Transport Phenomena in Biological Systems
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Lecture - 17
Pseudo Steady State Approximation

Welcome back. Today, we would look at something called a pseudo steady state approximation. In the last class, we looked at an unsteady case, an unsteady state case of mass flux. For our equation, it brought into a time derivative, a partial time derivative. And that complicated the solution significantly as you would see.

Whatever was probably about a page of analytical solution went to probably about what about five or six, seven pages of solution with especially when we wanted to look at flux rate, yeah I think it was flux we needed to differentiate an integral, we needed to use a Leibnitz rule and so on so forth. So mathematically, it became, rather involved. This pseudo steady state approximation may in certain situation help us with reducing the mathematical complexity.

And that is the reason why we are looking at it. In terms of a concept itself it is a very powerful concept. I have used it significantly, even in research recently. And that resulted in publishable work. Therefore, not to be taken lightly. A lot of benefits to this approximation. So, let us see what that is.

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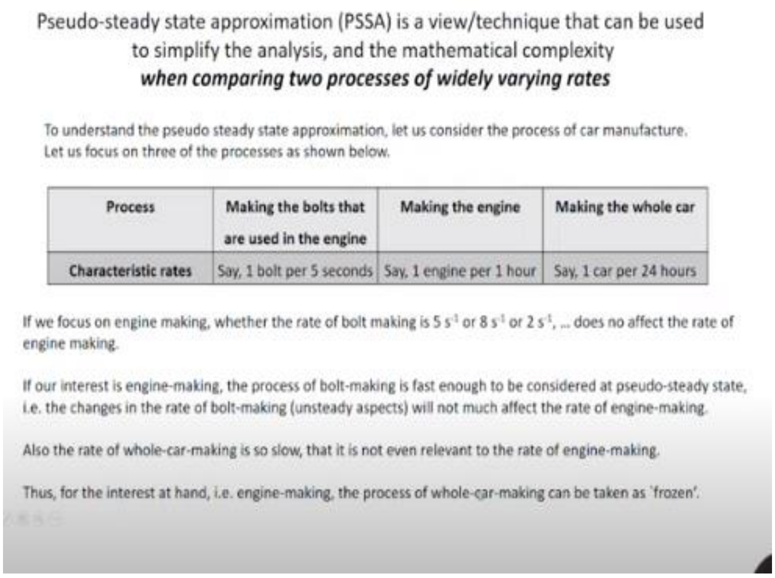

Formally or to put it clearly pseudo steady state approximation PSSA for short, is a view or a technique that can be used to simplify the analysis, as well as the mathematical complexity when comparing two processes of widely varying rates. This is the most important aspect, when comparing two processes of widely varying rates. Only when we are comparing, can we use this approximation at all.

To understand the PSSA better, let us consider the process of some parts in a car manufacturing process, okay. We are going to focus on three processes. They are the process of making bolts. You know these bolts probably thousands, more than that are used in a typical internal combustion engine-based car. So, making bolts that are used in the engine especially and the characteristic rate of making the bolt is one bolt in every five seconds, let us say. The second process that we are going to consider is making the engine and let us say that the characteristic rate is one engine per hour, okay. This is for understanding. So, let us take typical values or some values which gives us an idea. Do not hold me to these numbers. And then the third process of making a whole car let us say a car takes about 24 hours to make, okay.

So, these are the three processes that we are looking at. We would predominantly look at two at a time, we are comparing two processes with widely varying rates, okay. There is one bolt in every five seconds. There is one engine in every hour okay and this is one car in 24 hours. So, the rates are widely different. If we focus on engine making, okay this is an important aspect, we are focusing on engine making.

That is our process of interest. If you focus on engine making, whether the bolt making rate is five seconds or $5s^{-1}$ that is one every five seconds, one every two seconds, one every seven seconds, one every eight seconds, one every three seconds, it does not really matter, right. Or in other words, the change in the rate of bolt making does not affect the rate of engine making, okay.

We are focusing on rates of course; it does not affect the rate of engine making. This can even widely vary okay, but because the characteristic rate is much faster compared to the slower process, which is our interest, the rate of the faster process does not matter. So, if our interest is in engine making, the process of bolt making is fast enough to be considered at pseudo steady state.

In other words, the variation in rate whether you make a bolt every five seconds, two seconds, seven seconds, eight seconds, three seconds, five seconds, one second whatever it is, the variation in rate can be ignored okay. Or at this process can be considered to be at pseudo steady state. The changes in the rate of bolt making unsteady aspects will not much affect the rate of engine making.

And since our interest is in the engine making and the rate of engine making we can safely ignore the variation in the rate or the unsteady nature of this rate and therefore we can consider that to be at steady state. And you all know the mathematical advantage of steady state. The time derivatives can be set to zero okay.

So mathematically it becomes extremely helpful in reducing the complexity, the amount of effort that you need to spend in and so on and so forth, okay. So that is what we do. Also, this is when we consider engine making with bolt making. The rate of making the whole car is so slow that it is not even relevant here okay, it is kind of frozen. So, the much slower process can be ignored, okay.

Thus, for the interest at hand that is engine making, the process of whole car making can be taken as frozen.

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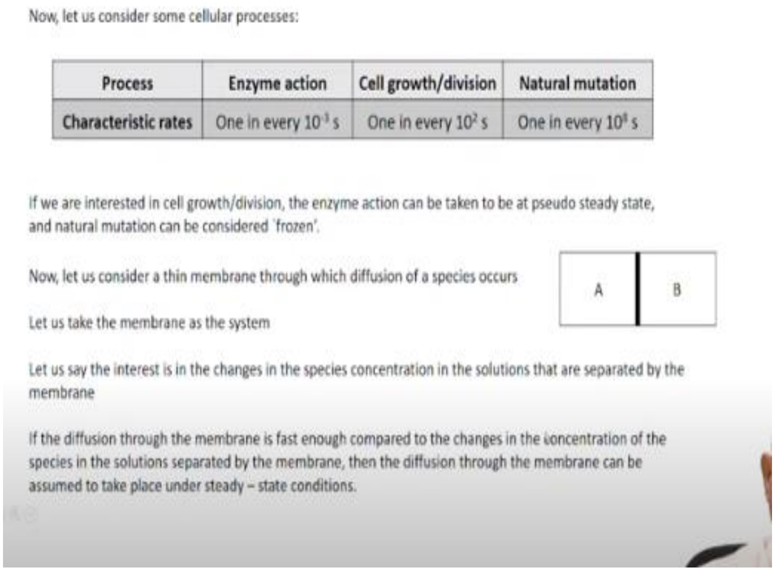

Okay, so that was for understanding. An essence, if our interest is in the rate of the slower process, the faster process can be assumed to be at a pseudo steady state whether it is actually at steady state or not does not really matter. It can be assumed to be at pseudo steady state because the faster rate does not impinge on the slower rate. Now let us move closer to our system of interest.

We will consider some cellular processes. We are going to consider three different cellular processes. One is enzyme action with a characteristic rate of 10^{-3} seconds, one every 10^{-3} seconds. Cell growth or division one every 10^{2} seconds i.e. 100 seconds let us say. And the third one, process of natural mutation which you know takes a long time for a mutation to happen, change in DNA that gets transferred from one population to the other and so on and so forth.

So, let that gets transferred and established and so on and so forth. So, let us say one in every 10^{8} seconds. So, three different processes of widely varying rates. And let us say, if that we are interested in cell growth or division. If you are interested in this, the enzyme action can be taken to be at pseudo steady state. And the natural mutation can be considered frozen.

In fact, do not even worry too much about frozen, we will mostly deal with faster and slower two processes and if the interest is in the slower process we will take the faster process to be at pseudo steady state. Okay, now let us consider a thin membrane through which a diffusion of a species occurs, okay. So, this is I am going to apply this concept, or I am going to illustrate this concept by applying it to a certain situation.

That situation is, let us say every standard situation, where you have a membrane, you have some species moving from one side of the membrane to the other side of the membrane, maybe from the extracellular space to the intracellular space of the membrane. I am going to represent that as this thick line, the membrane as this thick line here.

And A could maybe be the extracellular space and B could maybe be the intracellular space. I have just contained it here, but that is not necessary. Let us take the membrane as a system as the aspect on which we focus. And let us say our interest is in this changes in species concentration in the solution that is the extracellular space as well as the intracellular space that are separated by the membrane. That is our interest.

If the diffusion of the species through this membrane is fast enough compared to the changes in concentration of the species in the solutions that are separated by the membrane, then the diffusion through the membrane can be assumed to take place at steady state conditions, okay. Our interest is in how fast these concentrations change.

And if this process of diffusion through the membrane is much faster compared to the changes in concentration in the solutions on either side of the membrane, then we can take that diffusion to be at steady state whether it is at steady state or not.

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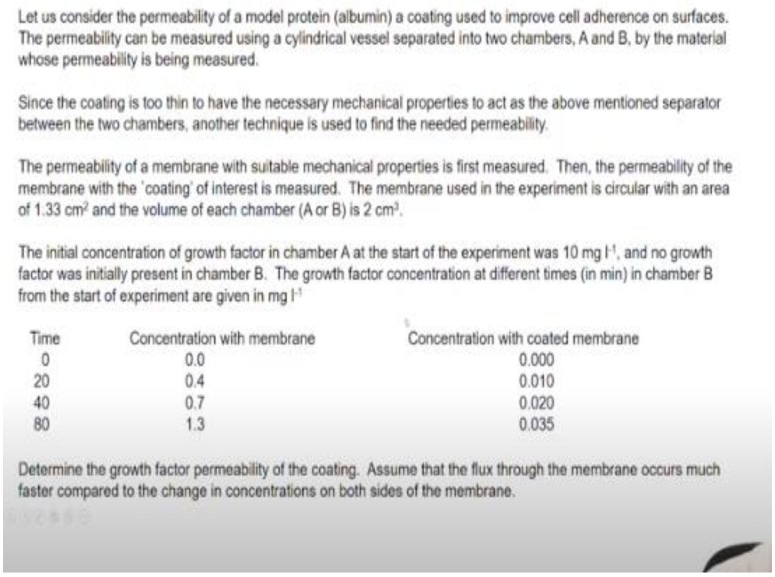

Let me illustrate this more specifically or in a more concrete fashion by applying it to a particular problem okay. Let me post the problem here and let us see. Let us see how we feel at that time. The problem reads something like this, it is a long problem. Let us consider the permeability of a model protein (albumin) through a coating that is used to improve the cell adherence on surfaces. The permeability can be measured using a cylindrical vessel separated into two chambers A and B by the material whose permeability is being measured, okay. Since the coating is too thin to have the necessary mechanical properties to act as the above mentioned separator between the two chambers, another technique is used to find the needed permeability okay.

We are interested in finding the permeability of albumin through something here. For that we use some sort of a device like this. We take a cylindrical chamber, or the membrane separates two chambers in a cylinder. This is the longitudinal section of the cylinder and therefore it will look like a rectangle, okay. If it is a cylinder you cut it like this, it is going to look like a rectangle.

And then this membrane let us say separates two chambers A and B, okay. This is the kind of device that we are talking about now. We are interested in measuring the permeability or estimating the permeability of a model protein through a coating, okay. I shall probably say that is okay, I think you can figure that out, that is through a coating. The permeability of a membrane with suitable mechanical properties is first measured.

Then the permeability of the membrane with the coating of interest is measured, okay. We are interested in the permeability through the coating, but unfortunately it does not have the necessary mechanical properties. Therefore, we piggyback it on to another membrane with suitable mechanical properties. And first the permeability through the backing or through the support is measured.

Then the permeability of the membrane and the coating are measured. The membrane used in the experiment is circular with an area of 1.33 cm^2 and the volume of each chamber A or B is 2 cm^3, both are equal. The initial concentration of growth factor in chamber A at the start of the experiment was 10 mg/l. And no growth factor was initially present in chamber B.

The growth factor concentration at different times in minutes in chamber B, from the start of the experiment are given in mg/l in the following table. Time, concentration in B with the membrane and concentration in B with the coated membrane. That is the membrane plus the coating of interest. So, at time 0, the concentration with the coated membrane is 0.

At 20 minutes 0.4, 0.01 in mg/l, At 40 minutes, 0.7 and 0.02. At time minutes 1.3, 0.035. Determine the growth factor permeability of the coating. Assume that the flux through the membrane occurs much faster compared to the change in concentrations on both sides of the membrane, okay. This is explicitly given here.

Many a time, we will have to look at the rates, we will have to estimate the rates and see whether we can easily use the pseudo steady state approximation that could happen. Here it is anyway explicitly given for illustration purposes, that the flux through the membrane occurs much faster compared to the change in concentrations on both sides of the membrane. Okay, let us start doing this.

I think you will need help to do this. Therefore, let us start doing this. Only thing is should we start now? I think the solution requires quite a bit of time. And we have just learnt a new concept pseudo steady state, which is for analysis. What I suggest is you think through this material, internalize the material. And when we begin the next class, I will give you the solution, okay.

That way I think it will help even out in the overload aspects, okay? Or it will not cause an overload. Okay, see you in the next class. Bye.