Transport Phenomena in Biological Systems

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Module - 1
Lecture - 2
Mass Conservation

Welcome to the next lecture in the course transport processes in biological systems, transport phenomena in biological systems. I am going to use these terms interchangeably. So, please do not get confused. Transport processes and transport phenomena are used in the same sense in this course. In the previous lecture, we saw some basic aspects of the course.

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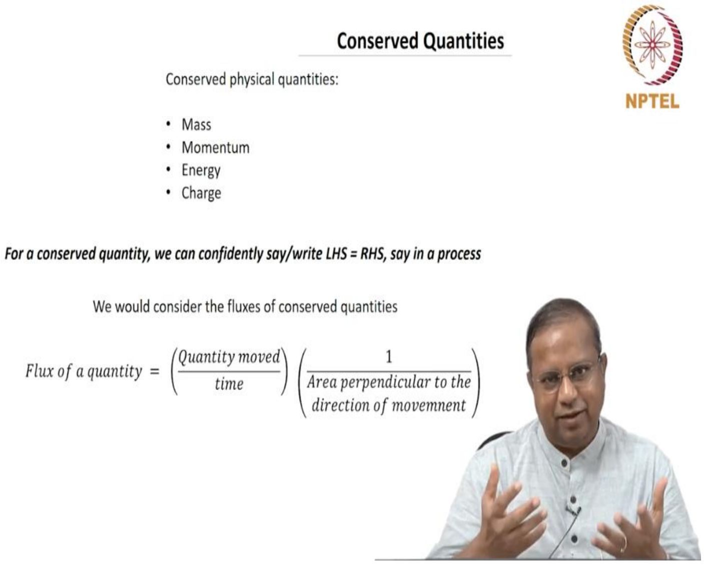

Then, we looked at, towards the end of the lecture, the conserved quantities. That is, we said that we know some physical quantities are conserved in a process. The left-hand side = the right-hand side is something that conservation principle gives to us, which is a very big thing for analysis. The conserved physical quantities could be mass, momentum, energy, charge and so on so forth.

There are other quantities, other physical quantities that are conserved. But we are going to look at these 4 in this course that are good enough for biological systems analysis at this stage. Whenever it is necessary, we could look at the other aspects also. Just to give you an example, the most other disciplines of engineering, for example: chemical, metallurgical, mechanical, aerospace, civil, and so on and so forth they typically look at momentum conservation, mass conservation, energy conservation predominantly. Charge conservation, predominantly the electrical engineers look at. The other engineers do not look at them. Whereas for biological engineering, to manipulate biological systems, we need to look at all these 4 conserved quantities.

We said we would look at the fluxes of these conserved quantities. That would be our major focus in this course. And we also defined the flux of a quantity. Flux of mass, for example: mass flux = mass per unit time that moves from one place to another per unit area in the perpendicular direction. Mass movement is in this direction; in the direction that is perpendicular to the direction of transfer.

$$Flux\ of\ a\ quantity = \left(\frac{Quantity\ moved}{Time}\right)\left(\frac{1}{\begin{matrix}Area\ perendicular\ to\ the\\ direction\ of\ the\ movement\end{matrix}}\right)$$

It could be momentum flux; it could be energy flux; it could be charge flux. Charge flux is the amount of charge transferred per unit time across a unit area or per unit area in the direction perpendicular to the direction of transfer.

$$Charge\ Flux\ = \left(\frac{Quantity\ of\ charge\ transferred}{Time}\right)\left(\frac{1}{\begin{matrix}Area\ perendicular\ to\ the\\ direction\ of\ the\ transfer\end{matrix}}\right)$$

This is where we finished up the last step.

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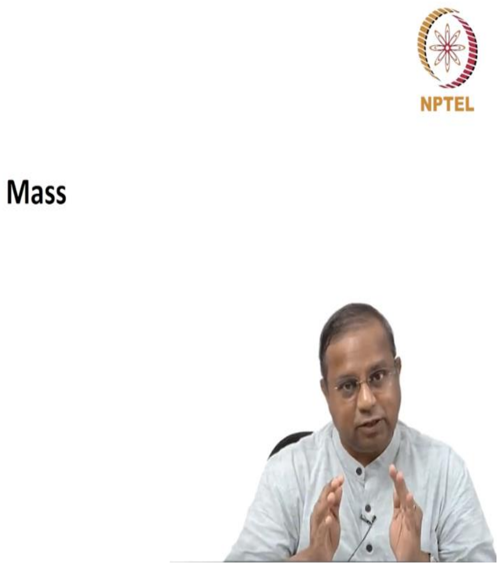

Let us move further. Mass conservation is so important. Let us first look at the conservation aspects for mass alone, because it happens to be such an important principle; and then, look at the flux aspects of mass. We will just do this for mass. We will not do this for momentum, energy and charge. Because it is so useful, we will look at mass conservation alone first.

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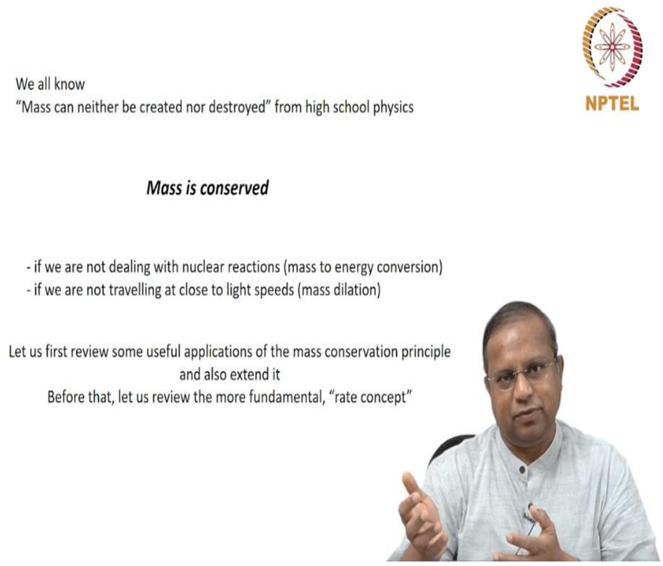

We all know that mass can neither be created nor destroyed. This we have known for a very long time. That is what we mean by saying that mass is conserved. There are a couple of riders. This assumes that we are not dealing with nuclear reactions. Because in nuclear reactions, mass to energy conversion is possible. In fact, that is the essence of nuclear energy, where mass gets converted to energy. And, it is assumed that we are not travelling at speeds close to that of light. Because we know that when speeds reach light speeds, there is mass dilation. And such situations are not considered in this case. Apart from these 2 aspects, the rest which is applicable a lot of times of interest to us. In those times, the mass conservation is definitely valid.

So, what we are going to do is: first, review some applications of the mass conservation principles. And, we will also extend it. You could have or you would have done some of this in the material and energy balances course that you would have taken earlier in the curriculum. So, if you have already done that, you could look at this as some sort of a review.

But, this is so important that I thought we should take a look at it in some sort of a review mode at least; so that, we are sure that we are at the same base level. That is the reason why I am doing this. If you are very comfortable with this part of the lecture, that does not matter. It will probably give you a different view of looking at mass conservation. Maybe you did not look at it in totality; and so on so forth.

So, even if you have done a course in material and energy balances, this could turn out to be useful. Just look for the viewpoint. For the others, just start picking up the principle. Before we even get into this mass conservation, let us spend a little bit of time on what is called the rate concept. This might seem trivial to some of you, but it is not. The misunderstanding of this aspect or not appreciating the fact that this is so central to any engineering analysis leads to a lot of confusion, even much after graduation.

You would have graduated with a degree, even done a higher degree, a PhD and so on so forth. But, somehow people have not internalized this. And that leads to a lot and a lot of confusion when people get into analysis. And therefore, let us spend a little bit of time right here, right in the beginning, to understand the importance of looking at rates of various different things in an engineering context.

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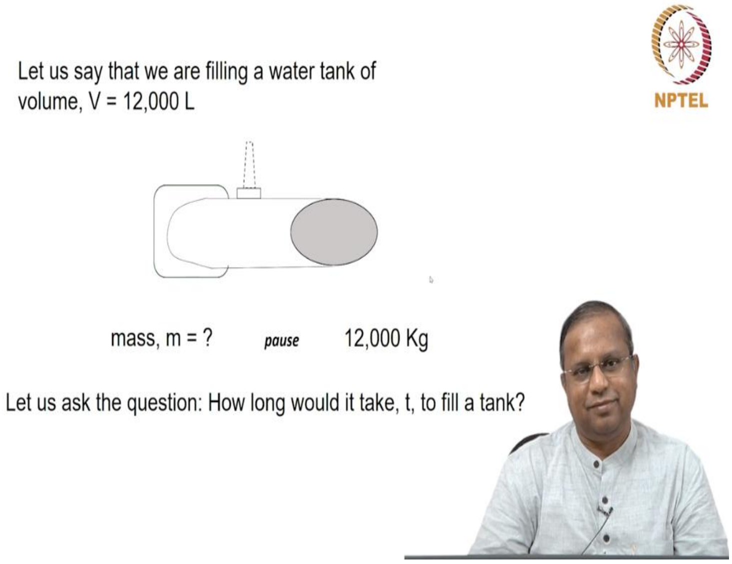

To do this, let me take the case of a typical odd scenario. This especially in Chennai, in many summers, there is water shortage. The water supply reduces, the planning still needs to be better and so on so forth. Some years it is not so bad; some years it is really bad. When the water supply gets bad, people use tankers. The government provides water to various localities in tankers.

Tankers are these lorries with cylindrical containers that have a capacity to carry water. These container lorries go to the water filling stations where water is filled. Water supply is there somehow. Water is filled. And then, they go and distribute the water to various localities, for a certain payment of course. You will have to pay for this in certain situations. In some cases, it is supplied free of cost, depending on the situation.

So, let us say that we are looking at that situation here. So, this you could see some representation of the container that contains water. And this is a part of the lorry, on which the container is. And this is the water stream that is flowing to fill this container. This is the situation that we have in our hand. Let us say that we are filling a water tank and the volume of this container or the tank is 12,000 liters.

Why did I choose 12,000 liters ? Because the smaller tankers in Chennai are 8000 liters and the larger tankers are 16,000 liters. So, I have chosen an average value of 12,000 liters. When the tank is full, what do you think is a mass of water in the tank? Can you work it out? It might be a good gelling of your brain to work things out. Let me give you probably a couple of minutes.

Why don't you work this out? What is the mass of water in the tank? And we know that the volume of water in the tank when it is full is 12,000 liters. I will give you some time to work it out. How would you go about that?

To find, mass(m)-?

Given, Volume(V)-12000 l

We are all engineers and what we know from school is that the density of water (ρ) is

1 gcm^{-3}. And if you convert that, that goes to 1000 kgm^{-3}.

That is the density of water. And what is the definition of density? Density is mass per unit volume.

$$Density = \frac{mass}{volume}$$

We know the density and the volume; we need mass. So, mass is nothing but density times volume(Density*volume). Density is 1000 $kgm^{-3}$and volume is 12,000 l. We know that

$1m^3$ = 1000 l

And therefore, 12,000 liters is 12 m^3. And therefore, mass of a water in this tank when it is full is 12,000 kgs.

$m=1000\ kgm^{-3} * 12\ m^3=12000$ kgs

When we know volume and density, we need to multiply density and volume to get mass $(m = v * \rho)$.

And that is what we did. There was some unit conversion here, because the values were given in different unit systems. That is what we did. Go ahead; do this; feel comfortable with this; pause the video till you feel comfortable; and then, start up the video. I am going to continue. My interest is this: How long would it take to fill the tank?

And that, I am going to call as time 't'. How long would it take to fill the tank? This is very typical engineering question. You have a tanker going and filling water. It is going into a filling station. How long should the tanker be there? It is a very practical question to ask in terms of design. Suppose somebody is designing the distribution of water system. This is an important data to have.

How long does it take to fill the tank? Right? It is a very engineering input to have. How would you go about doing it? Think about it. Pause the video here. Think about it. Hopefully, you had some ideas.

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r_{in}, Input rate (Kg s^{-1})	t, time (s)
10	1200 (20 min)
20	600 (10 min)
50	240 (4 min)

If we know the ***rate*** of water input, r_{in}, $t = m/r_{in}$

You would have realized that you need to have an idea of the rate at which the water is being filled, to figure out the time of filling. Hopefully, you got there. The data is given in the table below.

r_{in} Input rate (kg s^{-1})	**t Time(s)**
10	1200(20 min)
20	600(10 min)
60	240(4 min)

(1 minute = 60 seconds)

In other words, if we know the rate of water input, then the time is nothing but mass per rate. Because, rate is nothing but mass per time. We need the time, therefore, that is mass per rate. As simple as that. So, once we know the input rate, then, we can find out the time.

And it so happens in these situations, that the rate is typically about 20 kg s^{-1}, because it typically takes about 10 minutes to fill the tank. If it is smaller than that, you know, it takes too long to fill a tanker. That is not very practical. If it is, it needs to be larger than that, then the pumping power needs to be that much higher; you need to have much more expensive pumps; higher capacity, and therefore more expensive pumps and so on so forth. It may not be really helpful in the overall economic analysis. And that way, this becomes a very crucial information to have. Now we have the rate at which the water is filled to be about 20 kg s^{-1}. Let us get back to our discussion.

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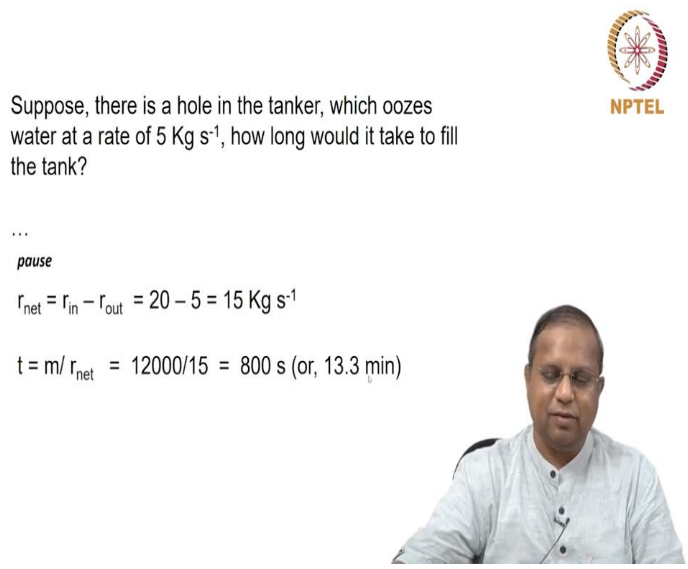

Suppose there is a hole in the tank which oozes water at the rate of 5 kg s^{-1}. How long would it take to fill the tank? Can you work this out? Pause the video here. Let me read it out again; or you could read it out as many times as you want, to understand completely.

Hope that you are able to work this out. I would go about it like this. We need to find the net rate. There is a rate at which water is being filled and there is a rate at which water is oozing out. Therefore, the net rate of water being filled is the input rate of water - the output rate of water.

Net rate=Input Rate – Output Rate

The input rate of water is 20 kg s^{-1}. The output rate of water is 5 kg s^{-1}.

Net Rate = 20 – 5= 15 kg s^{-1}

Once we have this; once we are focused on the rate, we have this number. And then, it becomes very simple that the time is nothing but mass per rate.

$\text{Time} = \frac{\text{mass}}{\text{Net Rate}}$. $\text{Time(s)} = \frac{12000\text{ kg}}{15\text{ kg/s}} = 800$ (or, 13.3 min)

If you had not focused on the rate concept, it is a rather confusing situation to be in. You would have probably experienced that. This much is coming in; this much is going out; how do I go about doing this? It is a standard confusion. Whereas, the minute you focus on rates; once you focus on the rate, it is a very simple, straightforward calculation. That is the need to look at the rate concept. Let me give you one more example just to drive home the point a little better.

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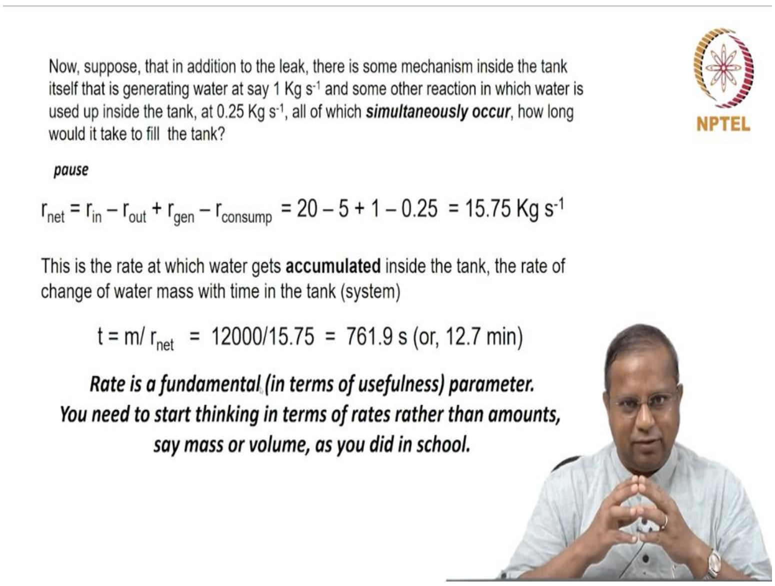

Now, suppose, that in addition to the leak, there is some mechanism inside the tank itself that is generating water. This is fiction. Therefore, we say some reaction generates water inside.

And the rate at which it generates water inside is 1 kg s^{-1}. And some other reaction in which water is used up inside the tank at 0.25 kg s^{-1}. All of which are happening simultaneously.

There is input rate filling in, there is oozing out; and then, there is generation of water and then, there is consumption of water. Everything is happening simultaneously inside the tank. Inside that, simultaneous thing is the killer. How long would it take to fill the tank? You pause the video here. Go forward and work it out. I hope you worked it out by looking at rates.

Again, if you focus in the rate, net rate. The net rate is the rate at which it comes in, rate at which it goes out; rate at which it gets, getting consumed by a reaction; rate at which something is being added; the water is being added by another reaction, okay. So, there are 2 things that are adding; there are 2 things that are subtracting. So, input and generation are adding; output and consumption are subtracting.

Input rate is 20 kg s^{-1}. Output rate is 5 kg s^{-1}. Generation rate is 1 kg s^{-1}. Consumption rate is 0.25 kg s^{-1}. So, you, the algebraic sum, 20 – 5 + 1 – 0.25 = 15.75 kg s^{-1}. So, this is the rate at which the water is getting accumulated inside the tank. And that is the rate of change of water mass with time, in the tank, which we can call as a system.

Net Rate = 20 – 5 + 1– 0.25 = 15.75 kg s^{-1}

System is something that we focus our attention on, as you would already know. Therefore, the time that it would take to fill under these conditions is mass divided by the net rate; 12,000 kgs divided by 15.75. That is 761.9 seconds or 12.7 minutes.

$\text{Time} = \frac{\text{mass}}{\text{Net Rate}}$. $\text{Time(s)} = \frac{12000 \text{ kg}}{15.75 \text{kg/s}} = 761.9 \text{ s(or, 12.7 min)}$

So, the moment you focus on the rate, it becomes a simple calculation. That is the reason for us to focus on rates. If you recall, when you were in high school, the focus would have predominantly been on mass, amount, volume etc. The moment you came into engineering, people started talking about rates. You would not have probably given too much attention, but you need to realize that there is a big need to talk in terms of rates. Once you start talking in terms of rates, everything engineering related becomes very simple.

Anything dynamic related for that matter, a dynamic system; cell is a dynamic system. Anything that is dynamic in nature, anything that changes with time, once you bring in rates, it becomes very simple to look at it. Or, you need to start looking at things only in terms of rates. So, rate as a fundamental in terms of usefulness, that is what I mean, parameter. You need to start thinking in terms of rates rather than amounts.

By amounts, I mean mass or volume as you did in school. That is the main point. That is the important aspect that I am trying to drive home here. What we will do is, we will take a break here. We have been at it for some time now. It is too long again; it becomes a little tiring. So, let us take a break here and continue in the next lecture. See you then.