Thermodynamics for Biological Systems: Classical and Statistical Aspects

Prof. Sanjib Senapathy
Department of Biotechnology
Indian institute of Technology - Madras

Lecture – 64
Thermodynamic Probability for Indistinguishable Particles

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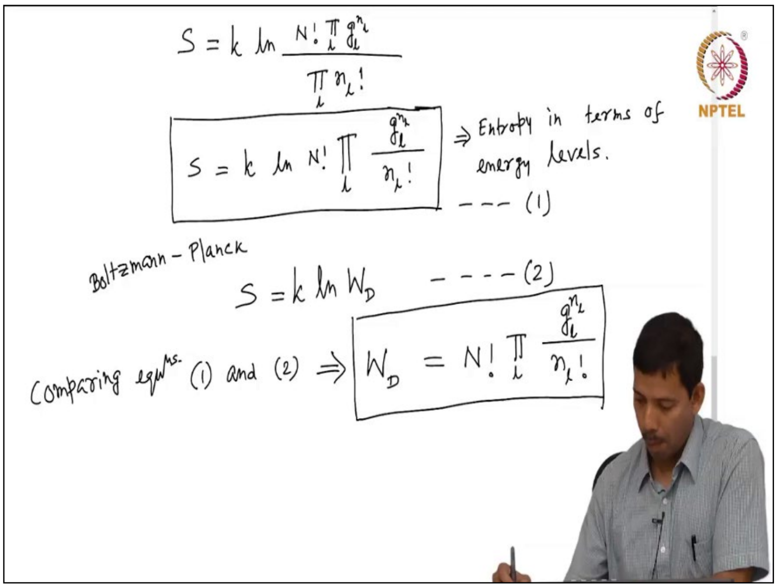

So, this is this was our first step that we got the thermodynamic probability of the distinguishable particles in terms of the energy levels.

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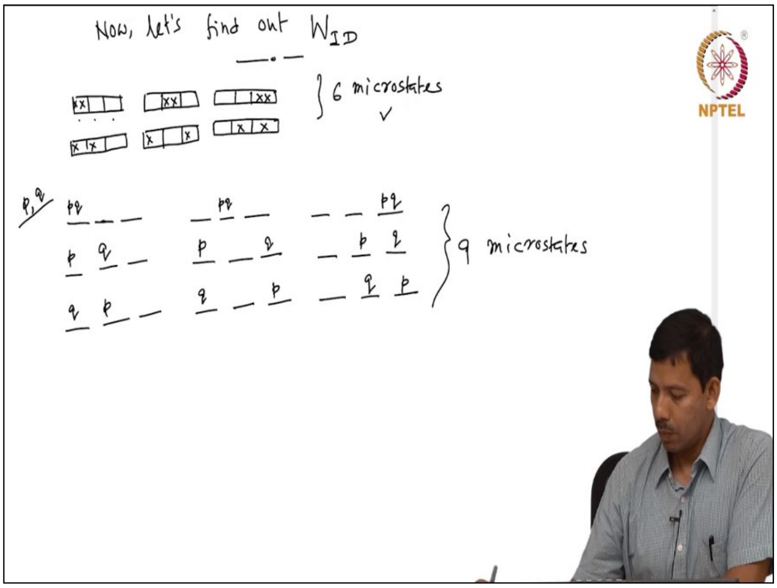

Now let us look at the W_{ID} now let us find out thermodynamic probability for the indistinguishable particles.

Let us take up a very simple system where we have two particles and my energy state is 3 fold degenerate. So, I have two particles and my energy state is 3 fold degenerate. So, I do have 6 different ways of distributing 2 indistinguishable particles in you know energy state having 3 fold degeneracy. So, in other words I have 6 microstates.

So, if you go back to our previous discussion of distinguishable particles what was the possibility how many number of microstates we could get if the particles were distinguishable. So, let us see so if our particles were p and q so we are we can distinguish the particles as p and q and we had 3 fold, degeneracy. So, here we got 9 microstates.

There was a problem due to which we came across the Gibbs paradox. So, when the particles are not distinguishable which is the case we get 6 microstates for this simple example of two particles distributed in an energy state with 3 fold degeneracy.

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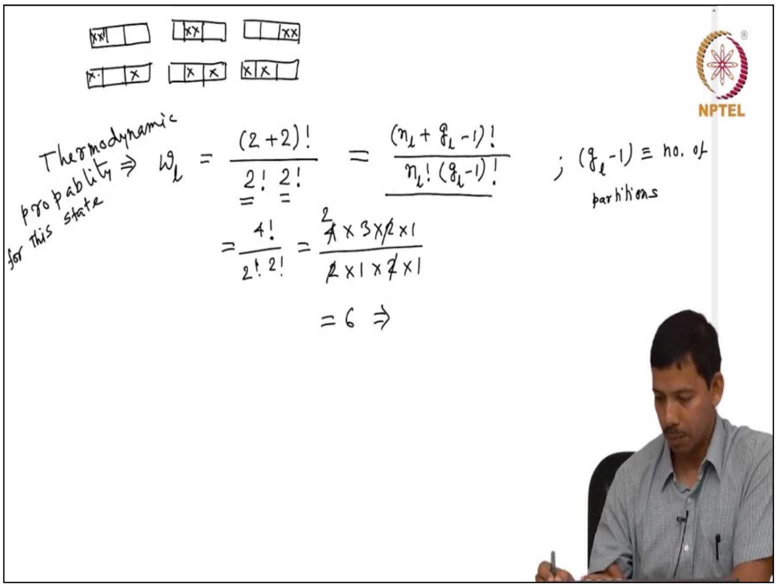

$$W_l = \frac{(2+2)!}{2!\,2!}$$
$$= 6$$

On generalisation:

$$W_l = \frac{(n_l + g_l - 1)!}{n_l!\,(g_l - 1)!}$$

So, here my 2 was the number of particles which is n_l in that particular energy level. So, we can write n_l factorial the second term we did we said this is the number of partitions here it is 2 because it was 3 fold degenerate see my degeneracy was g_l so my number of partitions require is g_l - 1 factorial so where g_l - 1 is nothing but the number of partitions and on the top we can write 2 + 2 as we written so n_l number of particles + g_l - 1 factorial. So this is the thermodynamic probability for that particular state in terms of energy levels.

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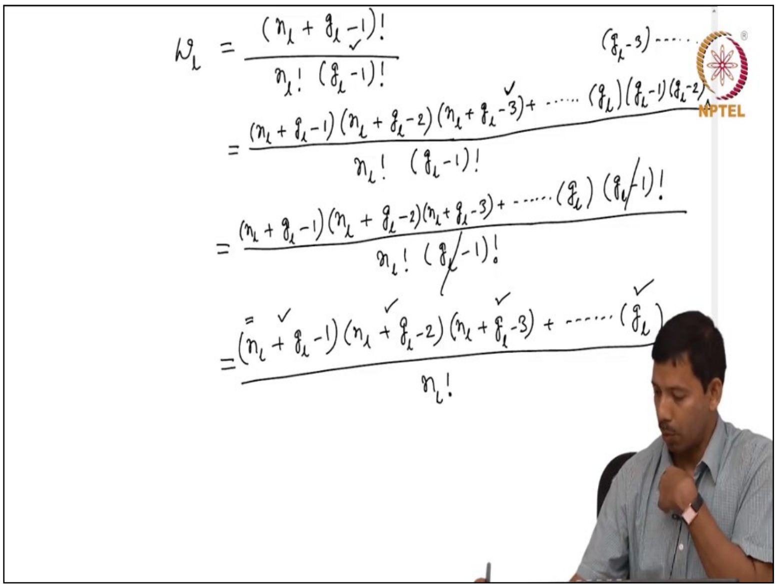

$$W_l = \frac{(n_l + g_l - 1)!}{n_l!\,(g_l - 1)!}$$

Now let us simplify so how we can simplify? We can expand the numerator and if we expand will get $n_l + g_l - 1$ we get $n_l + g_l - 2$ we get $n_l + g_l - 3$ plus this series goes on this series goes on with this number keeps increasing and at certain point this number becomes equal to nl.

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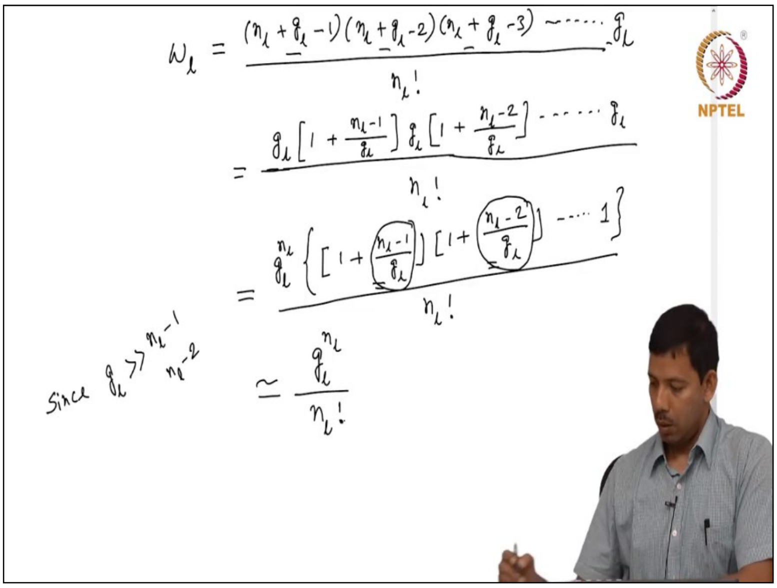

$$W_l = \frac{(n_l + g_l - 1)(n_l + g_l - 2)(n_l + g_l - 3) \dots \dots \dots .(g_l)}{n_l!}$$

Since $g_l \gg \ n_l - 1, n_l - 2 \dots$

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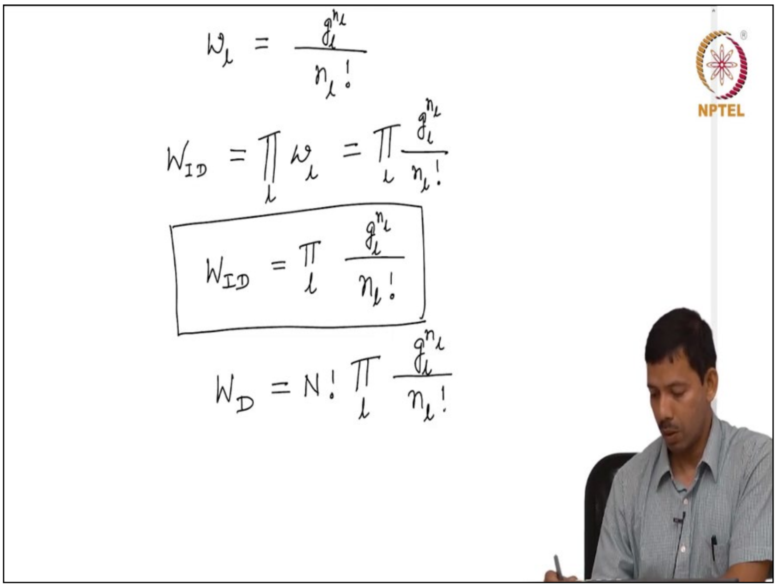

So

$$W_l \approx \frac{g_l^{n_l}}{n_l!}$$

$$W_{ID} = \prod_l W_l = \prod_l \frac{g_l^{n_l}}{n_l!}$$

$$W_D = \ N! \prod_l \frac{g_l^{n_l}}{n_l!}$$

So, in other words we like wl gl nl divided by nl factorial is what we get power we get for the thermodynamic probability of a particular energy state. Now if we want to get W we need to get the product of our all energy states and their energy levels and therefore we take the product over wl and that gives us the expression of W_D as gl nl divided by nl factorial. So, that this is our expression for W_D, W_{ID} thermodynamic probability for the indistinguishable particles in terms of energy levels is equal to product over all energy levels in gl to the power nl divided by nl factorial.

And what was our W_D the thermodynamic probability for the distinguishable particles was W_D is equal to n factorial and in the rest of the remaining part is similar same nl factorial.