Transport Phenomena in Biological Systems
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Lecture - 22
Fluid Flow Types

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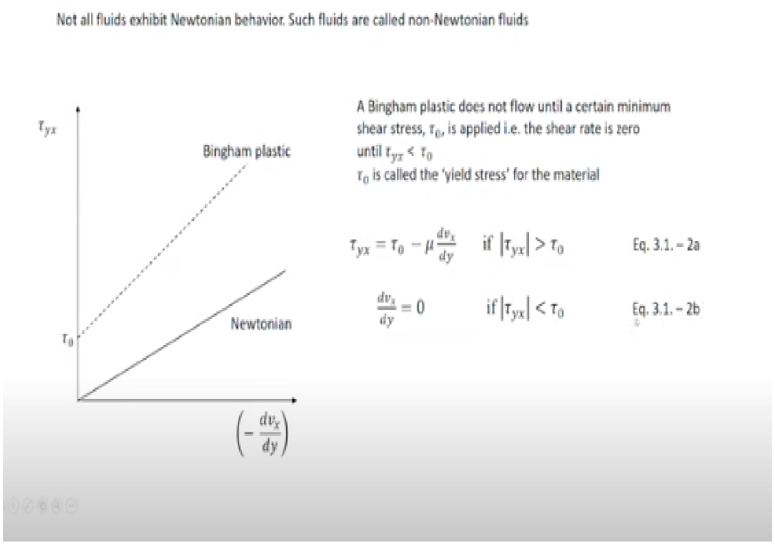

Okay, as is to be expected not all fluids exhibit Newtonian behavior. And the fluids that do not exhibit Newtonian behavior are called non-Newtonian fluids. This is our Newtonian fluid, a straight line passing through the origin of the τ_{yx} versus the shear rate - $\frac{dv_x}{dy}$ graph. You could have a fluid that is something like this, okay? It starts not from the origin or it does not pass through the origin but passes through a certain y intercept, okay.

So what do you think this kind of a fluid will behave as? Can you think of that? You see that the hint is the shear rate which is the variation of velocity with distance when it is zero there is a certain shear stress and only beyond that do you have some value for the shear rate itself, okay. So that is the hint here, think about it, pause the video here think about it, then come back we will discuss, okay.

Typically this interceptor is called τ_0 or the yield stress. The fluid which exhibits this kind of a behavior is called a Bingham plastic. And what Bingham plastics do is that they do not move. There is no velocity gradient, there is no change of velocity with distance until a certain τ_0 is applied. And beyond the τ_0, it behaves like a Newtonian fluid.

Therefore, there is a certain yield stress that is required to make it move. And after you make it move then it behaves like a Newtonian fluid okay. This is the characteristic of Bingham plastic. Bingham plastic does not flow until a certain minimum shear stress τ_0 is applied. The shear rate is zero until τ_{yx} < zero. Yield stress we talked about, okay.

And an example of such a fluid is the toothpaste that you use every morning okay. You take a toothpaste, you just keep it there, nothing is going to move out of it, paste tube. You press it, you provide a certain yield stress. Once it crosses a certainly yield stress, then it starts moving, okay. So that is a very standard example of a Bingham plastic remember that.

Mathematically, you know the Newtonian fluid we was described by the Newton's law of viscosity τ_{yx} equals μ(- $\frac{dv_x}{dy}$). So this Bingham plastic is described by the set of equations τ_{yx} equals τ_0 - μ($\frac{dv_x}{dy}$). If $\tau_{yx} > \tau_0$, if not ($\frac{dv_x}{dy}$) $= 0$, if $\tau_{yx} < \tau_0$ okay. So it does not move till this is applied and after it is applied, it starts moving, and it shows a typical Newtonian behavior.

A Bingham plastic fluid exhibits a rheology different from a Newtonian one. It does not flow until a certain minimum shear stress, τ_0, is applied i.e. the shear rate is zero until $\tau_{yx} < \tau_0$. τ_0 is called the 'yield stress' for the material.
It can be represented as

$$\tau_{yx} = \tau_0 - \mu \frac{dv_x}{dy} \quad \text{if } |\tau_{yx}| > \tau_0 \qquad (3.1\text{-}2a)$$

$$\frac{dv_x}{dy} = 0 \quad \text{if } |\tau_{yx}| < \tau_0 \qquad (3.1\text{-}2b)$$

Of course, it does not pass through the origin, it passes through τ_0. That is why you have an intercept here y = mx + C. So we call the first one 3.1-2a. Second one as 3.1-2b. Okay, so we saw a Newtonian fluid and a Bingham plastic. Those are not the only ones there. There are many more complex fluids in nature. Blood is a very complex fluid.

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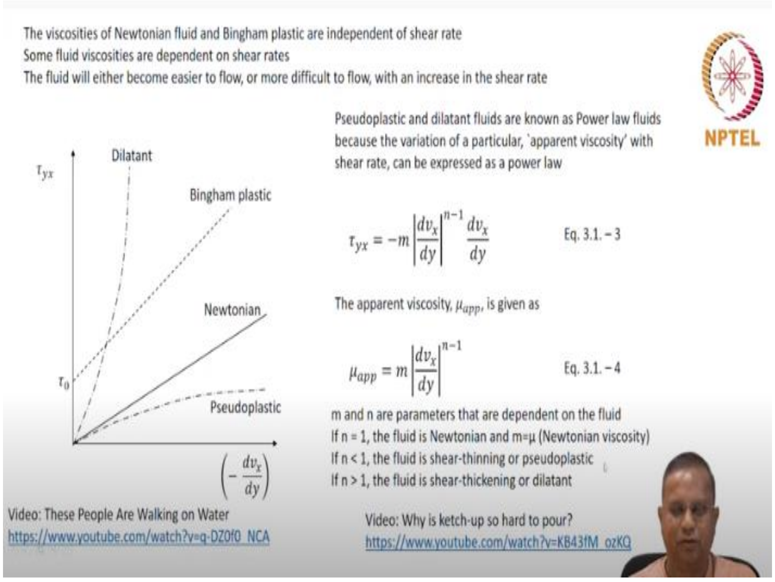

The viscosities of Newtonian fluid and Bingham plastic are independent of shear rate, they were constants, okay. They were not dependent on the shear rate. However, the viscosity could be dependent on the shear rate for certain fluids, it can always happen. The fluid will become either easier to flow or more difficult to flow with an increase in shear rate. That is what it means, okay. Which means if you stir harder certain things will become harder to stir, and certain things will become easier to stir as you increase the speed of stirring. That is what it means. So on the rheological characterization graph τ_{yx} versus (- $\frac{dv_x}{dy}$) is a Newtonian fluid and this is the Bingham plastic with τ_0 the yield stress and these fluids are either like this, which is shear thinning okay the viscosity decreases with increasing shear rate. So the faster you stir, the higher the variation of velocity with distance and lower is its proportionality with shear stress, which means it becomes shear thinning, okay. It becomes easier to stir if you increase the speed of stirring at higher and higher speeds of stirring, which is given as you move in this direction, it becomes a flatter curve. The shear thinning is called a pseudoplastic fluid.

It could also move in the other direction. As you stir it faster and faster it becomes more difficult to stir and that is represented by this curve. The shear thickening is called a dilatant fluid, okay.

Pseudoplastic fluids, typical paints are pseudoplastic. You stir faster it becomes easier and quicksand is a very good example of a dilatant fluid. The faster you try to move, the more difficult it gets to move okay because the viscosity increases with shear rate quite steeply.

Pseudoplastic fluids and dilatant fluids are known as power law fluids because of the way we mathematically represent them. Because the variation of a particular apparent viscosity with shear rate can be expressed as a power law. What I mean by that is the equation that describes this behavior either this or this is τ_{yx} equals -m($\frac{dv_x}{dy}$) *raised* to the power of n – 1 times ($\frac{dv_x}{dy}$).

$$\tau_{yx} = -m\left|\frac{dv_x}{dy}\right|^{n-1}\frac{dv_x}{dy} \qquad (3.1\text{-}3)$$

where the apparent viscosity, μ_{app}, is given as

$$\mu_{app} = m\left|\frac{dv_x}{dy}\right|^{n-1} \qquad (3.1\text{-}4)$$

where m and n are parameters that are dependent on the fluid.

- If $n = 1$, the fluid is Newtonian and $m = \mu$ (Newtonian viscosity)
- If $n < 1$, the fluid is shear-thinning or pseudo-plastic
- If $n > 1$, the fluid is shear-thickening or dilatant

So you have this term depending on the shear rate, okay. That is what I meant by the viscosity is being dependent on shear rates. The apparent viscosity which is this term is given as m($\frac{dv_x}{dy}$)$^{(n-1)}$ This negative actually goes with this, we all know that and equation 3.1-4 where m and n are the parameters that are dependent on the fluid.

If n equals 1, the dependence on shear rate goes away and you get back here Newtonian fluid m equals mu, then Newtonian viscosity. If n is less than 1, the fluid will be shear thinning or pseudoplastic which is this. If n is greater than 1, the fluid is shear thickening or dilatant, which is this, okay. So these are the mathematical representations which will become useful.

- If $n = 1$, the fluid is Newtonian and $m = \mu$ (Newtonian viscosity)
- If $n < 1$, the fluid is shear-thinning or pseudo-plastic
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What I would like you to do is watch this video, okay. I cannot show you this video here for obvious reasons. They are available from standard sources. It is a fun video and it will give you an idea of at least one type of fluids. This is called these people are walking on water. It is a fun video with a lot of peppy music associated with it and so on so forth. Please watch this. **(Video Starts: 08:42) (Video Ends: 11:18).**

And why is ketchup so hard to pour is another video, okay. This will explain how the molecular nature of the substance of ketchup determines why it becomes so difficult to pour, okay. So I would recommend that you watch these videos. **(Video Starts: 11:38) (Video Ends: 16:02).**

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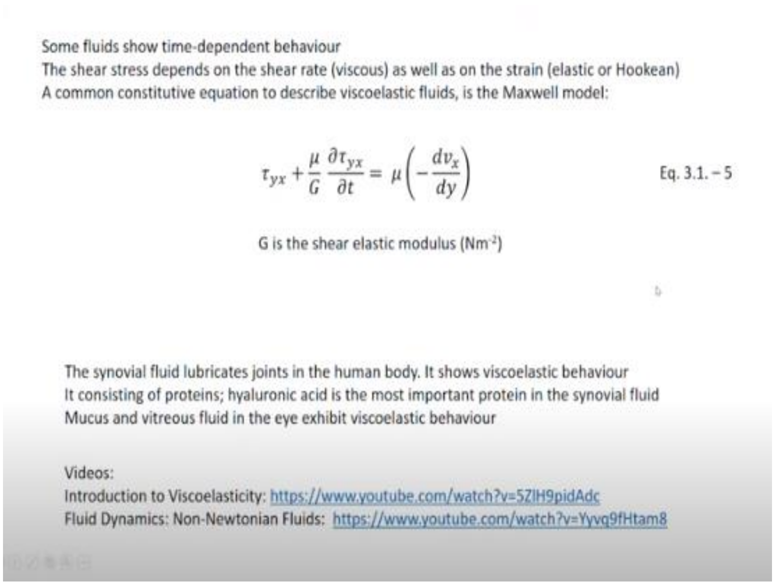

Apart from this, there are other fluids also okay and some of those are biologically very important. They show time-dependent behavior, okay. The shear stress depends on the shear rate as well as on something called a strain, which could be either elastic or Hookean. Do not worry too much about it. And a common constitutive equation to describe viscoelastic fluids is what is called the Maxwell model, which has given as this,

$$\tau_{yx} + \frac{\mu}{G}\frac{\partial \tau_{yx}}{\partial t} = \mu\left(-\frac{dv_x}{dy}\right) \qquad (3.1\text{-}5)$$

where G is the shear elastic modulus (Nm^{-2}).

This is the time variation that is brought in here and G is the shear elastic modulus (Nm^{-2}). And the synovial fluid that synovial fluid that lubricates joints in the human body is a viscoelastic fluid. Also I think the hyaluronic acid, yes the hyaluronic acid is which is present in the eye and so on so forth, which provides a lot of lubrication, which is the important protein in the synovial fluid also. This itself is a viscoelastic fluid.

Mucus and vitreous fluid in the eye exhibit viscoelastic behavior as hyaluronic acid present in it. At least the vitreous fluid has that.

You could watch these two videos, introduction to viscoelasticity as well as fluid dynamics non-Newtonian fluids to gain more appreciation of these kinds of fluids, viscoelastic fluids which will be nice. **(Video Starts: 17:53) (Video Ends: 27:11).**

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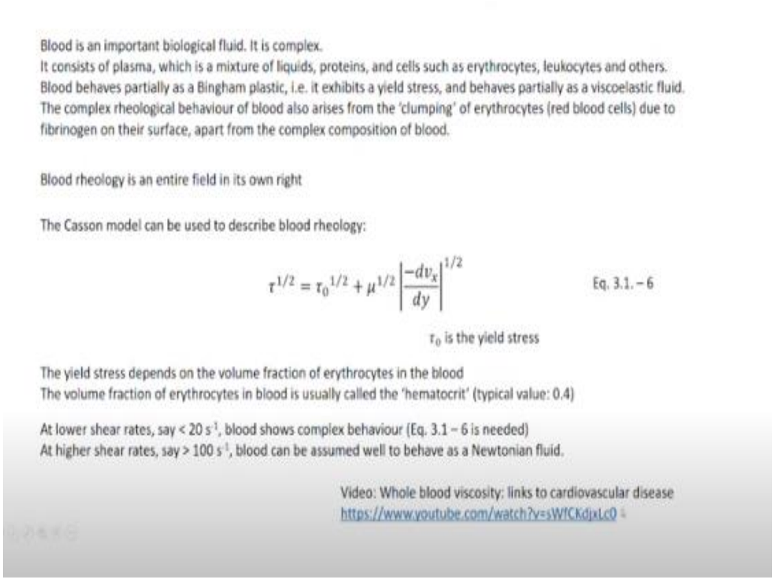

Blood is a very important biological fluid, synovial fluid is also important. Blood, we all know. It is a highly complex fluid. Books have been written on blood rheology. It consists of plasma which is a mixture of liquids, proteins and many different cells. We all know this; erythrocytes, leukocytes and so on so forth. Blood behaves partially as a Bingham plastic, it exhibits a yield stress and behaves partially as a viscoelastic fluid okay, time-dependent behavior and so on.

The complex rheological behavior of blood also arises from the clumping of RBCs erythrocytes due to the fibrinogen on their surface apart from the complex composition of blood. And as I said, there are books written on blood rheology. It is a field in its own right. And one of the models that is used to describe blood rheology is what I am just going to present in this introductory course and leave it at that. You need to know what kind of model is used. And that is the reason I am giving you this. We would not deal too much or delve too much into this model. The Casson model is something like this.

The Casson model can be used to describe blood rheology. It can be stated as

$$\tau^{1/2} = \tau_0^{1/2} + \mu^{1/2} \left| \frac{-dv_x}{dy} \right|^{1/2} \qquad (3.1\text{-}6)$$

where τ_0 is the yield stress.

This is the Casson model which describes blood rheology. τ_0 is the yield stress and rest you know. The yield stress actually depends on the volume fraction of erythrocytes RBCs in the blood, okay. So the yield stress is dependent on that. And the volume fraction of erythrocytes and blood is typically called the hematocrit. These are names that you need to know, it directly impinges on the blood rheology. A typical value for the hematocrit is 0.4.

Interestingly, at low shear rates of less than $20s^{-1}$, the blood shows complex behavior. So if the blood is flowing slow, then you need to worry about the complex nature of blood. If the blood is flowing fast in as in some blood vessels, then it behaves very close to a Newtonian fluid, okay. That is what happens. So if you are studying the blood flow in probably certain large arteries, you can approximate it to Newtonian flow and there would not be much of a problem at all. That is here at high shear rates greater than $100s^{-1}$, the blood can be assumed well to behave as a Newtonian fluid. You might want to watch this video to gain better understanding, appreciation and so on, whole blood viscosity links to cardiovascular disease.

Please watch this recommended video. **(Video Starts: 30:21) (Video Ends: 35:40).** Okay. So what we looked at so far is the relationship between shear stress and shear rate which is a standard characterization for fluids. The rheological characterization. The difference types of fluids Newtonian fluid first shear stress shear rate, straight line passing through the origin. Then we had a Bingham plastic, shear stress versus shear rate, again a linear curve, but not passing through the origin. Then, pseudoplastic fluid, a dilatant fluid and then we went into fluids whose viscosity depends on the shear rate, viscoelastic fluid. And also there could be a time dependent variation. And then we briefly looked at blood, which is a highly complex fluid rheologically speaking and even otherwise it is a highly complex fluid. And the Casson model is used to describe that. When we meet next we will take things forward. See you.