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Transport Phenomena in Biological Systems

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Lecture – 36

Turbulent Flow

Welcome up we are looking at momentum flux. We looked at rather rigorous way of analysis initially, we started looking at the microscopic level and taking things forward. And then somewhere in the middle I told you something about a dimensional analysis involving non dimensional numbers. That was that does not have too much depth let us say it is based on one mathematical theorem. The Buckingham pi theorem and then it is based on practice that happens to work. However, it gives useful results. Although not rigorous, it gives us useful results. And then we went back to some rigorous analysis to get some insights into complex situations, the unsteady flow aspects as well as the pulsatile flow aspects. Let us look at turbulent flow. This is we still surely start out with a rigorous way of looking at it, and then start making the various assumptions. we will start relaxing things a little bit so that it becomes the approach becomes helpful. Rather than be rooted in rigor alone. Then we will slowly move into more and more practice aspects. Our understanding may not be so complete, so much in depth, but it will be very useful. First today, let us look turbulent flow we all know what turbulent flow is, we said there are two major types of flows one is laminar flow, where the flow is in layers, the layers corresponding to the geometry of the system. The layers could be flat the layers could be cylindrical and so on or at turbulent flow where it is a chaotic flow with pockets of fluid tumbling over each other and flow. Layers of the we saw Reynolds number which is a means by which we could or we could predict distinguish between laminar flow and turbulent flow the laminar flow conditions flow in layers occurs if the laminar if the Reynolds number is less than a certain value and if it is greater than a certain value it has turbulent flow, that particular number depends on the situation. For example, for flow through circular pipes cylindrical pipes then number that we will be using this course is 2100 below which the flow is laminar 4000 above which the flow would be turbulent and that is well known.

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So, let us move forward many flows in the bioindustry are turbulent flow and pipes, flow in the bioreactor, flow in separation equipment are usually turbulent and they are preferred that way in the industry because it provides better mixing which is necessary for optimal production, optimal operation and so on so forth. Turbulent flow can occur near artificial valves of the heart sometimes, and that leads to wasteful expenditure of pumping energy. The heart is the one that is providing the energy for pumping and that is wasteful expenditure of pumping energy. I would like to invite you to watch this video, it is the title is whole blood viscosity links to cardiovascular disease, it is available. Free. Please watch this video this will give you a good idea between turbulent flow and cardiovascular disease. You know, it shows you nicely how the turbulent flow could cause cardiovascular disease. Please take a look at it. To appreciate where all the turbulent flow is relevant. You just need to look at it appropriately you will find turbulent flow where the appropriate flows are occurring. What you are going to see in this chapter is how to approach or how to approach the analysis of turbulent flow.

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Let us begin. Let us start very simple. The velocity v_z it could be this direction, this direction we will strict to this for the time being, v_z is the velocity at any point in turbulent flow. And that can be expressed as an average component and a fluctuating component, the sum of those two things. That is reasonably gentle. It is a good strategy and that is reasonably gentle and this works very well as you see. So, this would be one aspect of our approach to view the velocity at any point at any time as consisting of an average component and fluctuating component. If this is not very clear right now, it will become clearer maybe a couple of slides down the line. We will spend more time on this aspect. People researchers, through careful experimental measurements they have shown for the flow in a pipe, the following is valid in turbulent flow. You know the ratio of the average velocity to the maximum velocity they found the velocity to the maximum velocity ratio, they found it as proportional to $(1 - r / R)^{1/7}$ this is turbulent flow. This is through measures. They better measured the velocity at each point in a cross section and they found this to be valid the average component divided by the maximum velocity of the average component is related in this fashion raised to the power of one by seven. We know that in the laminar trace go back to your interior notes and look at it you will find that v_z by $v_{z\ max}$ was $1 – (r/R)^2$ * the parabolic velocity profile. So, v_z by $v_{z\ max}$ times $1 – (r/R)^2$. Therefore, this ratio is this. So, you can find the difference between laminar flow that we have already seen in great detail and the turbulent flow which we have started looking at in this chapter we will call this equation 3.8-1. The average of the mean component to the maximum velocity that ratio people found through measurements to be four fifths. Whereas, we recall from our earlier notes, what is it for laminar flow? Yes half. Go back into your notes and check it will be half also one more. Or, you know, even if you do not take that negative sign and so on, it is proportional to the flow rate per seven fourths. That is what people have found through experiments in turbulent flow, and laminar flow. If you recall the Hagenminus Poiseiulle equation, it was directly proportional to Q. Let us call this equation 3.8-3. So, there are differences by differences in the relationships between laminar and turbulent flow.

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	Turbulent flow		Laminar flow	
$\dfrac{\bar{v}_z}{\bar{v}_{z,\max}} \cong$	$\left(1-\dfrac{r}{R}\right)^{\frac{1}{7}}$	$=$	$\left[1-\left(\dfrac{r}{R}\right)^2\right]$	(3.8-1)
$\dfrac{\bar{v}_{z,\text{avg}}}{\bar{v}_{z,\max}} \cong$	$\dfrac{4}{5}$	$=$	$\dfrac{1}{2}$	(3.8-2)
$\Delta P \propto$	$Q^{\frac{7}{4}}$	$\propto$	Q	(3.8-3)

You could watch these videos. This gives you an idea of how the turbulent flow occurs when the flows open in the river or something like that. And in a tube, this video gives you how the flow appears. You will have a means of clicking these easily through I think there should be a file that you could download with these links and you can go and click these links. As we have already seen the visualization of turbulent flow the turbulent flow can be visualized as a random motion of pockets of fluids and those pockets of fluids are called Eddie's. Eddy singular Eddie's plural turbulent flow in a tube. The flow is entirely random at the center of the tube far away from the wall near the wall, the cylindrical wall, the fluctuation and velocity in the axial direction is greater than the fluctuations in the radial direction, we are looking at the tube, this is the radial direction all over this is the axial direction to the direction in which the flow is occurring. And the fluctuation velocity in the axial direction is greater than the fluctuations in the radial direction the v’ the fluctuating component is higher in the axial direction compared to the radial direction and at the wall the fluctuations are zero there is no the layer closest to the wall sticks to the wall does not move and the fluctuations are zero.

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Now, let us consider the fluid behaviour at one point in turbulent flow in the tube or the pipe we are going to use pipe and tube interchangeably in this course. Let us say as we are watching it know there is turbulent flow that is occurring in the pipe. As we are watching it, we are going to slowly reduce or decrease the pressure drop which is causing the flow. You could do that by changing the motor settings, pump settings and so on and so forth. So, we are the main velocity we are going to decrease the mean velocity by causing a pressure drop change or a decrease in the pump speed. The variation in the axial component of the velocity v_z at any point of the observation because this is velocity in various directions it has a predominant axial component, but it could also have other components because the fluctuations. The variation of the axial component v_z at the point of observation would be something like this in a plot of v_z this is the velocity at any time versus time. So, it is going to be like this and like this and so on and so forth then the pressure is being pressure drop is being slowly reduced and therefore, the velocity will also slowly reduce whereas, the actual velocity would fluctuate something like this. This would be the trace of the various measurements of the velocity v_z the axial component at any point. So, this is what I mean. So, if we draw an average of these various points, it will look something like the dotted line. The dotted line gives you the average of this profile of the values in this profile. So, this is what we mean by the average component v_z'. So, v_z' is called the time smoothed velocity that is the average v_z over a time interval a large enough with respect to the time of turbulent oscillation this the let us say that we are taking an average over this t a. So, at each point we take an average over a certain neighbourhood and this time interval is large enough with respect to the time of turbulent oscillation. But small enough with respect to the changes in the pressure drop that is causing the flow the pressure drop is slowly reducing the velocity the average velocity is slowly reducing, but the fluctuations are much much faster, this time t a is much higher is much larger compared to the characteristic time of the fluctuation but is much slower compared to the time of the change in pressure that is causing the flow and the velocity causing the flow that is causing the decrease in velocity that is.

The variation of the axial component of the velocity, v_z, at the point of observation, would look like that given in Fig. 3.8-1.

$\bar{v}_z$ is called the time-smoothed velocity, i.e. the average of v_z over a time interval large enough with respect to the time of turbulent oscillation, but small enough with respect to the time changes in the pressure drop causing the flow.

$$\bar{v}_z = \frac{1}{t_a}\int_t^{t+t_a} v_z dt \qquad (3.8\text{-}4)$$

Thus

$$v_z = \bar{v}_z + v_z' \text{ (average + fluctuation)} \qquad (3.8\text{-}5)$$

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The pressure at a point will also vary in a similar fashion

$$p = \bar{p} + p' \tag{3.8-6}$$

If we take the average of the fluctuations, $\bar{v}_z$, since the positive values will balance the negative values

$$\bar{v}'_z = 0 \tag{3.8-7}$$

Thus, we cannot use $\bar{v}'_z$ as a measure of turbulence. However, the average of the squares of the fluctuation values, $\overline{v_z'^2}$, will not be zero and can be used as a measure of turbulence. In fact

$$\text{Intensity of turbulence} \equiv \frac{\sqrt{\overline{v_z'^2}}}{\bar{v}_{z,\text{avg}}} \tag{3.8-8}$$

The intensity of turbulence is typically between 0.01 and 0.1.

The pressure at any point will also vary in a similar fashion. So, this is the pressure at any point in the fluid. So, we could write pressure in general as the sum of an average component and a fluctuating component equation 3.8-6 we are going to manipulate this average component and fluctuating components. So, let us try to understand them a little better. If we take the average of the fluctuations by the very definition of average the positive values will always cancel out the negative values. That is the whole point about the average. So, if you sum all the positive variations, they will cleanly cancel out the negative variations if we take the average, that is the definition of the average itself. So, we cannot use this the average of the fluctuations as a measure of anything because it is going to be zero. So, if you are looking at the measure of the fluctuations, measure the turbulence and so on so forth. We cannot do this we will have to do something else. The mathematical trick here is, v^2 we square the averages, we square the fluctuating components v_z'. When we square them, the negatives will be will also become positive, the square of negatives are positive, so we will have the sum of all positive fluctuations, and then take the average over all the squares. And then take the square root of it. So, that way we avoid the difficulty of the positive terms cancelling out the negative terms cleanly. And we will have a measure that we can work with and what better measure than the fluctuation and velocity itself. So, this or a modification of this can be used as a measure of turbulence. In fact, the intensity of turbulence, which is a measure of turbulence is defined as the square root of the fluctuating velocity squares the average of the fluctuating velocity square which means you take the fluctuating velocity you square it, then you add all the squares and then take the average this. And of course, if you take the average that will be a squared term the units would be velocity squared and therefore, you need to take the square root to get it in the units of velocity. So, once all that is done, you divided by the average velocity and that is called the intensity of turbulence equation 3.8-8. The typical values of the intensity of turbulence are between point 0.01 and 0.1, this will give you an idea and near the wall as we mentioned. The axial intensity of turbulence would be much greater than the radial intensity of turbulence at the center, there would be a lot of fluctuations all around totally chaotic flow, but closer to the wall the axial fluctuations are going to be greater than the radial fluctuations. At the center it is an isotropic conditions where they are compared.

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As long as the eddy size is greater than the mean free path of the molecule which means the concept of a continuum holds. The following fundamental aspects need to be applicable for turbulent flow if you are able to use or apply the continuum aspects the fundamentals of continuum aspects need to be applicable. So, whatever we have developed you know material balance, energy balance, material balance, momentum balance and so on so forth.

At the wall, since the fluctuations in the radial component will be different from those in the axial direction, we need to differentiate between the two. Researchers have found that near the wall

$$\text{(Axial)}\ \frac{\sqrt{\overline{v_z'^2}}}{\overline{v}_{z,\text{avg}}} > \frac{\sqrt{\overline{v_r'^2}}}{\overline{v}_{z,\text{avg}}}\ \text{(Radial)}$$

At the centre of the tube the above values are comparable (isotropic condition).

As long as the eddy size is greater than the mean free path of the molecules (continuum holds), the

- equation of continuity (mass balance)
- equation of motion (momentum balance)

They will generally be applicable here they are, you know universal principles, it does not really matter what kind of flow there is. So, that is the beauty of holding onto such principles. And that is what this course gives you. So, the equation of continuity that is based on mass balance, as well as the equation of motion that is based on momentum balance, both need to be valid. So, for turbulent flow, let us consider incompressible turbulent flow for illustration, if you are if you need to get into compressible turbulent flow, you can go and pick it up from some advanced techniques. So, incompressible turbulent flow, the equation of continuity can be written as, you know instead of wherever we had a v_x, we have a combination of an average component or a time smoothed component and a fluctuating component. That is all right the equation is reversed

Equation of Continuity

$$\frac{\partial}{\partial x}(\bar{v}_x + v'_x) + \frac{\partial}{\partial y}(\bar{v}_y + v'_y) + \frac{\partial}{\partial z}(\bar{v}_z + v'_z) = 0 \qquad (3.8\text{-}9)$$

Equation of Motion

x direction

$$\frac{\partial}{\partial t}\rho(\bar{v}_x + v'_x) = -\frac{\partial}{\partial x}(\bar{p} + p') - \left[\frac{\partial}{\partial x}\rho(\bar{v}_x + v'_x)(\bar{v}_x + v'_x) + \frac{\partial}{\partial y}\rho(\bar{v}_y + v'_y)(\bar{v}_x + v'_x) + \frac{\partial}{\partial z}\rho(\bar{v}_z + v'_z)(\bar{v}_x + v'_x)\right] + \mu\nabla^2(\bar{v}_x + v'_x) + \rho g_x \qquad (3.8\text{-}10)$$

And the equation of motion again, let us look at only the x direction, then you can extend it to the other directions is going to be something like this.

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Taking the time average of the velocity components, i.e. $\bar{v} = \frac{1}{t_a}\int_0^{t_a} v\, dt$ over t_as that are large with respect to turbulent oscillations but small with respect to macro variations, the time-smoothed equation of continuity can be written as follows (note that the time averaged fluctuations will tend to zero)

$$\frac{\partial \bar{v}_x}{\partial x} + \frac{\partial \bar{v}_y}{\partial y} + \frac{\partial \bar{v}_z}{\partial z} = 0 \qquad (3.8\text{-}11)$$

Now, if we take the time average of the velocity components, this is a very standard thing that we do. And we are going to take time average over t_a that are large with respect to turbulent oscillations, but small with respect to the micro variations is not microwave, small with respect to the sorry, the macro variations, the change in pressure and so on so forth with respect to the macro variations. And if we time smoothed the equation, that is what this is called. If we do this procedure, the time average fluctuations will go to zero, because we are taking a simple average. And the time smoothed equation of continuity, please write this and check this. I am not going to derive that here. But this is how this equation, when you take the time smooth average will become whatever equation I am going to write. The variation and these would go to zero when you take the time average that time smoothing when you do when you take the time average, that is going to go to zero

If you have difficulty you can get back to me by the forum that was the course forum or an email. The time smooth equation of motion would be something like this. It will turn out to be something like this because of the time smoothing of the fluctuating components going to zero.

Similarly, the time-smoothed equation of motion can be written as

$$\frac{\partial}{\partial t}\rho\overline{v}_x = -\frac{\partial\overline{p}}{\partial x} - \left[\frac{\partial}{\partial x}\rho\overline{v}_x\overline{v}_x + \frac{\partial}{\partial y}\rho\overline{v}_y\overline{v}_x + \frac{\partial}{\partial z}\rho\overline{v}_z\overline{v}_x\right] - \left[\frac{\partial}{\partial x}\rho\overline{v'_x v'_x} + \frac{\partial}{\partial y}\rho\overline{v'_y v'_x} + \frac{\partial}{\partial z}\rho\overline{v'_z v'_x}\right] + \mu\nabla^2\overline{v}_x + \rho g_x \tag{3.8-12}$$

The third term in brackets on the RHS of Eq. 3.8-12 is the only extra term when compared to the equation of continuity for laminar flow.

So, apart from this term, the rest of the equation is the same as that we got earlier. Only this term is the extra term compared to the equation of continuity for laminar flow here in this case it does not matter it is simple time smoothing that we do the fluctuating components will go to zero here since the product happened and the average of the products of the fluctuations happen that is not going to go to zero and that term will remain.

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Now, since $\rho\vec{v}\vec{v}$ = momentum flux or stress, let us say that

$$\bar{\tau}_{xx}^{(t)} = \rho\overline{v_x' v_x'}$$

$$\bar{\tau}_{xy}^{(t)} = \rho\overline{v_x' v_y'}$$

and so on.

These are the components of the turbulent momentum flux tensor $\bar{\tilde{\tau}}^{(t)}$. The stresses are also known as Reynolds stresses.

In vector notation, the time-smoothed equation of continuity is

$$\vec{\nabla}.\vec{\bar{v}} = 0 \tag{3.8-13}$$

and the time-smoothed equation of motion is

$$\rho\frac{D\vec{\bar{v}}}{Dt} = -\vec{\nabla}\,\bar{p} - \left[\vec{\nabla}.\bar{\tilde{\tau}}^{(l)}\right] - \left[\vec{\nabla}.\bar{\tilde{\tau}}^{(t)}\right] + \rho\vec{g} \tag{3.8-14}$$

This is equation 3.8-14 and compared with the laminar flow situation, this is the only extractor. (refer time: 17:19)

The above Eqs. (3.8-9) to (3.8-14) are valid for an incompressible flow. Similarly, it can be shown that the earlier equations and the tables for laminar flow are valid if we replace

$$v_i \text{ by } \bar{v}_i$$

$$p \text{ by } \bar{p}$$

and $$\tau_{ij} \text{ by } \bar{\tau}_{ij}^{(l)} + \bar{\tau}_{ij}^{(t)}$$

However, to get the velocity profile, we need a relationship between τ and the velocity gradient.

For laminar flow, we had a theoretical base in terms of constitutive equations. For turbulent flow, we do not have that luxury. Nevertheless, many expressions based on experimental observations have been proposed. Two are given below.

Now, to get a velocity profile, we need a relationship between the shear stress and the velocity gradient. In the case of laminar flow and a Newtonian fluid especially, we had our Newton's law of viscosity, which gave relationship between shear stress and the velocity gradient. We do not have that luxury here. Let me read this out for laminar flow. We had a theoretical base in terms of constitutive equations, the Newtons law of viscosity, we do not have that kind of a thing here. However, what people have done is based on a large number of experimental studies, relevant expressions have been proposed. Relevant expressions for the relationship between tau and the velocity gradient have been proposed and there is going to present two here just give you a flavour, but people did was familiarity is nice and therefore, they first formulated one that is similar to a Newtonian fluid. But this is a turbulent case.

For laminar flow, we had a theoretical base in terms of constitutive equations. For turbulent flow, we do not have that luxury. Nevertheless, many expressions based on experimental observations have been proposed. Two are given below.

The first is on the same lines as for the laminar case.

$$\bar{\tau}_{yx}^{(t)} = -\mu^{(t)} \frac{d\bar{v}_x}{dy} \qquad (3.8\text{-}15)$$

where $\mu^{(t)}$ is termed as 'eddy viscosity' and its value could be hundreds of times the molecular viscosity.

Another popular expression was formulated by Prandtl. For this expression, it is assumed that the eddies in the fluid move around in a fashion similar to that of the molecules in a gas. A 'mixing length', l, which is a function of position represents an idea similar to the 'mean free path' in the kinetic theory of gases. The relationship is given as

This looks similar to a Newtonian fluid. However, this $\mu^{(t)}$ is nowhere close to that of a molecular viscosity, the form is fine it works fine. Only thing is that you cannot rely on molecular viscosities. In fact, turbulent viscosity it could be hundreds of times in molecular viscosity, typically equation 3.8-15 but this formulation allows us to have certain insights.

This $\mu^{(t)}$ is called the eddy viscosity and it could be hundreds of times of molecular viscosity. Second relationship, a popular formulation was given by Prandtl a long time ago. And for doing that, he assumed that the eddies in the fluid move around in a fashion similar to that of molecules in a gas. And therefore, the gas kinetic theory principles can be brought in here, mixing length l, which is a function of position represents an idea similar to the mean free path in the kinetic theory of gases mixing length l in the liquid is equal to the mean free path. This is a viscosity component times the velocity gradient there.

$$\bar{\tau}_{yx}^{(t)} = -\rho l^2 \left|\frac{d\bar{v}_x}{dy}\right| \frac{d\bar{v}_x}{dy} \qquad (3.8\text{-}16)$$

Equation 3.8-16 and let us not spend too much, much more time just to formulations to tell you that these are available many more are available equal use.

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What is a typically used is an empirical relationship for flow in pipes tubes the velocity profile and turbulent flow can be obtained through Deissler's empirical formulation through a large number of experiments Deissler came up with these relationships and these can be used to get the velocity profile and turbulent flow in pipes.

For flow in pipes/tubes, the relationship between velocity and distance (velocity profile) in turbulent flow through Deissler's empirical formulation is as follows:

If we define

$$v^+ = \frac{\bar{v}_z}{\sqrt{\frac{\tau_0}{\rho}}}$$

and

$$s^+ = s\left(\sqrt{\frac{\tau_0}{\rho}}\right)\frac{\rho}{\mu}$$

where $s = R - r$ i.e. the radial distance from the wall and τ_0 is wall shear stress at $s = 0$.

This as you can if you can work out the unit dimensions. You will find this dimensionless. R is the wall radius - the radial distance or the radial distance from the wall is represented by s. What Deissler found was that for s^+ greater than 26 this is by experiments, therefore, do not worry about these odd numbers this is what was found. This is what was revealed by experimental data.

For $s^+ > 26$

$$v^+ = \frac{1}{0.36}\ln s^+ + 3.8 \qquad (3.8\text{-}17)$$

For $0 \le s^+ \le 5$

$$v^+ = s^+ \qquad (3.8\text{-}18)$$

And for $0 \le s^+ \le 26$

$$v^+ = \int_0^{s^+} \frac{ds^+}{1 + n^2 v^+ s^+ (1 - \exp\{-n^2 v^+ s^+\})} \qquad (3.8\text{-}19)$$

where n is Deissler's constant for tube flow, near the wall. It was found empirically to be equal to 0.124.

So, this empirical formulation can be used to get the velocity distribution in turbulent flow in pipes. I do not know where this is, from a set of experimental data therefore, to be valid only to that condition. We do not have the luxury of fundamental understanding, where we can apply with confidence with our eyes close to any situation that is what we could do earlier. We cannot do that here because we do not have an understanding however, we need to move forward and therefore, we use such aspects which could have certain limitations, but which are useful if the conditions fall into those limitations. I think that is all I have here. Before that n is the Deissler's constant for tube flow near the wall, empirically found experimentally found to be 0.124.That is good enough and introduction to approaching turbulent flow, starting from fundamental aspects and then some empirical aspects we look. Let stop here for this class. We will meet again in the next class.