Transport Phenomena in Biological Systems

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Lecture – 30
Capillary Flow

Welcome to this lecture. Today let us start looking at capillary flow which has again wide applications. Wherever you see a capillary, you could use this. I think I have formally listed down a few, so let me go forward.

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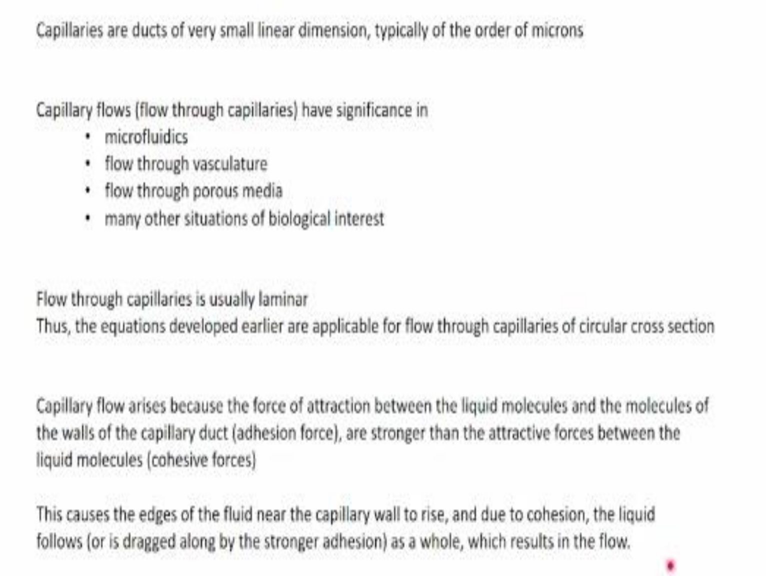

Before that, capillaries are ducts of very small linear dimension okay, very thin diameter if you want to call it so, typically of the order of microns that is what a capillary is. Capillary flows or flow through capillaries, capillaries have significance in various different aspects. Some of those are microfluidics, a lot of microfluidics is capillary flow. Flow through vasculature, a lot of bronchioles, arterioles are all capillaries.

Flow through porous media okay. Flow through porous media has very wide application, so they want to look at flow through soil flow in wastewater treatment aspects and through beds and so on so forth through clear filters or you could even look at organs as some sort of porous media in the body or some tissues as far as media in the body and so on and so forth. So wide applications of these fundamental aspects that here that we are going to look at in this lecture. And of course many other situations of biological interest. We are only limited by your visualization of capillaries in the system that you are working with if there is a need. Flow through capillaries is usually laminar why because you know Reynolds number $\rho vd/\mu$ velocities are usually small, the diameter is small okay, so diameter being of the order of microns 10^{-6} meters.

You will have to you know that ρ , v and μ that combination needs to be very, very high for you to get into a Reynolds number of greater than 2100 because even if you consider a cylindrical capillary, capillaries are usually cylindrical. So let me just say for completeness cylindrical capillaries, so usually the flow is laminar and we have already looked at laminar flow okay.

So we just have to go and see what we have for laminar flow, which aspect of it is different for capillary, how can we illustrate or how can we bring out the capillary nature of whatever variable that we are dealing with and whether we can represent capillary flows through that modification that is the strategy that we are going to use here. So let me just read this out. Thus, the equations developed earlier are applicable for flow through capillaries of circular cross section.

It is good to understand some physics of capillary flows. Capillary flow arises because the force of attraction between the liquid and the wall of the capillary duct, there is a certain force that force is called an adhesion force, that force happens to be greater than the attractive forces between liquid molecules which are called cohesive forces. The attraction between liquid molecules is a cohesive force that holds it like this.

The attraction between the liquid and the wall of the capillary is an adhesion force that is greater than this and therefore it starts moving in the direction of this, and once it starts moving the cohesion force takes the liquid along with the rest of it, so that is what happens. So this causes the edge of the fluid near the capillary wall to rise and due to cohesion the liquid follows or is dragged along by this stronger adhesion as a whole which results in the flow.

So this causes the edges of the fluid near the capillary wall to rise because of the adhesive force and there is a cohesion force between the liquid molecules therefore the liquid follows or is dragged along by the stronger adhesion as a whole, which results in the flow.

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Cohesion results in a force that is usually represented as a force per unit length, or surface tension, γ

The capillary pressure due to surface tension at that point, or the meniscus, in a capillary of radius, r, is given by the appropriate simplification of the Young-Laplace equation (For the derivation, see Berg JC. 2009. Introduction to Interfaces and Colloids. World Scientific)

$$p_{st} = \frac{2\gamma}{r}\cos\theta \qquad \text{Eq. 3.4.2.1.} - 1$$

where θ is the contact angle (wetting angle) between the liquid and the capillary wall

Note: this pressure is inversely proportional to the radius of the duct
This pressure becomes predominant in capillaries, and provides the 'driving force' for the bulk flow of the liquid through the capillary, even if other 'driving forces' are absent
When other 'driving forces' such as those provided by a liquid column, or an external pump are present, the pressures can be added to get the total pressure difference for the flow ($-\Delta P$).

To obtain the flow rate in pure capillary flow (when no other 'driving forces' for the flow are present) we can use the Hagen- Poiseuille relationship, Eq. 3.4.2. – 18

$$Q = \frac{\pi}{8\mu L} r^4 \left(\frac{2\gamma}{r}\right)\cos\theta = \frac{\pi\gamma}{4\mu L} r^3 \cos\theta \qquad \text{Eq. 3.4.2.1.} - 2$$

Cohesion results in a force that is usually represented as force per unit length or surface tension okay. So surface tension force we look at slightly differently when we analyze things. So you have already come across this I am sure in your high school and so on **so** or high secondary. So cohesion results in a force that is represented as force per unit length or surface tension γ.

The capillary force, capillary pressure due to surface tension at that point or the meniscus in a capillary of radius r has been already given by what is called the Young-Laplace equation okay. You simplify the Young-Laplace equation you will get an expression for the capillary pressure, we are not going to do that in this course. If you are interested or the interested students can look at JC Berg, there he would have derived how this comes about.

The book is called Introduction to Interfaces and Colloids published by World Scientific 2009, you can go and look at that. For now, we are just going to take that simplification and go forward. The simplification says that the capillary pressure due to surface tension can be given as 2 γ, γ is the surface tension here, force per unit length, where θ is the contact angle or the wetting angle between the liquid and the capillary wall okay.

$$p_{st} = \frac{2\gamma}{r}\cos\theta \qquad (3.4.2.1\text{-}1)$$

where θ is the contact angle (wetting angle) between the liquid and the capillary wall.

Let us say it is forming a lower meniscus as water, higher meniscus as mercury, so whatever the angle between the tangent to the surface and the wall at the point of contact that is called the contact angle or the wetting angle okay that is this θ here. This is equation 3.4.2.1 – 1 in the textbook. This tells you that the pressure is inversely proportional to the radius of the duct, this r is the radius of the duct.

And this pressure becomes predominant in capillaries and provides the driving force for bulk flow of the liquid through the capillary even if other driving forces are absent okay. Even if there is no other driving force, there is no other pressure difference, there is no pumping action or work that has been put in and so on and so forth, even if nothing is present just by the nature of substances the liquid starts going up.

When other driving forces such as those provided by liquid column or an external pump are present, the pressures can just be added to get the total pressure for the flow which is $-\Delta P$. The $-\Delta P$ you know comes from a convention $P_L - P_0$ is ΔP and therefore you take the negative of that to give you a part of the pressure driving force. Now to obtain the flow rate in pure capillary, for now let us not look at these other driving forces.

When no other driving forces for the flow are present we can directly you see Hagen-Poiseuille equation. To recall the Hagen-Poiseuille equation gave us the flow rate as a function of pressure drop, it was $\pi r^4/(8 \mu L)$, r is the radius of the pipe, times $-\Delta P$ okay, so that is what was the expression for flow rate. In this case, instead of the bulk pressure there, difference there we have the capillary pressure due to surface tension. So, if we replace ΔP by $(2 \gamma/r) \cos\theta$, we get an expression for the flow rate through capillary action okay or capillary flow which turns out to be when you simplify this,

$$Q = \frac{\pi}{8\mu L} r^4 \left(\frac{2\gamma}{r} \right) \cos\theta = \frac{\pi\gamma}{4\mu L} r^3 \cos\theta \qquad (3.4.2.1\text{-}2)$$

This is equation number 3.4.2.1 – 2.

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Flow rate = penetration velocity (v_p) X C. S. area

Thus

$$v_p = \frac{Q}{C.S.area} = \frac{Q}{4\pi r^2}$$

$$v_p = \frac{dL}{dt} = \frac{\gamma}{4\mu L} r\cos\theta \qquad \text{Eq. 3.4.2.1.-3}$$

The above equation can be integrated to get the position of the liquid front along the capillary as a function of time, in microfluidic situations

Now the flow rate is nothing but velocity times cross section area. In the context of capillary flow, the velocity is called the penetration velocity because it penetrates into the matrix that is probably why it is called the penetration velocity. Penetration velocity is indicated by the symbol V_p times the cross sectional area equals the flow rate. Therefore, the penetration of velocity is nothing but the flow rate divided by the cross-sectional area.

In this case flow rate by $4\pi r^2$, it is a circular cross section and if we substitute the value for the flow rate from the Hagen- Poiseuille equation that we just found we get

Since the flow rate is a product of the cross-sectional area and the penetration velocity, the penetration velocity (v_p) can be obtained by dividing the above equation by the cross-sectional area πr^2

$$v_p = \frac{dL}{dt} = \frac{\gamma}{4\mu L} r\cos\theta \qquad (3.4.2.1\text{-}3)$$

So, you can work this out and see whether you get this, this is the straightforward thing, you could do this. You also know that V_p is nothing but $\frac{dL}{dt}$, rate of change of distance with time. The distance in this case the distance traversed in capillary flow by the liquid 3.4.2.1 – 3 and therefore you could integrate this expression to get the position of the liquid front along the capillary as a function of time in microfluidic situations which is an important thing to know for design, operation and so on okay.

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Capillary flow in porous media:

Porous media is a term that refers to any medium that has a solid matrix with interconnected interstitial spaces, through which there is movement of some species of interest.
For example, soil is a porous medium through which water, pollutants, fines, etc., can travel.
Sometimes, the interstitial spaces are considered as a set of capillary tubes, and thus capillary flow through porous media is an area with wide applications.

Many substances of biological interest can be considered as porous media.

- Any tissue, including whole organs such as liver, kidney, heart, brain, etc., can be treated as porous media because they contain cells that are dispersed, and connected voids through which nutrients, drugs and other substances travel to reach the cells.
- Tissue regeneration, which is used to grow artificial organs, typically happens on a scaffold, and this system can be considered a porous medium.
- The biological pollution treatment system such as the trickling filter, or the matrix in which cells immobilized in a type of bioreactor, can be treated as porous media.

Now capillary flow and porous medium. There is a medium, the flow through the medium is capillary flow. The porous media is a term that refers to any medium that has a solid matrix with interconnected interstitial spaces through which there is movement of some species of interest. So the species of interest moves through the interconnected interstitial spaces and it can be considered as capillary flow because the interconnected interstitial spaces are very small in their characteristic dimension.

For example soil in a porous medium through which water, pollutants and fines, etc., can travel is an example of capillary flow through porous medium and sometimes the interstitial spaces are considered as a set of capillary tubes. We hypothesize the interconnected initiation spaces to be represented by a set of capillary tubes in parallel that is typically how we view the situation. We model the situation as a set of capillary tubes.

Thus capillary flow through the porous media is an area with wide applications. Many substances of biological interest can be considered as porous medium as I mentioned earlier. For example any tissue including whole organs such as liver, kidney, heart, brain, so on can be treated as porous media because they contain cells that are dispersed and connected voids through which nutrients, drugs and other substances travel to reach the cells okay.

So this is of direct interest to us. Tissue regeneration which is used to grow artificial organs typically happens on a scaffold, you all know this. We have a scaffold and then we grow cells to provide the support for the growth of cells and then the scaffold is either removed or it becomes a part of the artificial tissue and this system can be considered as a porous medium.

The biological pollution treatment system such as trickling filter or the matrix in which cells are immobilized in the type of a bioreactor can be treated as porous medium. So, you see the applications are very wide.

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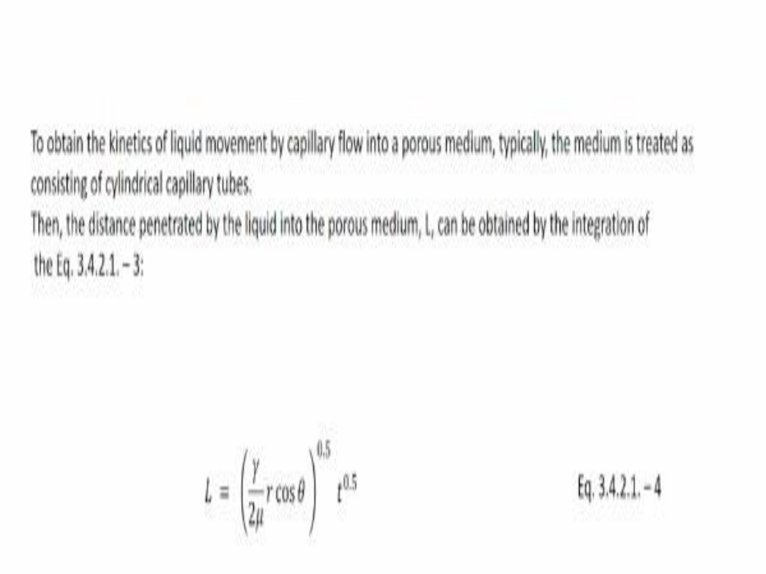

To obtain the kinetics of liquid movement by capillary flow into a porous medium and that would become important when you are trying to model things to understand further so that you can use it for improved design in operation. Typically, the medium is treated as consisting of cylindrical capillary tubes. Then the distance penetrated by the liquid into the porous medium, the distance is L.

This can be obtained by the integration of this equation 3.4.2.1-3. So you take L this side and dt this side, you can integrate this quite straightforward integration, you would get

$$L = \left(\frac{\gamma}{2\mu} r\cos\theta\right)^{0.5} t^{0.5} \qquad (3.4.2.1\text{-}4)$$

So you have an expression for the distance penetrated by the liquid into the porous medium through assimilated capillary flow or representative capillary flow okay. This is very useful. I think that will give you some idea as to how to start looking at capillary flows, this is just the starting point, how you use it is entirely up to you according to your requirements in the future. I think we can stop here, yeah the number is 3.4.2.1 – 4 which is important by the way, and when we meet next let us get into something else in momentum flux. See you then.