Transport Phenomena in Biological Systems
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Lecture - 38
Friction Factor for Flow through a Straight Horizontal Pipe

Welcome we are looking at the engineering Bernoulli equation or macroscopic approaches to flow situations analyze flow situations design flow aspects. And we said that we need to find a friction factor for each situation for us to easily make sense of what is happening and to use whatever we know towards design and operation. So let us start looking at the friction factor of a flow through a straight horizontal pipe. Let me show you how to get to that.

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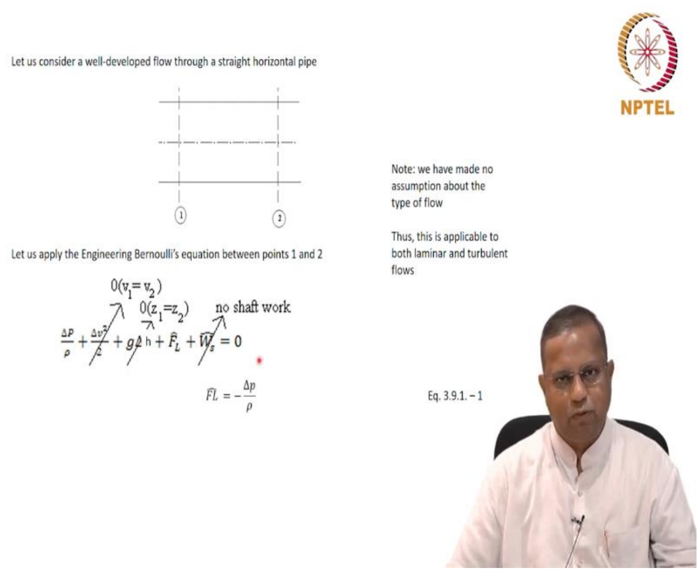

I am going to consider 2 major aspects before I get to that or to get to that let us consider a well-developed flow through a straight horizontal pipe well developed flow the velocity at any point at any radial position does not vary with length. So this is our let us say our horizontal pipe you know the axis of symmetry cross section 1 cross section 2 the engineering Bernoulli equation is typically applied between cross sections macroscopically applied between cross sections.

Here if we apply the engineering Bernoulli equation between points 1 and 2

Let us apply the engineering Bernoulli equation between cross-sections 1 and 2

$$\frac{\Delta p}{\rho} + \overbrace{\cancel{\frac{\Delta v^2}{2}}}^{0(v_1 = v_2)} + \overbrace{\cancel{g\Delta z}}^{0(z_1 = z_2)} + \hat{F}_L + \overbrace{\cancel{\hat{W}_s}}^{\text{no shaft work}} = 0$$

Thus

$$\widehat{FL} = -\frac{\Delta p}{\rho} \qquad (3.9.1\text{-}1)$$

that is in engineering Bernoulli equation. This you can close your eyes and write down and now if we cancel the terms it is well developed flow the velocity here is the same as velocity here. Therefore this term goes. There is no difference in height between the center point here and center point here from let us say a datum level and therefore $z_1 = z_2$ and Δz is 0.

There is no shaft work that has been included there is no pumping work that has been put in for example any such work that is introduced is called shaft work to understand it to start understanding shaft work let us say if there is a pump in the system you could say that there is shaft work that needs to be considered tell them no.

Now even before I move forward this is powerful. Note that we made no assumption about the type of flow it could be laminar it could be turbulent whatever we never said or we never considered anything that represents the type of flow all energetic aspects there is no relationship to the mechanics of flow. It is all energetic of flow so this is applicable irrespective of the type of flow are applicable both to laminar and turbulent flows. That way it is powerful in terms of use us it is very powerful.

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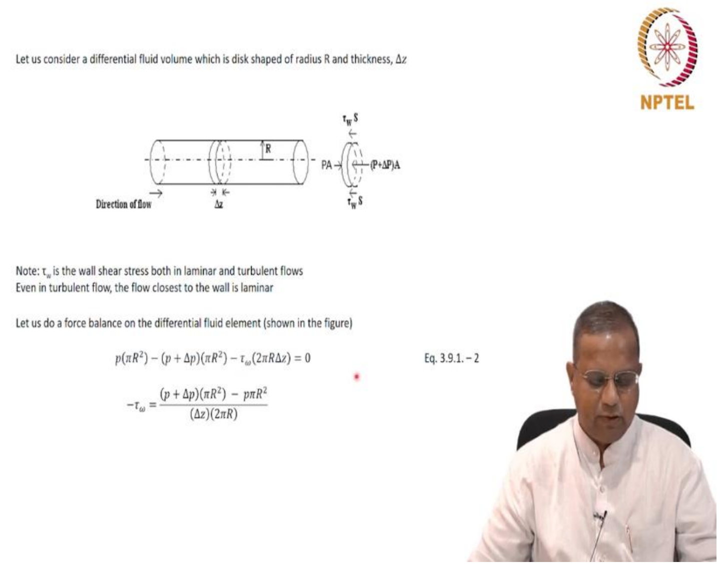

Now we are getting to friction factor to understand the process let us consider a differential fluid volume in this pipe a differential fluid volume which is representative the shape of the differential element must be representative. Therefore the fluid the differential fluid element that would be representative would be a thin disk of the same radius as that of the pipe the thickness has to be very small radius has to be that of the pipe and that disk is going to be a differential element for this flow situation.

So a disk shaped differential fluid volume is disk shaped of radius R and thickness Δz. In other words if this is a flow happening this is the direction of flow we take a disk of thickness Δz and radius R and that would be a representative volume element a differential volume element. If we draw the force balance if we do a force balance on this liquid element there is a pressure acting here in this direction.

This is a free body diagram therefore a pressure acting in this direction a pressure a different pressure acting in this direction which is different because of the length being different and of course. There is a wall shear that opposes the motion and that was shear is meant to act on along the circumference. So you have a pressure force acting this way and a shear force one shear force that is acting in the in this direction which is opposite to the direction of motion.

So τ_w is the wall shear stress both in laminar and turbulent flows even in turbulent flow the layers closes to the wall will not flow at very high velocities therefore there is a laminar layer a boundary layer it is called a laminar layer closes to the wall in fully turbulent flow. That is what said here even turbulent flow the flow closes to the wall is going to be laminar. So let us do a force balance on the differential fluid element that is shown here.

And therefore if you do force balance in a free body diagram as drawn for the purpose this is the direction in the positive z direction. So this gives us a force balance that must be equal to 0 when there is no net force acting and then you have very defined flow conditions or well developed flow conditions. So there is no net force that is acting on any fluid element a representative fluid element equation 3.9.1 -. 2

A force balance on the differential fluid element yields

$$p(\pi R^2) - (p + \Delta p)(\pi R^2) - \tau_\omega (2\pi R \Delta z) = 0 \qquad (3.9.1\text{-}2)$$

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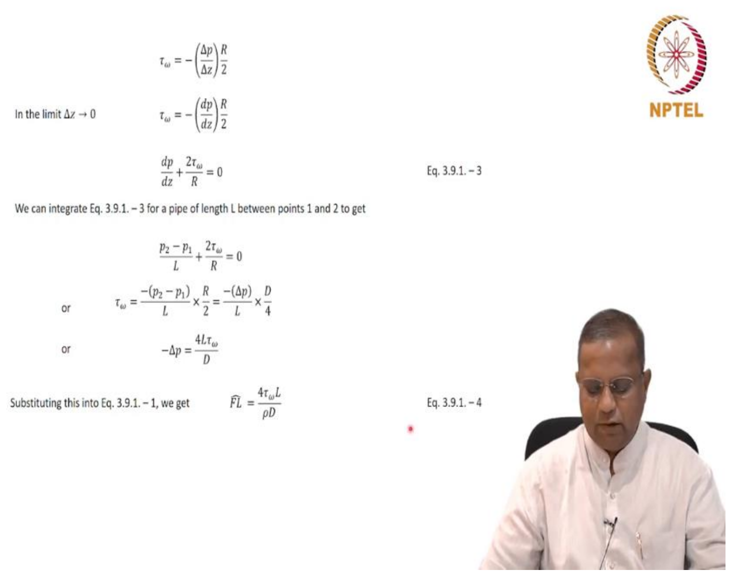

$$-\tau_{\omega} = \frac{(p+\Delta p)(\pi R^2) - p\pi R^2}{(\Delta z)(2\pi R)}$$

$$\tau_{\omega} = -\left(\frac{\Delta p}{\Delta z}\right)\frac{R}{2}$$

In the limit $\Delta z \to 0$

$$\tau_{\omega} = -\left(\frac{dp}{dz}\right)\frac{R}{2}$$

$$\frac{dp}{dz} + \frac{2\tau_{\omega}}{R} = 0 \qquad (3.9.1\text{-}3)$$

And we can integrate this equation for a pipe length L between points 1 and 2 to get this relationship with this more use. So because these are measurable quantities we can substitute or most of them are measurable quantities.

For a pipe of length L between points 1 and 2, Eq. 3.9.1-3 can be integrated to yield

$$\frac{p_2 - p_1}{L} + \frac{2\tau_{\omega}}{R} = 0$$

or

$$\tau_{\omega} = \frac{-(p_2 - p_1)}{L} \times \frac{R}{2} = \frac{-(\Delta p)}{L} \times \frac{D}{4}$$

or

$$-\Delta p = \frac{4L\tau_{\omega}}{D}$$

Substituting this into Eq. 3.9.1-1, we get

$$\widehat{FL} = \frac{4\tau_{\omega} L}{\rho D} \qquad (3.9.1\text{-}4)$$

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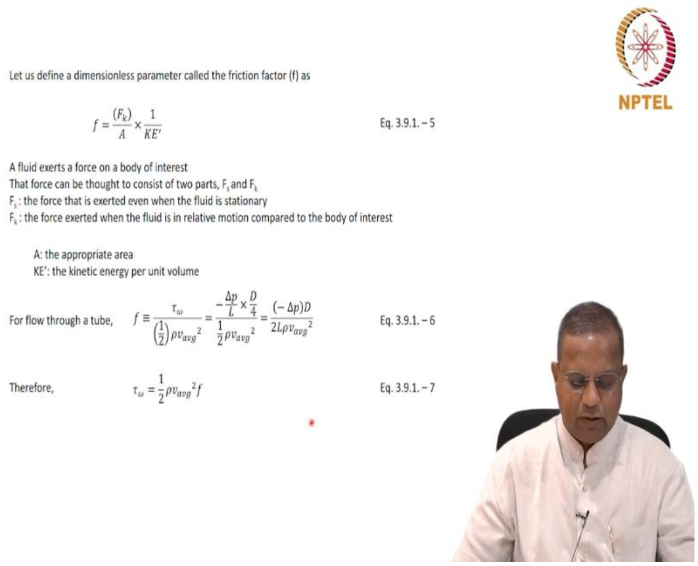

Now let us say we are ready to define a dimensionless parameter called the friction factor. We said this friction factor kind of unifies or gives a unified approach to various different situations. That is why we are interested in the friction factor.

Let us define a dimensionless parameter, f, as

$$f = \frac{(F_k)}{A} \times \frac{1}{KE'} \qquad (3.9.1\text{-}5)$$

where f is the friction factor, F_k is the force exerted by a fluid due to its motion on the body of interest, A is the appropriate area and KE' is the kinetic energy per unit volume.

(A fluid exerts a force on a body in contact with it and of interest. That force can be thought to consist of two parts, F_s and F_k. F_s is the force that is exerted even when the fluid is stationary. F_k is the force exerted when the fluid is in relative motion compared to the body of interest.)

So you take the you normalize it with respect to the area and then normalize that with respect to the kinetic energy per unit volume please correct this is kinetic energy per unit volume KE‘. For a flow through a tube the friction factor is F_k / A is nothing but the τ_w the shear wall shear is representative of this kinetic energy per unit volume is nothing but what is kinetic energy mass into the energy related to the motion and then you divided by the volume a $(1/2)mv^2$ is the kinetic energy.

In our case of tube flow, f can be conveniently defined as

$$f \equiv \frac{\tau_\omega}{\left(\frac{1}{2}\right)\rho v_{avg}^2} = \frac{-\frac{\Delta p}{L} \times \frac{D}{4}}{\frac{1}{2}\rho v_{avg}^2} = \frac{(-\Delta p)D}{2L\rho v_{avg}^2} \qquad (3.9.1\text{-}6)$$

Thus

$$\tau_\omega = \frac{1}{2}\rho v_{avg}^2 f \qquad (3.9.1\text{-}7)$$

Substituting Eq. 3.9.1-7 in Eq. 3.9.1-4, we get

$$\widehat{FL} = \frac{4\left(\frac{1}{2}\rho v_{avg}^2 f\right)L}{\rho D} = 4f\left(\frac{L}{D}\right)\left(\frac{v_{avg}^2}{2}\right) \qquad (3.9.1\text{-}8)$$

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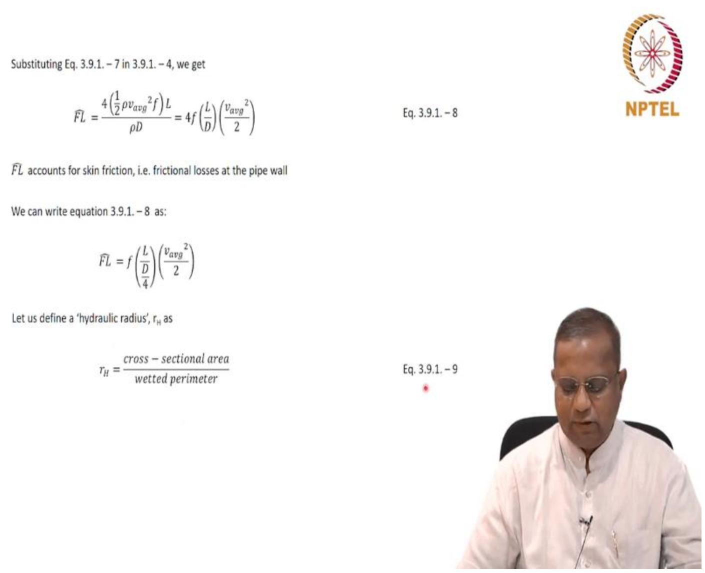

So $\widehat{FL}$ per unit mass transfer skin friction are the frictional losses at the pipe point. So the pressure drop that is being applied is used to overcome the friction losses the friction losses at the wall. And so we need to provide energy for the flow to occur. It is one small insight that you can draw from this intuitive but this one of the tests so we can write 3.9.1 - 8 as I am going to rewrite this to get it of the form that is needed.

If we define a 'hydraulic radius', r_H as

$$r_H = \frac{\text{Cross-sectional area}}{\text{Wetted perimeter}} \qquad (3.9.1\text{-}9)$$

for our pipe

$$r_H = \frac{\pi\left(\frac{D^2}{4}\right)}{\pi D} = \frac{D}{4}$$

Thus

$$\widehat{FL} = f\left(\frac{L}{r_H}\right)\left(\frac{v_{\text{avg}}^2}{2}\right) \qquad (3.9.1\text{-}10)$$

This equation, in practice, can be extended to all cross-sectional geometries.

Why I am doing this? I am going to define something called a hydraulic radius as the cross sectional area by the wetted perimeter. Once I define a cross section area by the wetted perimeter there is a possibility of extending this relationship to other geometries there will certainly be a cross sectional area for any geometry there will be a wetted perimeter for any geometry.

So if we can write this in terms of the hydraulic radius then it is no longer limited to circular cross section you could have square cross section as the case of some AC ducts and rectangular cross section as in the case of some AC ducts any cross section and then you could apply a similar equation. So let us write this in terms of the hydraulic radius hydraulic radius definition equation 3.9.1 - 9.

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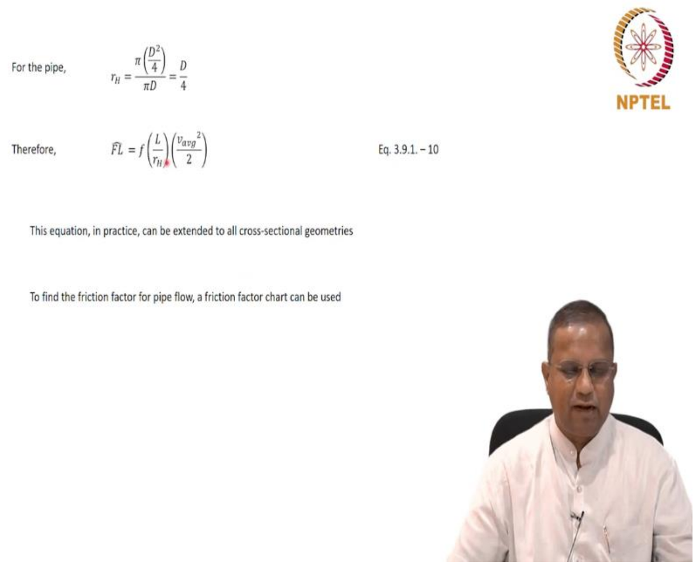

Then this equation can be used for any geometry it can be extended to all cross sectional geometries so to find the friction factor for pipe flow. We typically use a friction factor chat a fanning friction factor chat that we normally use civil engineers use something called a Moody is chart which is somewhat similar. The definition of the friction factor is slightly different in the moody stat I think it is 4F something like that. But everything else remains the same.

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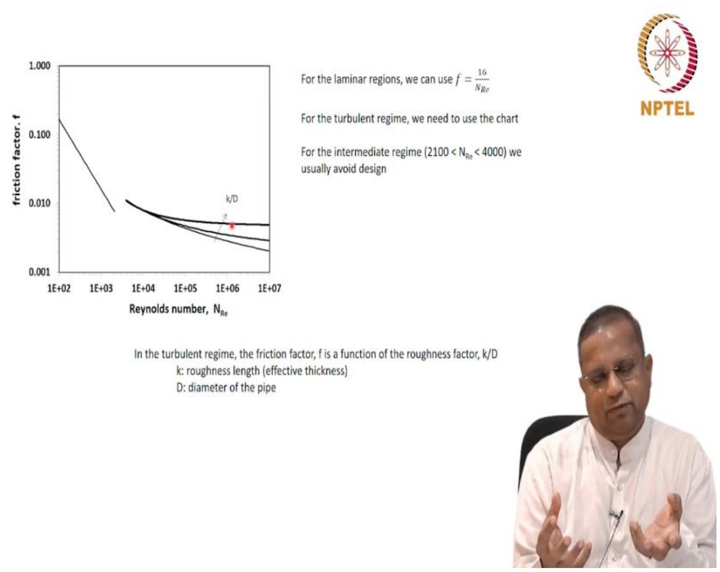

So this is the friction factor chart that you will find in the appendix of many fluid flow books. A simplified version of this has been given in your textbook just barely necessary for our own needs. So here you have the friction factor F that we have defined for the flow through a pipe in the y axis on a logarithmic scale and the Reynolds number of flow in on the x axis again on a logarithmic scale so it is a log-log plot till 2100. You can see this is 1000 this is 2000.

So in our log scale this is around to say this is 1000 this is 10000, 2000 somewhere here it is a log scale therefore it will be in the middle of this linear scale somewhat. So this is the reason for laminar flow. And beyond 4000 is the reason for turbulent flow so in the laminar flow you have this relationship between the friction factor and the Reynolds number that relationship is 16 / N_{Re}. So you could use friction factor at 16 / N_{Re}.

And directly get the friction factor f you know the Reynolds number for the turbulent regime we definitely need to use a chart we do not have this nice relationship you could also use the chart here the chart is the same as 19 / N_{Re}. So no problem there the for the turbulent regime we if you know the Reynolds number and if you know something called the roughness factor in the pipe roughness factors the thickness of the roughness level in the pipe the ratio of that to the diameter is what is called the roughness factor.

For 1 roughness factor for smooth pipe you have the lowest one for one roughness factor you have this for a higher roughness factor k / D you have this and so on so forth. And you can interpolate between this because they are reasonably close by on a logarithmic graph for an estimate. So for the turbulent regime to find out the friction factor for a given set of flow conditions as quantified by the Reynolds number you find the Reynolds number then go to this chart for the particular k / D value you find out the friction factor.

That is what we normally do for the intermediate regime 2100 to 4000 you see a gap here that gap is a gap we it is not well understood sometimes it is laminar sometimes turbulent as we have already seen. We normally avoid design in this regime we would not like to operate when the Reynolds number is between 2100 and 4000 because we do not we are not really sure of the nature of flow. In turbulent regime the friction factor is a function of the roughness factor.

Now some more explanation of the roughness factor k is the roughness length or the effect of thickness and D is the diameter of the pipe and do that is you have pipe and if you can idealize the roughness as some sort of projections you take the projection length that becomes k and you divided by the diameter of the pipe that becomes your reference factor. So that is what it is. So this is the way we approach things and when we meet next I will show you the first application through a problem itself.

As I said problems are integrated with the learning itself here you are not just given a separate problem as to solve as a part of this class. You are given that as those assignments but you are not given those as a part of this class. This class integrates the problems. You do some I do some sometimes you work out a lot more so that you pick up. They have been designed such that the learning can be more effective. Let us stop here let us meet in the next class and take things for see you.