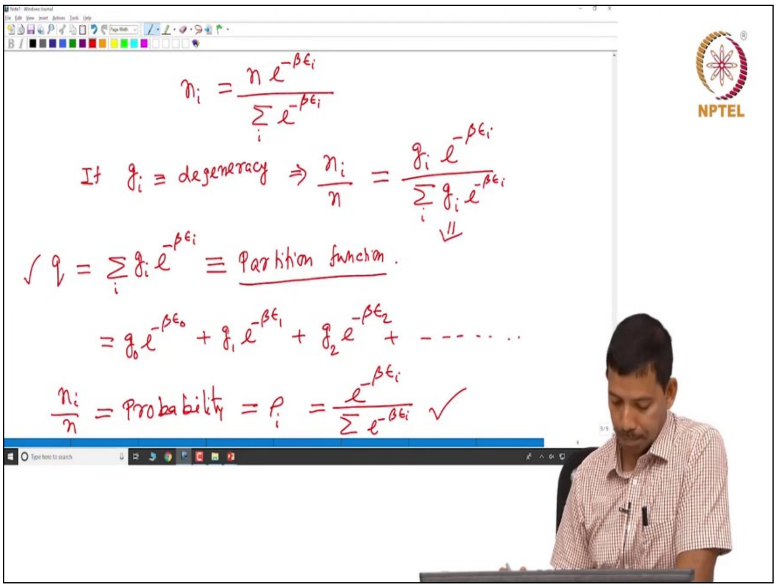

g_i is degeneracy so we can write as

$$g_i = \frac{n_i}{n} = \frac{g_i e^{-\beta \varepsilon_i}}{\sum g_i e^{-\beta \varepsilon_i}}$$

And partition function (q)

$$q = \sum g_i e^{-\beta \varepsilon_i}$$

Hence

$$probaility(\rho_i) = \frac{n_i}{n} = \frac{e^{-\beta \varepsilon_i}}{\sum e^{-\beta \varepsilon_i}}$$

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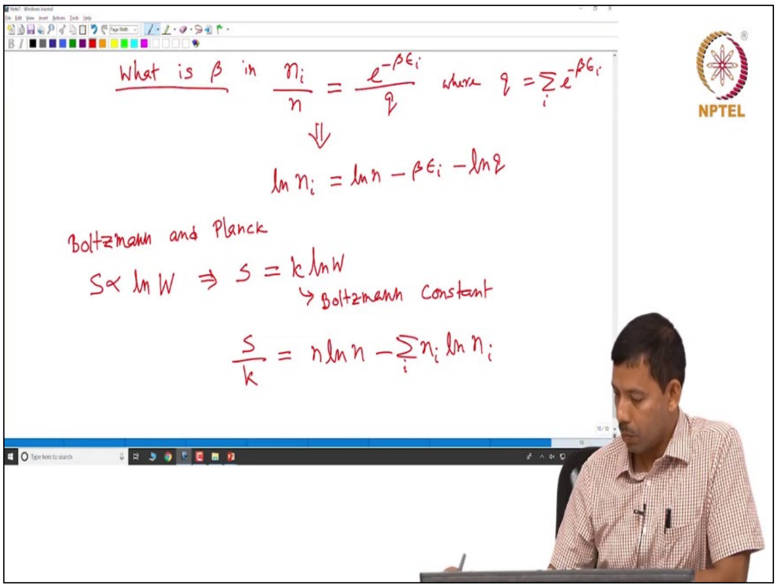

So now what is β in Boltzmann distribution law? From the above equation we can write

$$\frac{n_i}{n} = \frac{e^{-\beta\varepsilon_i}}{q}$$

Taking Lon on both side

$$\ln n_i = \ln n - \ \beta\varepsilon_i - \ln q$$

From Boltzmann and Planks law we know that

$$S = k \ln W$$

Where k is Boltzmann constant

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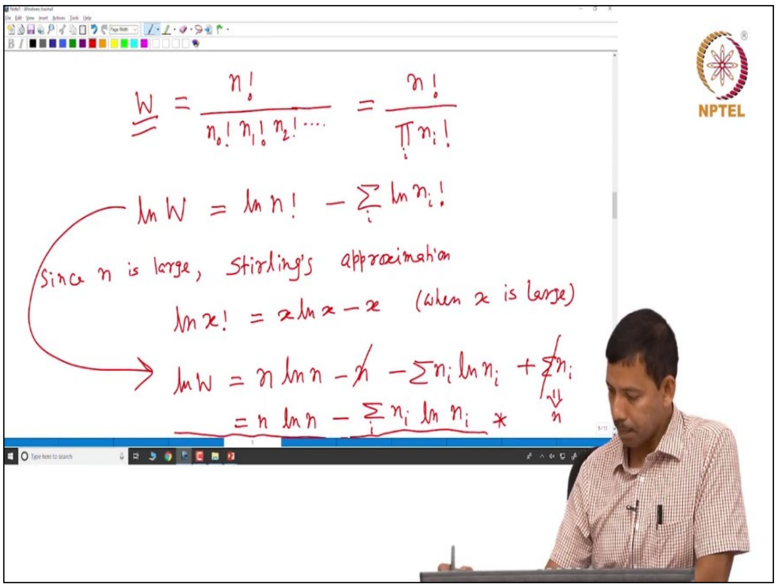

From our derivations we know that

$$\ln W = \text{n} \ln\, n\, - \sum_i n_i \ln n_i$$

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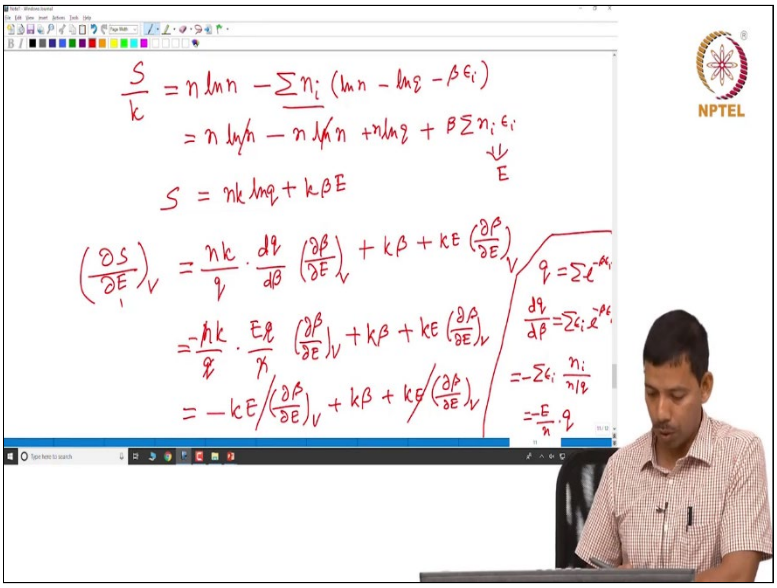

Hence we can write as

$$\frac{S}{k} = \ \ln W = \text{n} \ln\, n\, - \sum_i n_i \ln n_i$$

By substituting the value of $\ln n_i$ from the above equation we will get

$$S = nk \ln q\, + k\beta E$$

Where E is $\sum_i n_i \varepsilon_i$

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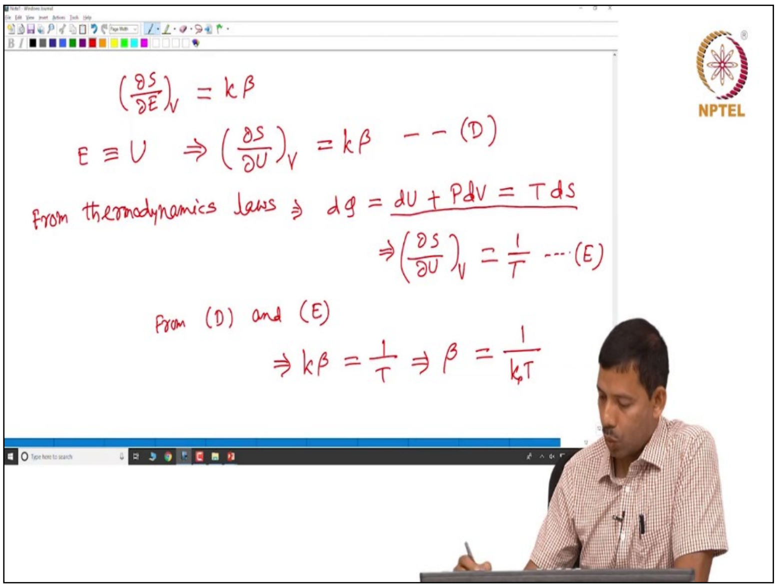

From here we will get

$$(\frac{\partial S}{\partial E})v = k\beta$$

del S by del E at constant volume is $k\beta$

Here we have E = U

Hence

$$(\frac{\partial S}{\partial U})v = k\beta$$

And from first law of thermodynamics we know that

$$(\frac{\partial S}{\partial U})v = \frac{1}{T}$$

By equating above two equations we get

$$k\beta = \frac{1}{T}$$

Or

$$\beta = \frac{1}{kT}$$

Here K is Boltzmann constant.

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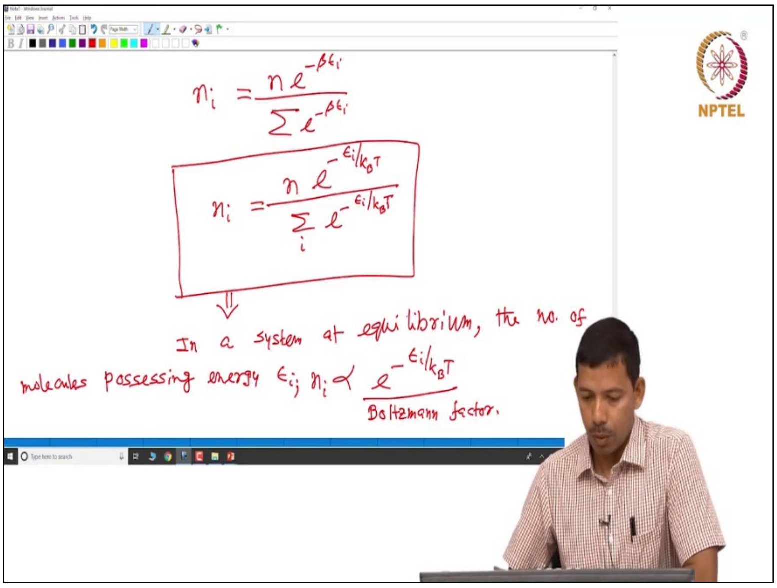

$$n_i = \frac{ne^{-\beta\varepsilon_i}}{\sum_i e^{-\beta\varepsilon_i}}$$

$$n_i = \frac{ne^{-\varepsilon_i/k_\beta T}}{\sum_i e^{-\varepsilon_i/k_\beta T}}$$

Now, if we go back to our Boltzmann distribution law ni is equal to n e to the power -beta epsilon i divided by sum over e to the power minus beta epsilon i we can now write as n is equal to n e to the power - epsilon i by KBT divided by sum over i e to the power -e to the power minus Epsilon i by KBT. So, this is that Boltzmann distribution law. So, so what is the significance of this equation is that, in a system at equilibrium, the number of molecules, the number of molecules possessing energy epsilon i is basically proportional to the Boltzmann factor which is, this is called the Boltzmann factors.