Transport Phenomena in Biological Systems
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Lecture – 31
Couette Flow

Welcome back. Today let us look at tangential annular flow. What do we mean by that? There are two concentric cylinders okay and we are looking at a liquid that is placed in the annulus or the annular space between these two cylinders and one of those cylinders or both of those cylinders could rotate. When that happens the liquid is forced to move in a tangential direction, right.

The velocity at any point is tangent to the radius and that is why we call this tangential annular flow. This has very many interesting applications. We have used it in our own research some years ago. I will probably give touch upon that also in this lecture.

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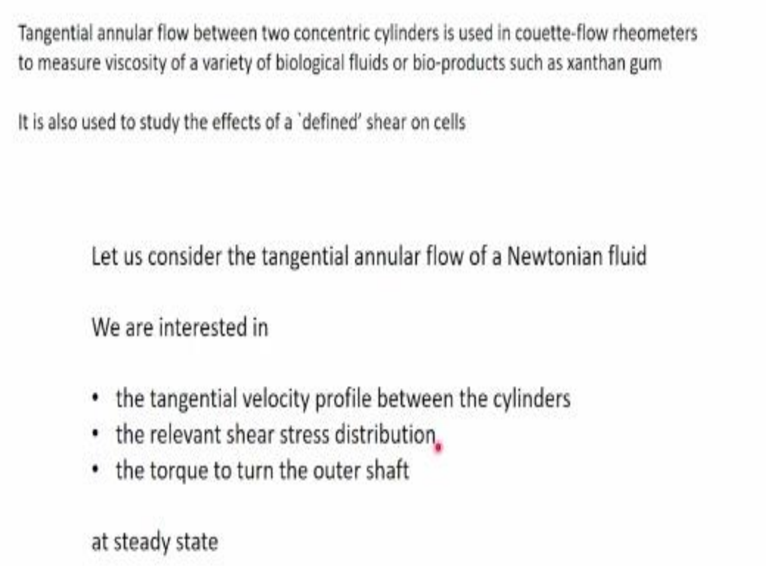

Tangential annular flow between two concentric cylinders is used in something called couette-flow rheometers, such a flow is called couette flow or is one of the examples of couette flow. So, this flow is used in couette flow rheometers to measure the viscosity of a variety of biological fluids or bio-product such as xanthan okay. The couette flow viscometer works on this principle. It is also used to study the effects of a defined shear on cells.

The shear range is rather narrow when we used this kind of a device and that is the reason why we used this kind of a device when we wanted to study the effect of defined shear stress on cells over the entire cultivation over a period of about 48 hours plus and so on. So let me tell you that in a little while from now. So, let us consider the tangential annular flow of a Newtonian fluid, we will you deal with Newtonian fluid.

You very well know that if it is not a Newtonian fluid, you will have to use the other complex equation, you just cannot use the simplified one with constant ρ and μ for Newtonian fluid. So that is the main difference in terms of approach. What we are interested in this as usual we are interested in the tangential velocity profile between the cylinders because we get a lot of relevant information if we know the tangential velocity profile.

And the relevant shear stress distribution, this is directly relevant if you are looking at the effect of shear stress on cells as we did in some of our studies and we are also interested in here to find out the torque to turn the outer shaft. So, our analysis is kind of focused towards these aspects okay. I had also mentioned earlier that we look very many different things. We are working out problems as a part of learning itself okay.

Then of course you will have your assignment aspects to take care of to strengthen your understanding in the given context or within the limitations of the given context. So, if you want more problems, please look at the end of the textbook, you have quite a few problems there which you can work out. However, the problems have been integrated into the class itself.

In fact, tutorials as they are called are a part of class itself in this class design or in this course design okay. So look for it and do not feel that there are no separate problems. I have been projecting problems first and then giving you solutions in the context, giving you very general solution in the context as well as solving the problem in the process okay. So, we are doing all that as a part of this course.

Coming to this, I have posed the problem as this. We are interested in the velocity profile, the shear stress distribution and the torque required to turn the outer shaft and all these we want to do at steady state because the flow will be at steady state most of the time. Initially, probably a few minutes it will take to stabilize and then when you shut it off maybe it might take a few minutes for it to dissipate and so on and so forth.

Then it may not be at steady state, otherwise it is going to be at steady state. The properties of interest which are the velocity, the shear stress and so on and so forth at a particular point in the flow do not change with time and that is how pretty much the entire flow is going to be over the entire period of cultivation as it was in our case.

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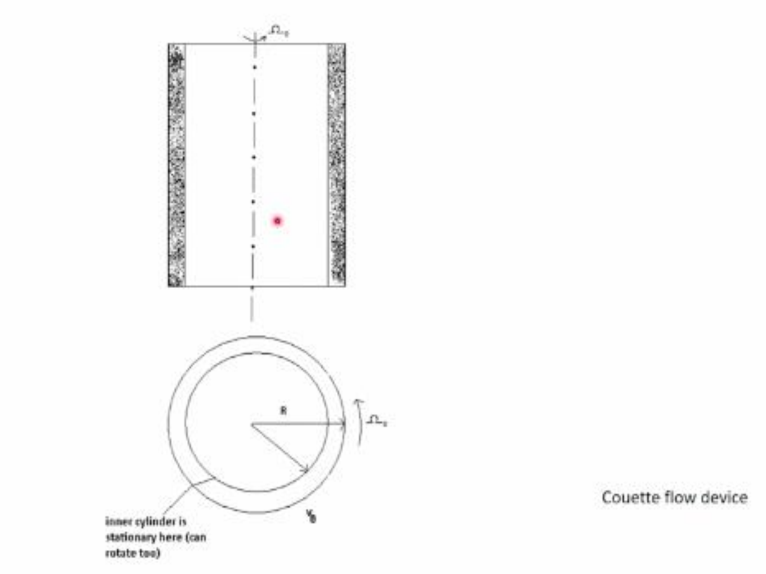

So, this is the figure here schematic here, this is the longitudinal section. You have the axis of these cylinders here. You have the inner cylinder represented by these 2 lines. You have the outer cylinder represented by these 2 lines okay. It is a cylinder you cut it like this, it's going to look like this and this is rotating in this direction with an angular velocity of Ω_0. If you look at it from the one of the ends, it is going to look like a concentric set of circles.

The terminology is going to be like this. Let us take the outer radius as capital R and the inner radius as some κ times R, so κ gives you the fraction of the outer radius to the inner radius. So, this is rotating at an angular velocity of Ω_0. The linear velocity is v_θ, the tangential velocity is v_θ. We are going to take in this case that the outer cylinder alone rotates.

There are some advantages to that geometry or that scenario where the outer cylinder rotates. Theoretically, any other cylinders can rotate or both of the cylinders can rotate okay. However, there are some wide ranges of Reynolds numbers over which you can assume the conditions that are relevant for us if the outer cylinder rotates. The inner cylinder is stationary here. So this is what we call as the couette flow device.

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This is a cylindrical system, and hence it is most convenient to use cylindrical coordinates for analysis
From Equation A2 of Table 3.4. – 2 for the *r*-component

=0, SS $v_r = 0$ =0, $v_r = 0$ =0, $v_r \neq f(\theta)$ =0, $v_z = 0$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta{}^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$

=0, $v_r = 0$ =0, $v_r = 0$ =0, $v_\theta \neq f(\theta)$ =0, $v_r = 0$ =0, $g_r = 0$

$$+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right) + \rho g_r$$

$$-\rho\frac{v_\theta{}^2}{r} = -\frac{\partial p}{\partial r} \qquad \text{Eq. 3.4.3.} - 1$$

As you can see this is a cylindrical system and therefore the most convenient set of equations to use from the tables, hopefully you made a copy, is the equation A2 from table 3.4 – 2 okay and this is for the r-component, let us start with the r-component. Let us apply that momentum balance to the system and let us see what we are going to get. This is the r-component, check the equation, go back and see.

This is a cylindrical system, and hence it is most convenient to use cylindrical coordinates for analysis. From Eq. A2 of Table 3.4-2, we get the equation of motion in the *r* direction as

= 0, SS $v_r = 0$ = 0, $v_r = 0$ = 0, $v_r \neq f(\theta)$ = 0, $v_z = 0$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right)$$

= 0, $v_r = 0$ = 0, $v_r = 0$ = 0, $v_\theta \neq f(\theta)$ = 0, $v_r = 0$ = 0, $g_r = 0$

$$= -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right) + \rho g_r$$

We are doing a steady state analysis therefore any time derivative is set to 0, so this goes out. There is only a v_θ here, right, you can imagine this, there is no v_r, there is no v_z. So v_r is 0, so this term goes to 0, v_θ exists but v_r is 0, so this term goes to 0. The v_θ of course is very much there, v_z is 0 and therefore this term goes to 0. The $\frac{\partial p}{\partial r}$ let us keep this, we do not know much about this.

There is no v_r, therefore this goes to 0. This goes to 0 for the same reason and here v_θ is the same at any θ, constant angular velocity right therefore v_θ is not different at different θ, in other words v_θ is not a function of θ. Therefore, the derivative with respect to θ of v_θ goes 0 and v_r is 0, so this goes to 0, and g is acting in this direction, therefore g_r which is perpendicular to that, there is no component of g in the r direction, therefore this goes to 0.

Thus

$$-\rho\frac{v_\theta^2}{r} = -\frac{\partial p}{\partial r} \qquad (3.4.3\text{-}1)$$

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From equation B2 of Table 3.4. – 2 (θ component)

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta$$

$p \neq f(\theta)$ because the points at different angles at the same radial position cannot have different pressures

Therefore, $$0 = \mu\left[\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r v_\theta)\right)\right] \qquad \text{Eq. 3.4.3. – 2}$$

Since r is the only variable, the partial derivatives have been converted into ordinary derivatives

Now, from equation B2 of the same table for the θ component, the first one was for the r-component, this one is for the θ component. You get this equation and if you see which terms are relevant, this is a steady state, there is no variation with time, so this goes to 0. The v_r is 0, so this entire term goes to 0, v_θ is there, but v_θ is not a function of θ, therefore this derivative goes to 0.

From Eq. B2 of Table 3.4-2 (θ component)

$$\rho\left(\underbrace{\frac{\partial v_\theta}{\partial t}}_{=0\ (SS)} + v_r \underbrace{\frac{\partial v_\theta}{\partial r}}_{(v_r=0)\ 0} + \frac{v_\theta}{r}\underbrace{\frac{\partial v_\theta}{\partial \theta}}_{v_\theta \neq f(\theta)\ 0} + \underbrace{\frac{v_r v_\theta}{r}}_{(v_r=0)\ 0} + v_z \underbrace{\frac{\partial v_\theta}{\partial z}}_{(v_z=0)\ 0}\right) = -\frac{1}{r}\underbrace{\frac{\partial p}{\partial \theta}}_{=0\ (p \text{ not a } f(\theta))}$$

$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\underbrace{\frac{\partial^2 v_\theta}{\partial \theta^2}}_{v_\theta \neq f(\theta)\ 0} + \frac{2}{r^2}\underbrace{\frac{\partial v_r}{\partial \theta}}_{v_r \neq f(\theta)\ 0} + \underbrace{\frac{\partial^2 v_\theta}{\partial z^2}}_{v_\theta \neq f(\theta)\ 0}\right] + \underbrace{\rho g_\theta}_{(g_\theta=0)\ 0}$$

So what we have remaining? Please note this, I think this is a left over from some previous thing, this the certainty is there and p is not a function of θ because the points at different angles at the same radial position cannot have different pressures okay.

In the above equation, $p \neq f(\theta)$ because the points at different angles at the same radial position cannot have different pressures.

Thus

$$0 = \mu\left[\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r v_\theta)\right)\right] \qquad (3.4.3\text{-}2)$$

Since r is the only variable, the partial derivatives have been converted into ordinary derivatives.

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The z- component

$$\rho\left(\underbrace{\frac{\partial v_z}{\partial t}}_{0} + v_r \underbrace{\frac{\partial v_z}{\partial r}}_{(v_r=0)\ 0} + \frac{v_\theta}{r}\underbrace{\frac{\partial v_z}{\partial \theta}}_{v_z \neq f(\theta)\ 0} + v_z \underbrace{\frac{\partial v_z}{\partial z}}_{(v_z=0)\ 0}\right) = -\frac{\partial p}{\partial z}$$

$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\underbrace{\frac{\partial v_z}{\partial r}}_{(v_z=0)\ 0}\right) + \underbrace{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}}_{(v_z=0)\ 0} + \underbrace{\frac{\partial^2 v_z}{\partial z^2}}_{(v_z=0)\ 0}\right] + \rho g_z$$

Therefore, $$0 = -\frac{\partial p}{\partial z} + \rho g_z \qquad \text{Eq. 3.4.3.-3}$$

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The boundary conditions (BCs) for integrating Eq. 3.4.3. – 2:

At $r = kR$, $v_\theta = 0$ (inner cylinder is stationary)

At $r = R$, $v_\theta = \Omega_0 R$

$$\left[\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rv_\theta)\right)\right] = 0$$

$$\frac{1}{r}\frac{d}{dr}(rv_\theta) = C_1$$

$$\frac{d}{dr}(rv_\theta) = C_1 r$$

Let $(rv_\theta) = m$

$$\frac{d}{dr}m = C_1 r$$

$$m = \frac{C_1 r^2}{2} + C_2$$

$$rv_\theta = \frac{C_1 r^2}{2} + C_2$$

$$v_\theta = \frac{C_1 r}{2} + \frac{C_2}{r}$$

Using the first BC, we get

$$0 = \frac{C_1}{2}(kR) + \frac{C_2}{kR}$$

$$C_1 = -\left(\frac{C_2}{kR}\right) \times \frac{2}{kR} = -\frac{2C_2}{k^2R^2}$$

Since r is the only variable, the partial derivatives have been converted into ordinary derivatives.

For the z component

$$\rho\left(\overset{=0\ (SS)}{\frac{\partial v_z}{\partial t}} + v_r \overset{(v_r = 0)\ 0}{\frac{\partial v_z}{\partial r}} + \frac{v_\theta}{r}\overset{v_z \neq f(\theta)\ 0}{\frac{\partial v_z}{\partial \theta}} + v_z \overset{(v_z = 0)\ 0}{\frac{\partial v_z}{\partial z}}\right)$$

$$= -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\overset{(v_z = 0)\ 0}{\frac{\partial v_z}{\partial r}}\right) + \frac{1}{r^2}\overset{(v_z = 0)\ 0}{\frac{\partial^2 v_z}{\partial \theta^2}} + \overset{(v_z = 0)\ 0}{\frac{\partial^2 v_z}{\partial z^2}}\right] + \rho g_z$$

Thus

$$0 = -\frac{\partial p}{\partial z} + \rho g_z \qquad (3.4.3\text{-}3)$$

Now, the boundary conditions for integrating 3.4.3. – 2 which is this equation okay. This is the differential equation, yes at r = kR, the inner radius of κ or let us call it it k, r = kR, the inner cylinder radius, $v_\theta = 0$ because the inner cylinder is stationary and therefore the liquid that we are considering the layer that is closest to the inner cylinder will stick to the inner cylinder and not move.

Therefore v_θ is 0 and at r = R, the velocity v_θ = the angular velocity times the radius that is the definition of linear velocity, angular velocity times the radius and that is the situation at r = R. So we have 2 boundary conditions. We can solve that second-order differential equation. So you have this equals 0 and therefore if we can take this as this being a constant, then this goes to 0, this equals to zero is irrelevant here.

Integrating Eq. 3.4.3-3 with the boundary conditions (BCs) given below

At $r = kR$, $v_\theta = 0$ (inner cylinder is stationary)

At $r = R$, $v_\theta = \Omega_0 R$

$$\left[\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rv_\theta)\right)\right] = 0$$

$$\frac{1}{r}\frac{d}{dr}(rv_\theta) = C_1$$

$$\frac{d}{dr}(rv_\theta) = C_1 r$$

Let $(rv_\theta) = m$

$$\frac{d}{dr}m = C_1 r$$

$$m = \frac{C_1 r^2}{2} + C_2$$

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Using the second BC, we get

$$\Omega_0 R = -\frac{2C_2}{k^2R^2} \times \frac{R}{2} + \frac{C_2}{R}$$

$$\Omega_0 R = C_2\left(\frac{1}{R} - \frac{1}{Rk^2}\right) = \frac{C_2}{R}\left(1 - \frac{1}{k^2}\right) = \frac{C_2}{R}\left(\frac{k^2}{k^2 - 1}\right)^{-1}$$

$$C_2 = \frac{\Omega_0 R^2 k^2}{k^2 - 1}$$

$$C_1 = -\frac{2}{k^2R^2}\left(\frac{\Omega_0 R^2 k^2}{k^2 - 1}\right) = \left(-\frac{2\,\Omega_0}{k^2 - 1}\right)$$

Thus, $$v_\theta = -\frac{2\Omega_0}{k^2 - 1}\cdot\frac{r}{2} + \frac{\Omega_0 k^2 R^2}{(k^2 - 1)r}$$

$$= \frac{\Omega_0 R^2}{(1 - k^2)}\left(\frac{r}{R^2} - \frac{k^2}{r}\right)$$

$$v_\theta = \frac{\Omega_0 k R^2}{(1 - k^2)}\left(\frac{r}{kR^2} - \frac{k}{r}\right)$$

$$= \frac{\Omega_0 R^2}{\frac{1}{k} - k}\frac{1}{R}\left(\frac{r}{kR} - \frac{kR}{r}\right)$$

$$v_\theta = \frac{\Omega_0 R\left(\frac{kR}{r} - \frac{r}{kR}\right)}{\left(k - \frac{1}{k}\right)}$$ Eq. 3.4.3. – 4

$$rv_\theta = \frac{C_1 r^2}{2} + C_2$$

$$v_\theta = \frac{C_1 r}{2} + \frac{C_2}{r}$$

Using the first BC, we get

$$0 = \frac{C_1}{2}(kR) + \frac{C_2}{kR}$$

$$C_1 = -\left(\frac{C_2}{kR}\right) \times \frac{2}{kR} = -\frac{2C_2}{k^2R^2}$$

Using the second BC, we get

$$\Omega_0 R = -\frac{\cancel{2}C_2}{k^2R^{\cancel{2}}} \times \frac{\cancel{R}}{\cancel{2}} + \frac{C_2}{R}$$

$$\Omega_0 R = C_2\left(\frac{1}{R} - \frac{1}{Rk^2}\right) = \frac{C_2}{R}\left(1 - \frac{1}{k^2}\right) = \frac{C_2}{R}\left(\frac{k^2-1}{k^2}\right)$$

$$C_2 = \frac{\Omega_0 R^2 k^2}{k^2-1}$$

$$C_1 = -\frac{2}{k^2R^2}\left(\frac{\Omega_0 R^2 k^2}{k^2-1}\right)$$

Using the second BC, we get

$$\Omega_0 R = -\frac{\cancel{2}C_2}{k^2R^{\cancel{2}}} \times \frac{\cancel{R}}{\cancel{2}} + \frac{C_2}{R}$$

$$\Omega_0 R = C_2\left(\frac{1}{R} - \frac{1}{Rk^2}\right) = \frac{C_2}{R}\left(1 - \frac{1}{k^2}\right) = \frac{C_2}{R}\left(\frac{k^2-1}{k^2}\right)$$

$$C_2 = \frac{\Omega_0 R^2 k^2}{k^2-1}$$

$$C_1 = -\frac{2}{k^2R^2}\left(\frac{\Omega_0 R^2 k^2}{k^2-1}\right)$$

$$= -\left(\frac{2\Omega_0}{k^2-1}\right)$$

Therefore

$$v_\theta = -\frac{2\Omega_0}{k^2-1}\cdot\frac{r}{2}+\frac{\Omega_0 k^2 R^2}{(k^2-1)r}$$

$$=\frac{\Omega_0 R^2}{(1-k^2)}\left(\frac{r}{R^2}-\frac{k^2}{r}\right)$$

$$=\frac{\Omega_0 k R^2}{(1-k^2)}\left(\frac{r}{kR^2}-\frac{k}{r}\right)$$

$$=\frac{\Omega_0 R^2}{\frac{1}{k}-k}\frac{1}{R}\left(\frac{r}{kR}-\frac{kR}{r}\right)$$

$$v_\theta = \frac{\Omega_0 R\left(\frac{kR}{r}-\frac{r}{kR}\right)}{\left(k-\frac{1}{k}\right)} \qquad (3.4.3\text{-}4)$$

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The relevant shear stress distribution can also be obtained by using the expression for the shear stress components in cylindrical coordinates as given in the Table 3.4 – 5

From the Eq. A in that Table

$$\tau_{r\theta} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$0\ (v_r \neq f(\theta))$

$$\tau_{r\theta} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

Since r is the only independent variable, the partial derivatives can be replaced with the total derivatives
Using eq. 3.4.3. – 4, we can get

$$\tau_{r\theta} = -\mu\left[r\frac{d}{dr}\left\{\frac{\Omega_0 R\left(\frac{kR}{r}-\frac{r}{kR}\right)}{r\ \left(k-\frac{1}{k}\right)}\right\}\right]$$

We said we are after velocity distribution shear stress distribution and the torque, one done, so 2 more to go, but they are quite straightforward. The relevant shear stress distribution can be obtained using the expression for shear stress components itself directly in cylindrical coordinates given in table 3.4. – 5. So from equation A, you go and look at that table $\tau_{r\theta}$ is what we are looking at

The relevant shear stress distribution can also be obtained by using the expression for the shear stress components in cylindrical coordinates as given in Table 3.4-5. From Eq. A

$$\tau_{r\theta} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

In this case

$$\tau_{r\theta} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \overset{0\,(v_r \neq f(\theta))}{\frac{1}{r}\frac{\partial v_r}{\partial \theta}}\right]$$

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$$\tau_{r\theta} = -\frac{\mu\Omega_o R}{\left(k-\frac{1}{k}\right)}\left[r\frac{d}{dr}\left(\frac{1}{r}\left\{\frac{kR}{r}-\frac{r}{kR}\right\}\right)\right]$$

$$= -\frac{\mu\Omega_o R}{\left(k-\frac{1}{k}\right)}\left[r\frac{d}{dr}\left(\frac{kR}{r^2}-\frac{1}{kR}\right)\right]$$

$$= -\frac{\mu\Omega_o R}{\left(k-\frac{1}{k}\right)}\left[r\left(-\frac{2kR}{r^3}\right)\right]$$

$$= -\frac{2\mu\Omega_o R^2 k}{\left(k-\frac{1}{k}\right)}\left(\frac{1}{r^2}\right)$$

$$\tau_{r\theta} = -2\mu\Omega_o R^2\left(\frac{k^2}{k^2-1}\right)\left(\frac{1}{r^2}\right) \qquad \text{Eq. 3.4.3.-5}$$

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The torque that is needed to turn the outer cylinder = force x lever arm distance

Torque = $-\tau_{r\theta}|_{r=R}$ X area X lever arm distance

negative sign before $\tau_{r\theta}$ is to overcome the shear stress by the fluid on the wall

$$= -\tau_{r\theta}|_{r=R} \times (2\pi RL) \times R$$

$$= 2\mu\Omega_o R^2\left(\frac{k^2}{1-k^2}\right)\left(\frac{1}{R^2}\right) \times 2\pi RL \times R$$

$$= 4\pi\mu\Omega_o L R^2\left(\frac{k^2}{1-k^2}\right) \qquad \text{Eq. 3.4.3.-6}$$

Using Eq. 3.4.3-4 (since r is the only independent variable, the partial derivatives can be replaced with total derivatives), we get

$$\tau_{r\theta} = -\mu\left[r\frac{d}{dr}\left\{\frac{\Omega_0 R\left(\frac{kR}{r} - \frac{r}{kR}\right)}{r\left(k - \frac{1}{k}\right)}\right\}\right]$$

$$= -\frac{\mu\Omega_0 R}{\left(k - \frac{1}{k}\right)}\left[r\frac{d}{dr}\left(\frac{1}{r}\left\{\frac{kR}{r} - \frac{r}{kR}\right\}\right)\right]$$

$$= -\frac{\mu\Omega_0 R}{\left(k - \frac{1}{k}\right)}\left[r\frac{d}{dr}\left(\frac{kR}{r^2} - \frac{1}{kR}\right)\right]$$

$$= -\frac{\mu\Omega_0 R}{\left(k - \frac{1}{k}\right)}\left[r\left(-\frac{2kR}{r^3}\right)\right]$$

$$= -\frac{2\mu\Omega_0 R^2 k}{\left(k - \frac{1}{k}\right)}\left(\frac{1}{r^2}\right)$$

$$\tau_{r\theta} = -2\mu\Omega_0 R^2\left(\frac{k^2}{k^2 - 1}\right)\left(\frac{1}{r^2}\right) \qquad (3.4.3\text{-}5)$$

The torque that is needed to turn the outer cylinder

= Force × Lever arm distance

= $-\left.\tau_{r\theta}\right|_{r=R}$ × Area × Lever arm distance

(the negative sign before $\tau_{r\theta}$ is to overcome the shear stress by the fluid on the wall)

$$= -\left.\tau_{r\theta}\right|_{r=R} \times (2\pi RL) \times R$$

$$= 2\mu\Omega_0 R^2\left(\frac{k^2}{1-k^2}\right)\left(\frac{1}{R^2}\right)\times 2\pi RL \times R$$

$$= 4\pi\mu\Omega_0 LR^2\left(\frac{k^2}{1-k^2}\right) \qquad (3.4.3\text{-}6)$$

Now, okay so we have an expression for the angular velocity, the tangential velocity as well as the relevant shear stress and now let us look at the torque that is needed to turn the outer cylinder. We all know from basic physics that torque is nothing but force times the lever arm distance. So torque is the force, the force here is used to overcome the shear stress, therefore the shear force. So the shear force is nothing but the shear stress times the area.

This can be directly used to design the couette flow device and thereby decide on the motor characteristics that you need, the type of motor that you need and so on and so forth to provide this required torque plus more usually design with a safety factor and that would give you the appropriate choice of the motor for this application. What I am going to also show you is where we used this particular device okay.

Of course, this is used in a viscometer as Bostwick viscometer, the viscometer that we mentioned earlier the couette flow viscometer to measure viscosities okay. There is certain way by which you can get the viscosity in terms of the other measured variables that is what is used for as the basis for that was committed design. Here, I am going to show you something else okay.

Here, I think I need to show you that paper. If you go to my webpage, look for a paper that was published in 2003 I think, the first author would be Susmita Sahoo and look at I think it is called macro level and genetic level effects of shear stress on sets. We would have used this kind of a device and appropriately modified it, we did it for the first time to be able to grow cells over an extended period.

The earlier, I mean the use of couette flow device for studying the effect of defined shear was known for a long time, only thing is that people could not use it for more than a few minutes because the oxygen would become limiting. We found a way of providing oxygen continuously through air but not affecting the flow profiles so that the shear stress profiles would remain unaffected.

The advantage of using couette flow for exposing the cells through defined shear stress would be available okay that is a contribution which is published in biotechnology progress way back in 2003. Please take a look at that for one of the applications of couette flow device this kind of a flow situation. We had to go through all this, some of the expressions were available, some of the expressions we had to derive in order to design one such device.

The device design was complex from a mechanical engineering point of view. You know the space between those two steam cylinders is 2 mm that is the annular space that we are looking at and the outer cylinder rotates at 1500 rpm okay, huge rpm, and therefore it should not shake. It was of this about foot plus height, about 30-35 centimeters of height okay. So, there cannot be any motion in any other direction except for this motion.

To achieve that, Vespan industries the person I worked with at the industry that I worked with at Mumbai when I was at IIT Bombay, he had actually gone to auto bearings to provide the needed mechanical properties so that it would rotate without any vibration okay. So that is the kind of situations that required, high precision and so on and so forth that provides that we have done it ourselves and we have used essentially the same principles to design that is the same equations were used to design okay.

Okay, I think we will stop here. I would have shown you that paper, I do not think I have that paper here on this computer, but there is a reference that I have given, you can go and see the paper. See you then, let us take things forward in the next class. Bye.