Transport Phenomena in Biological Systems
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Lecture - 20
Momentum Flux - Introduction

Welcome to the next chapter in the course on Transport Phenomena in Biological Systems. In the previous class, we completed the second chapter on mass flux. The first chapter was of course, more on review mode. Half of it was in review mode. A little bit we extended to microscopic systems and then saw the derivatives that we would be using. And then we looked at mass flux. And now we are going to look at momentum flux.

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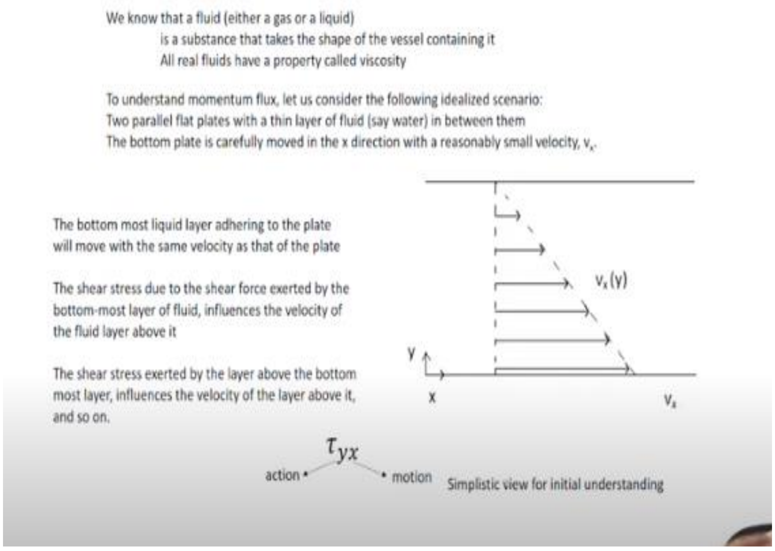

We know that a fluid is either a gas or a liquid here. And that we know from high school, early science classes. And it is a substance that takes the shape of the vessel containing it. And all real fluids have a property called viscosity. To understand momentum flux, what you are going to do is, we are going to consider first an idealized scenario, okay? This is for didactic purposes, I would like you to imagine what will be happening here, based on the figure that I am going to show you. But first, some description. There are two parallel plates here with a thin layer of fluid. And that fluid could be water to begin with, a simple enough fluid. And that layer of fluid is in between them. So it is a very thin layer of fluid, the plates are placed close to each other.

I am going to blow up this gap and I show you the picture and all that but they are very close to each other. Let us say semi-infinite plates or something like that, two flat plates. And at a certain time say times zero, the bottom plate is carefully moved in the positive x direction okay, slowly carefully moved in the positive x direction with reasonably small velocity v_x, okay I said slowly, the small velocity v_x, okay.

For you, I think I need to stick with my positive direction which is this. Therefore, the bottom plate is moved slowly in the positive x direction. So this is the situation. Here you have the top plate, you have the bottom plate. In reality the space between them is very small. Just a layer of liquid in between them. I have just blown that up to show you what is happening here.

We are interested in drawing insights into what is happening here, okay. So this is the x direction, this is the y direction. And we will stick to two dimensions to begin with. And then we will understand things much better here because it is more intuitive and then we will shift to three dimensions.

Now the situation here is that the top plate is stationary, the bottom plate is moved and there is a thin layer of fluid in between the plates. The bottom-most liquid layer okay, let us say the bottom most liquid layer is somewhere here. This will adhere to the plate and let us say it adheres to the plate and therefore it moves with the same velocity as that of the plate.

That is what is the situation here. I am asking you to imagine this. Sticking to the plate and moving at the velocity of the plate is a very good assumption that we will keep doing often. But the rest is idealized. We all know what shear stress is or we all know what stress is, force per unit area. Shear, we have an idea as we are all engineers, we know what a shear stress is. It distorts the surface and so on, okay.

So the shear stress due to the shear force exerted by the bottom most layer of the fluid that is going to influence the velocity of the fluid layer above it, okay. It is going to pull it along. There are shear forces between them. Therefore, this is a shear stress and that is going to have an effect on the layer just above it. And because of that effect, the next layer is going to start moving with a certain velocity.

And that layer is going to have a certain effect, shear stress on the layer above it, and that is going to start moving with a certain velocity, okay. So the shear stress exerted by the layer above the bottom layer influences the velocity of the layer above it, and so on and so forth, okay. So, the bottom most layer of the plate, the bottom plate is moving in the positive x direction with a velocity, small velocity v_x.

The layer closest to the bottom plate attaches to it and moves with a velocity of v_x. That is going to exert a shear force on the layer just above it and make it move with a certain velocity. The second layer is going to exert a shear stress on the layer above it and make it move with a certain other velocity and so on and so forth, okay. So this is the scenario here. I hope you have a mental picture of that clear.

We have been talking about shear stress and we need to understand it clearly, define it clearly and so on so forth. We know it is shear force per unit area. However, we use this term τ with two subscripts $_{yx}$. In this case, typically we have shear stress with two subscripts in this situation, τ_{yx}. And we need to understand what these are. y the first subscript gives the direction of the action. The direction of the action of this movement is in the y direction and therefore, that is what is indicated first. And x is the direction of motion. The motion in the x direction is having an effect in the y direction. So the first subscript is for the action, the direction of the action. The second subscript is for the direction of the motion. This is good enough to start out with.

I think you can take it for an initial understanding and once it becomes second nature to you then you would know how to write it appropriately. For starters, let us say it is good to see the first subscript as a direction of action and the second subscript as a direction of motion, okay. So in this case, τ_{yx} is the only relevant shear stress. There is motion in the x direction because of this in the fluid, there is an effect that is caused in the y direction and that shear stress is given as τ_{yx}. Okay, as I said simplistic view. Be ready to expand this view, make it a lot more complete as we go along. To start out with it is fine.

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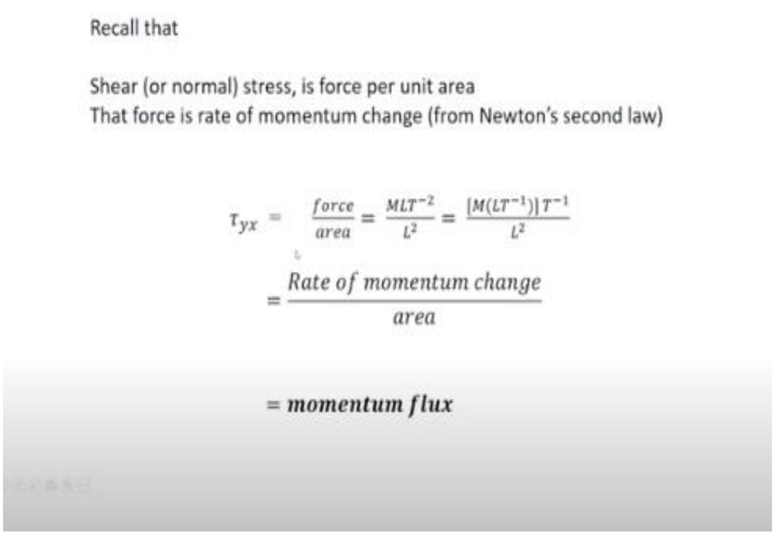

Recall that shear or there is something called normal stress, which we will come to in a little bit. It is nothing but force per unit area, we know this. That force is the rate of momentum change from Newton's second law, okay. Rate of change of motion is directly proportional to the force acting on it is Newton's second law for a body. So the force is nothing but the rate of momentum change.

So τ_{yx} is nothing but force per unit area. We said shear stress in this case. Let us write the dimensions of this to understand a little better or let us see whether we can express this in as a flux that we are looking at in our running theme of development is the flux. Let us see whether we can get into a flux form. Force is mass times acceleration MLT^{-2} (mass * acceleration = kg m s^{-2}). Area is L^2.

And let us group MLT^{-2} as $M(LT^{-1})\ T^{-1}$okay. Why am I doing this? This as you can recall is mass(M) times velocity(LT^{-1}), which is momentum. And there is a T^{-1} remaining here(per unit time), divided by L^2. So this is the numerator is nothing but the rate of momentum change. The denominator is nothing but the area. So what is this? The rate of momentum change per area.

$$\tau_{yx} = \frac{\text{Force}}{\text{Area}} = \frac{\text{MLT}^{-2}}{\text{L}^2} = \frac{[\text{M}(\text{LT}^{-1})]\text{T}^{-1}}{\text{L}^2}$$
$$= \frac{\text{Rate of momentum change}}{\text{Area}}$$
$$= \text{Momentum flux}$$

Do you recall what such a quantity is? Yes, it is momentum flux. Earlier we saw mass flux, mass rate with respect to time per unit area. In this case we have momentum. So the rate of momentum change with respect to area that is momentum flux. So here we have our momentum flux in the form of stresses. And our development will be in terms of these stresses. That will be the historical development. We are following the same thing here to understand the area better.

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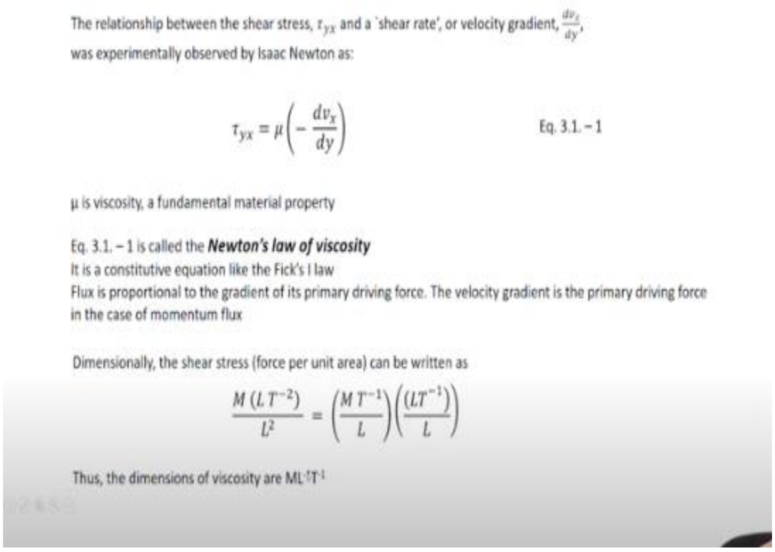

The relationship between the shear stress τ_{yx} and something called a shear rate. Shear rate is nothing but the velocity gradient. The rate of change of velocity with respect to distance, $\frac{dv_x}{dy}$ may be okay because the velocity in x direction changes as you move in the y direction, okay. That is what is called the shear rate. So velocity gradient is nothing but the shear rate $\frac{dv_x}{dy}$ is the relevant shear rate in this case.

This was experimentally studied by Isaac Newton who found that for simple fluids such as water, this relationship is something like this, okay. It is the shear stress or the momentum flux is let us say shear stress for the time. Then we will switch to momentum flux. The shear stress is proportional to the negative of the velocity gradient, okay. What does it mean by the negative here?

What is the velocity gradient if you take a slightly larger piece? It is $(v_2 - v_1)/(y_2 - y_1)$. In this case it is minus and therefore, it is $(v_1 - v_2)/(y_2 - y_1)$. That is essentially what this negative means, okay. So the shear stress is proportional to the negative of the shear rate and the constant of proportionality is called μ. μ is nothing but the viscosity of the solution.

The relationship between the shear stress, τ_{yx} and a 'shear rate', or velocity gradient $\frac{dv_x}{dy}$ was experimentally observed by Isaac Newton as

$$\tau_{yx} = -\mu \frac{dv_x}{dy} \qquad (3.1\text{-}1)$$

This is equation 3.1-1. μ is the viscosity and viscosity happens to be a fundamental physical property of the fluid, fundamental material property of the fluid. This equation 3.1-1 is called the Newton's law of viscosity. Does this remind you of something that you have already seen earlier? It is the same as Fick's first law. Fick's first law said that mass flux is directly proportional to the negative of the concentration gradient. And diffusivity was the constant of proportionality. Here, you have a momentum flux being proportional to the velocity gradient, negative of the velocity gradient and the constant of proportionality is your viscosity, okay. So same kind and it is valid for a class of fluids and therefore it is a constitutive equation, just like the Fick's first law, okay. So formally speaking, flux is proportional to the gradient of its primary driving force.

The velocity gradient happens to be the primary driving force in the case of momentum flux. The concentration gradient was the primary driving force in the case of mass flux. Here, the velocity gradient is the primary driving force in the case of momentum flux. So dimensionally the shear stress with this force per unit area, it can be written as ($M LT^{-2} / L^2$) which is I am just grouping this differently to represent this, right. It is LT^{-1} is v_x and L is y ($\frac{dv_x}{dy}$). Therefore, if you take this out what remains is $MT^{-1} L^{-1}$ and that is the unit of viscosity.

$$\left(\frac{MT^{-1}}{L}\right)\left(\frac{(LT^{-1})}{L}\right)$$

Thus, the dimensions of viscosity are $ML^{-1}T^{-1}$.

Remember this $ML^{-1} T^{-1}$ are the dimensions of viscosity. Dimensions of viscosity not units, dimensions of viscosity. So this is the way we normally find out dimensions of a new quantity. We take an equation that contains the quantity and work out the units of the other things. And whatever remains turns out to be the unit of the quantity that we are looking at. This is nice to remember $ML^{-1}T^{-1}$ itself. If you do not remember this equation which is now easy to remember. It is a constitutive equation of the same form as the Fick's first law and then you can work out the dimensions whenever needed. And you need the dimensions quite often, okay.

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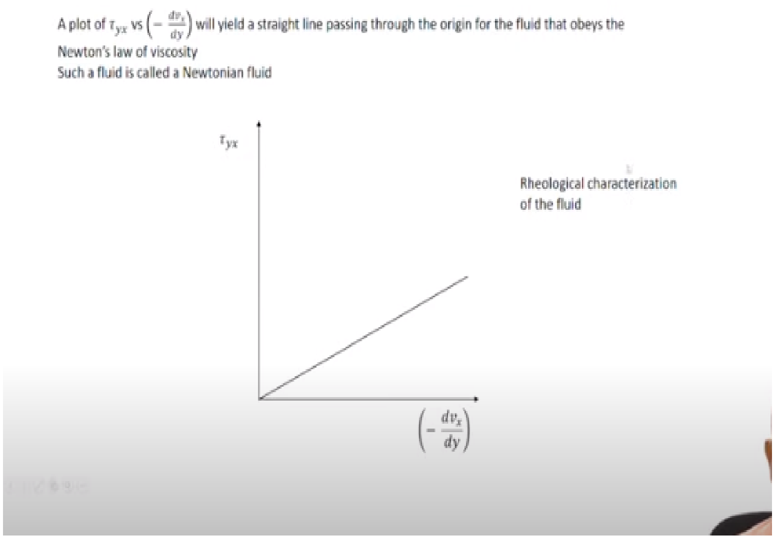

It is quite obvious that a plot of the shear stress τ_{yx} versus the shear rate $-\frac{dv_x}{dy}$ will yield a straight line passing through the origin, okay, y versus x for a fluid that obeys the Newton's law of viscosity and such a fluid is called the Newtonian fluid. It is very common term Newtonian fluid. So this is the graph of τ_{yx} versus $-\frac{dv_x}{dy}$. It is a straight line that is passing through the origin, okay?

And of course, the slope will give you the viscosity, the slope of this line, y equals mx slope will give you the viscosity. This kind of a characterization of shear stress versus shear rate is a very standard characterization in fluids, fluid mechanics, and so on so forth. It is called rheological characterization. The rheological behavior explains how or describes how the shear stress is related to the shear rate and this characterization is called the rheological characterization. And rheology is a huge area by itself. There are books very many books written on rheology of fluids. When we meet next we will take things forward. See you.