Transport Phenomena in Biological Systems

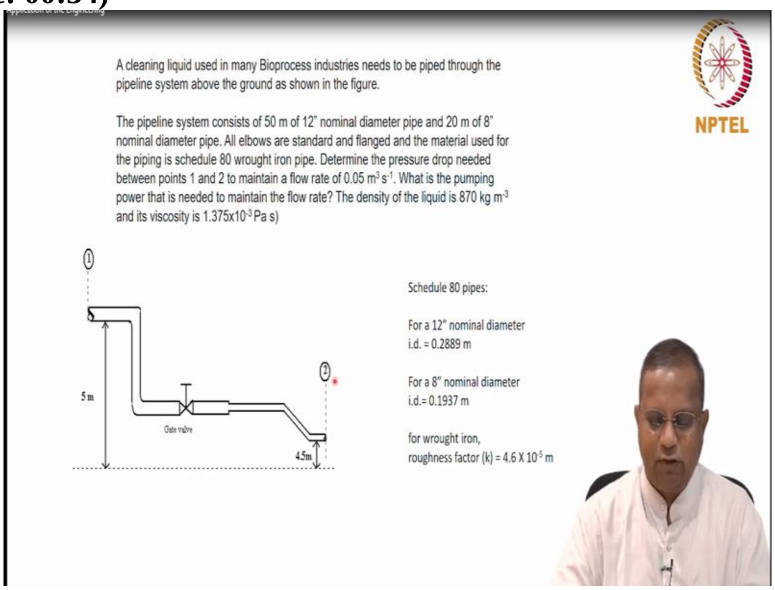
Prof. G.K. Suraishkumar
Department of Biotechnology Bhupat and Jyoti Mehta School of Biosciences Building
Indian Institute of Technology – Madras

Lecture - 39

Application of the Engineering Bernoulli Equation to a Piping Network

Welcome, let us look at the application of the engineering Bernoulli equation to a piping network in this class and we are to do that through an actual problem.

(Refer Slide Time: 00:34)



Let me state the problem first and then tell you how to go about solving that. And it is just application of Bernoulli equation with associated aspects and you will be able to pick up how to use the engineering Bernoulli. In this situation that is the piping network. A cleaning liquid used in many bioprocess Industries needs to be piped through the pipeline system about the ground as shown in the figure, this is the ground and this is the piping network or the pipeline system.

The pipeline system consists of 50 meters of 12 inch nominal diameter pipe this entire length thicker is 50 meters and 20 meters of 8 inch nominal diameter pipe. Do not worry about these terms as yet I will come to that. All elbows are our standard and flange and the material used for the piping is schedule 80 wrought iron pipe determine the pressure drop needed between points 1 and 2. To maintain a flow rate of 0.05 m^3/s. what is the pumping power that is needed to maintain.

The flow rate the density is given and the response it is given off the fluid of the cleaning fluid that is used that is pumped through the piping network here you have some terms that you may not have seen earlier. There is something called a nominal diameter. There is something called a schedule 80. These are terms that are widely used in the practice that involves pipes. So these are terms that emerged out of the practice convenience in a convenient means of practicing things. The nominal diameter especially refers to something that would help people put together pipes.

That would fit each other very long time back in the day need not measure the outer diameter or the inner diameter and the check whether it is going to fit and so on and so forth. Because the way of attaching it pipes is you put pipe together and put something on top of it a sleeve maybe a nipple maybe and so on and so forth. That is one way of connecting pipes. So, if you need to connect pipes then you need to worry about the relationship between their diameters and you could easily connect pipes of the same nominal diameter so the nominal diameter is some measure.

Which we not refer to the actual diameter in any way mean actual diameter of the pipe that is so a 12 inch nominal diameter is 12 inch nominal diameter 8 inch nominal diameter is smaller the 12 inch nominal diameter. For the for example for the 12 inch nominal diameter the internal diameter is actually 0.2889 meters for 8 inch nominal diameter it is 0.1937 meters and this you can get listed in handbooks in tables of buy properties and handbooks.

For a 12" nominal diameter

$$\text{id} = 0.2889 \text{ m}$$
$$\therefore \text{ cs area} = 0.066 \text{ m}^2$$

For a 8" nominal diameter

$$\text{id} = 0.1937 \text{ m}$$
$$\therefore \text{ cs area} = 0.0297 \text{ m}^2$$

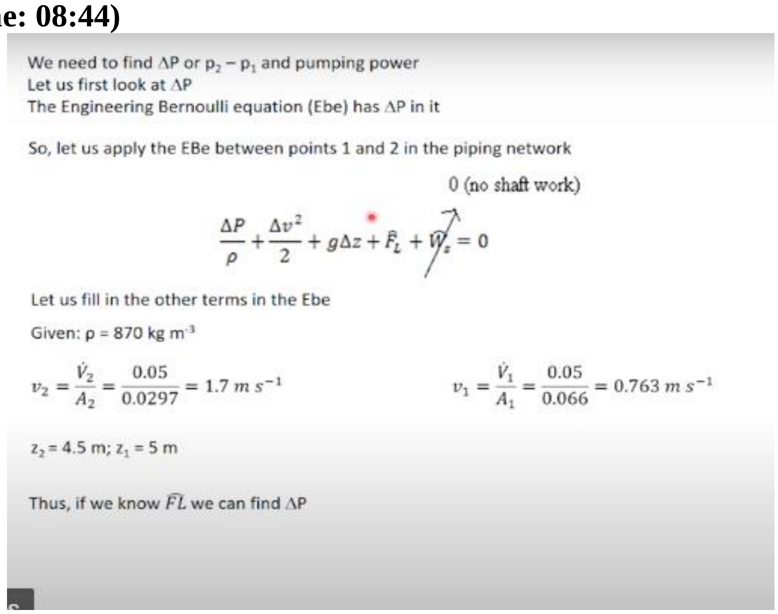
Also, for wrought iron, roughness factor $(k) = 4.6 \times 10^{-5}$ m.

It is given in fluid flow books at the end some tables are given it is given various chemical engineers handbook and so on and so forth. For our situation I will give you the numbers here. Otherwise people just talk about a 12 inch nominal diameter, you need to go back to this table to figure out what exactly the inner diameter of the pipe is for our calculations, for our calculations we need this inner diameter area and so on so also these are valid only for schedule 80 pipes. Get schedule number relates to the strength of the pipe in some way. Let us not get too much into it.

It is some pressure versus the ability to take pressure and so on so forth. So schedule 80 let is just take it as schedule 80 it has certain sense have properties and for a schedule 80 pipes for a 12 inch nominal diameter i.d. would be this much in the 8 inch nominal diameter schedule 80 pipe the i.d. with this much. So, the scheduled number refers to certain mechanical properties of the pipe the nominal diameter refers to some relevant dimension of the pipe if you can see that it may not refer to the actual dimension of the pipe.

That being clear also, you recall while if you want to find the friction factor from the fanning friction factor chart or friction factor versus the Reynolds number you need to know the roughness factor or the roughness lengths and so on so forth roughness factor interchangeably use. So, the reference factor for a wrought iron pipe typically is $4.6 * 10^{-5}$ meters. So the k / d needs to take to be able to find out the friction factor. So that is a situation here. So let us go through it step by step.

(Refer Slide Time: 08:44)



Let us apply the engineering Bernoulli equation between points 1 and 2 in the piping network shown in Fig. 3.9.1-4.

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z + \hat{F}_L + \hat{W}_s = 0$$

0 (no shaft work)

$$\frac{p_2 - p_1}{\rho} + \frac{(v_2^2 - v_1^2)}{2} + g(z_2 - z_1) + \widehat{FL} = 0$$

We need to find $p_2 - p_1$.

We need to find Δp or p_2 - p_1 and the pumping power that is what we need to find here. That is what is being given in this problem. Determine the pressure drop needed between points 1 and 2 to maintain a flow rate of 0.05 m^3. And also what is the pumping power that is needed to maintain the flow rate these are the 2 things that are needed that is what is given here Δp and pumping power we first look at data from the engineering Bernoulli equation we can get Δp that you know. So, let is apply the history Bernoulli equation between points 1 and 2 in the piping network.

We know that $\rho = 870 \text{ kg m}^{-3}$

$$v_2 = \frac{\dot{V}_2}{A_2} = \frac{0.05}{0.0297} = 1.7 \text{ m s}^{-1}$$

$$v_1 = \frac{\dot{V}_1}{A_1} = \frac{0.05}{0.066} = 0.763 \text{ m s}^{-1}$$

$$z_2 = 4.5 \text{ m}; \; z_1 = 5 \text{ m}$$

$$\widehat{FL} = ?$$

For a pipe, and different pipe fittings (valves, etc., which are piping network components), $\widehat{FL}$ can be calculated as $\widehat{FL} = K_f \frac{v_{avg}^2}{2}$ for each fitting, and added together to get the total $\widehat{FL}$. K_f values for some common fittings are given in brackets next to the fitting: straight pipe $\left(4f\frac{L}{D}\right)$; 180° bend (2.2); 90° elbow (0.9); 45° elbow (0.4); tee (1.8); wide open globe valve (15); wide open gate valve (0.2). In addition, the K_f values for a sudden contraction and a sudden expansion can be evaluated as follows

$$\text{Sudden contraction: } 0.4\left(1 - \frac{A_b}{A_a}\right)$$

$$\text{Sudden expansion: } \left(1 - \frac{A_b}{A_a}\right)^2$$

where b is smaller diameter and a is larger diameter; v_{avg} is taken at b.

We can apply it across cross sections as you know, there is no pump as a part of the pumping network here if you see carefully there is no pump as a part of the system. So, in this is we are focusing on the part that does not have a pump. Therefore, there is no shaft work only when a pump is present typically are when something else is present you need to worry about shaft work so here there is no shaft work and therefore let is start filling in the other terms in the engineering Bernoulli equation.

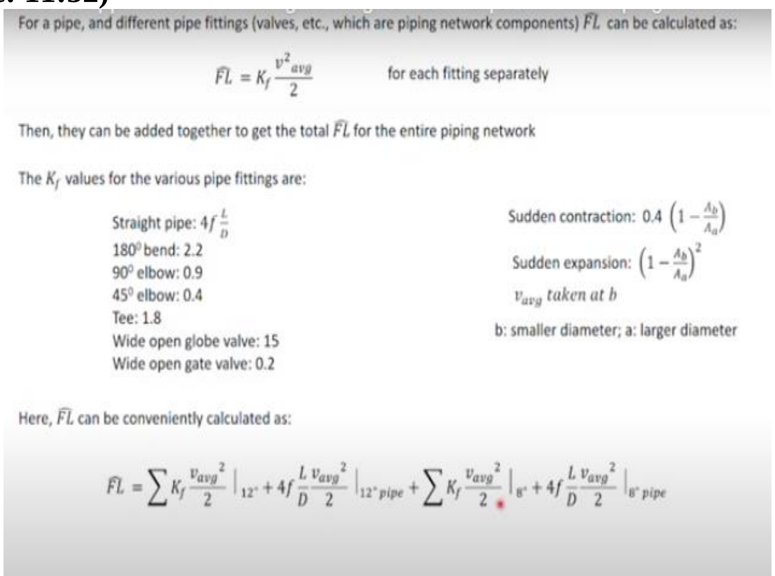
We know the density is 70 kg/m^3 and therefore the velocity is the volumetric flow rate and therefore, the velocity is nothing but the volumetric flow rate by the area, you know the area times the velocity is the volumetric flow rate volumetric flow rate required is 0.05 area if you can calculate from the inner diameter that was given there by d^2/ 4 that will turn out to be 0.0297 m^2 and therefore, v_2 turns out to be 1.7 m/s.

v_1 similarly turns out to be for the same volumetric flow rate, the area πd^2/ 4 for that particular diameter, the 1 inch nominal diameter pipe turns out to be 0.763 meter per second and this one was 4.5. If you recall the one end, the other one was 5 meters if you recall that figure, this is z_1

from the datum level this is z_2. So we are filling in the various terms, we have fill done we have discounting are listening whatever we know, we know that z_2 is 4.5 and z_1 is 5 meters.

So, we have a handle on the term we have a handle on this term. And therefore if we know this, then we can find Δp which is one of the things that is needed we know the density so straightforward. It seems straightforward. Let us see how to do that.

(Refer Slide Time: 11:52)



For a pipe and the different pipe fittings, valves and so on and so forth. You know you have a gate valve globe valve 90 degree bend 45 degree bend, a 180 degree bend all these things valves, bends and other aspects of the piping network apart from the straight pipe these are called pipe fittings. So for pipe fittings the frictional losses will be very different in when the fluid is changing its direction of flow that is going to cause a lot of frictional losses, especially 180 degrees and all that it is going to cause lot more friction losses may be.

And when it goes through a valve it is going to cause a lot more efficient losses compared to maybe a straight slight difference situation straight pipe and the frictional loss for a pipe fitting is given with this formulation, a certain K_f which depends on the pipe fitting times $v^2_{average}$ / 2 has been given the same form so that you could integrate various things together. So, you take the frictional loss for each pipe fitting separately and these are energy losses and as a scalar.

In this case

$$\widehat{FL} = \sum K_f \frac{v_{\text{avg}}^2}{2}\bigg|_{12''} + 4f\frac{L}{D}\frac{v_{\text{avg}}^2}{2}\bigg|_{12''\text{ pipe}} + 4f\frac{L}{D}\frac{v_{\text{avg}}^2}{2}\bigg|_{8''\text{ pipe}} + \sum K_f \frac{v_{\text{avg}}^2}{2}\bigg|_{8''}$$

To find *f*, let us use the friction factor chart for which we need the Reynolds numbers.

$$N_{\text{Re},12''\text{ pipe}} = \frac{\rho v_{av} D}{\mu} = \frac{870 \times 0.763 \times 0.289}{1.375 \times 10^{-3}} = 1.39 \times 10^5$$

Therefore, you can add each energy together each energy loss together to get the total energy loss. So it is here it is here. So, the K_f values you need to know to be able to do this when we can know the v average from the volumetric flow rate the area and so on and so forth, but you need to know the K_f values, the K_f values for the various pipe fittings of relevance here and some more are as follows.

So K_f is nothing but 4f (L / D) 180 degree bend is 2.2 90 degree elbow is 0.945 degree elbow is paying for a Tee joint is 1.8 a wide open globe valve it is 15 a wide open gate valve as point to a sudden contraction when pipe suddenly changes diameter to the lower diameter a certain contraction is 0.4 times 1 - A_b / A_a these are the areas and tell you what b and a are in a minute for a certain expansion is $(1 - A_b / A_a)^2$, this can actually be derived by using by applying the engineering Bernoulli equation $v_{average}$ you know you need $K_f\, v^2_{average} / 2$ to get the efficient passes.

$$N_{\text{Re},8''\text{ pipe}} = \frac{870 \times 1.7 \times 0.194}{1.375 \times 10^{-3}} = 1.47 \times 10^5$$

Both are turbulent flows.

Now, for reading the appropriate curve on the friction factor chart in the case of a turbulent flow, we need $\frac{k}{d}$.

$$\frac{k}{d_{12''}} = \frac{0.000046}{0.2889} = 1.6 \times 10^{-4}$$

$$\frac{k}{d_{8''}} = \frac{0.000046}{0.194} = 2.37 \times 10^{-4}$$

From the friction factor chart, $f_{12"} = 0.0045$; $f_{8"} = 0.00445$.

Pipe fittings: 2(12", 90°) + 2(8", 45°) elbows, 1(12") gate valve, 1(8") sudden contraction

$$\sum K_f \Big|_{12"} = 2\times 0.9 + 1\times 0.2 = 2.0$$

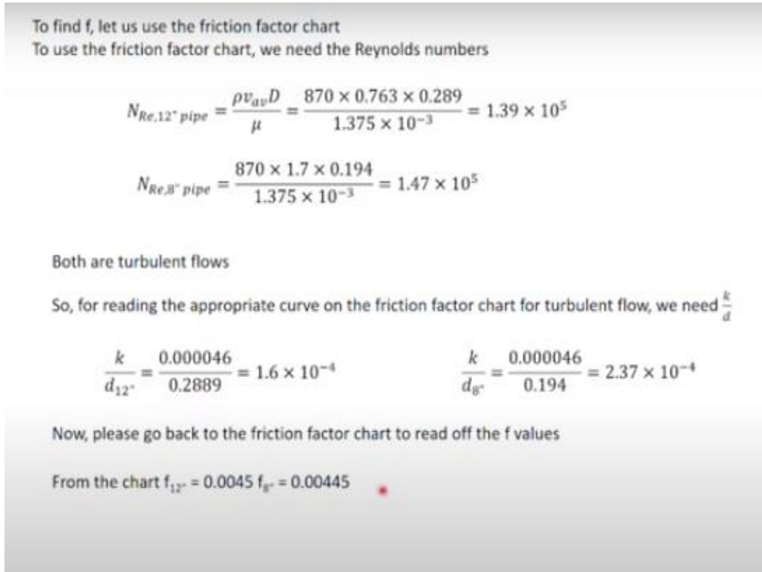
$$\sum K_f \Big|_{8"} = 2\times 0.4 + 0.4\left(1 - \frac{0.0297}{0.066}\right) = 1.02$$

This $v_{average}$ is actually taken at b. So, if you take the average at b this formulation holds, b happens to be the smaller diameter, a happens to be the larger diameter irrespective of whether it is a contraction or an expansion. So, this is the way to apply the formula to find out the frictional losses in this case is nothing but whatever comes out of you are engineering but not the Bernoulli equation application. And here the friction loss can be conveniently calculated as you recall the figure.

See, you have a gate valve here it straightforward. In the 12 piping network, you have a 90 degree bend here and a 90 degree bend here. You have a sudden contraction here, you have a 45 degree bend here you have another 45 degree bend here. So, this is the these are the various pipe fittings there is a 1990 gate valve certain contraction 45 45 that is those are the pipe fitting here so in our case, the friction losses, K_f average, $v^2_{average} / 2$ for the 12 months alone, plus 4f (L / D)($v^2_{average} / 2$) for the 12 inch pipe I just separated out the straight pipe from the rest, that is all.

You could also consider the straight pipe as one of the regular fitting, and then it will just be a sum of K_f $v^2_{average} / 2$. So if I consider the other fittings to be separate, then sum over the fittings of K_f $v^2_{average} / 2$ plus the straight pipe loss 4f (L / D)($v^2_{average} / 2$). Of course the 12 inch losses are going to be different from the 8 inch losses. Therefore we have separated the 2 so pipe fittings for the 12 inch loss the loss there plus the loss in the straight pipe region of the 12 inch pipe plus the loss in the pipe fittings in the 8 inch pipe plus the loss in the straight pipe section of the 8 inch expect. This is the total loss.

(Refer Slide Time: 17:14)



To find f, let us use the friction factor chart
To use the friction factor chart, we need the Reynolds numbers

$$N_{Re,12" pipe} = \frac{\rho v_{av} D}{\mu} = \frac{870 \times 0.763 \times 0.289}{1.375 \times 10^{-3}} = 1.39 \times 10^5$$

$$N_{Re,8" pipe} = \frac{870 \times 1.7 \times 0.194}{1.375 \times 10^{-3}} = 1.47 \times 10^5$$

Both are turbulent flows

So, for reading the appropriate curve on the friction factor chart for turbulent flow, we need $\frac{k}{d}$

$$\frac{k}{d_{12"}} = \frac{0.000046}{0.2889} = 1.6 \times 10^{-4} \qquad \frac{k}{d_{8"}} = \frac{0.000046}{0.194} = 2.37 \times 10^{-4}$$

Now, please go back to the friction factor chart to read off the f values

From the chart $f_{12"} = 0.0045$ $f_{8"} = 0.00445$

Now to find f the friction factor for finding the loss in the straight pipe we can use the Reynolds number the friction factor chart which uses the Reynolds number, the Reynolds number for the 12 inch pipe if you substitute the density the average velocity diameter by ρ it turns out to be $1.39*10^5$. Definitely turbulent. More than 4000 differently turbulent. That is what I mentioned earlier. In the industry predominantly with the flows are turbulent.

For an 8 inch pipe, the same Reynolds number turns out to be 1.47 times 10^5. And so for the reading the appropriate curve on the friction factor chart, we need k / d we have the x coordinate here, these 2 are the relevant x coordinates that we need to use. And once we know which part which curve to read on, by fixing the k / d, we can directly get the friction factor. So, k / d is nothing but point $4.6*10^{-5}$ divided by the diameter of the 12 inch nominal diameter pipe which happens to be 0.2889.

And therefore, k / d happens to be $1.6 *10^{-4}$, so, whichever is closest to that you could take and k / d for the 8 inch pipe turns out to be $2.37 *10^{-4}$ So, what I would like you to do is go back to the friction factor chart, take a copy of whatever was given whatever is shown here, or you can go to the book and check then read off the friction factor from the friction factor chat. Can you go ahead and do it, stop the video here.

Go back to that whatever time it takes 5 minutes 10 minutes and then come back and continue the week good. From the friction factor chart, you would have found for these Reynolds numbers and these k / d values. The friction factor for the chart pipe happens to be 0.0045 and the friction factor for the 8 inch pipe happens to be 0.00445.

(Refer Slide Time: 19:47)

Pipe fittings: 2(12", 90°) +2(8", 45°) elbows, 1(12") gate valve, 1(8") sudden contraction

$$\sum K_f \Big|_{12"} = 2 \times 0.9 + 1 \times 0.2 = 2.0 \qquad \sum K_f \Big|_{8"} = 2 \times 0.4 + 0.4\left(1 - \frac{0.0297}{0.066}\right) = 1.02$$

Substituting all the above values in the EBe

$$\frac{\Delta p}{870} + \frac{1.7^2 - 0.763^2}{2} + 9.8(45-5) + \left(4 \times 0.0045 \times \frac{50}{0.289} \times \frac{0.763^2}{2}\right) + \left(4 \times 0.0045 \times \frac{20}{0.194} \times \frac{1.7^2}{2}\right)$$
$$+ \left(2 \times \frac{0.763^2}{2}\right) + 1.02 \times \frac{1.7^2}{2} = 0$$

$$\frac{\Delta p}{870} = 4.9 - 1.154 - 0.91 - 2.65 - 0.582 - 1.474$$

$$\Delta p = -1626.9\ Pa \quad or \quad -1.63\ KPa$$

Pipe fittings we have as I told you earlier to in the 12 inch the 90 degree bends to 45 degree elbows, the 8 inch, 1 gate valve we can consider that to be fully open 1, 8 inch center and straight and sudden contraction. So, the K_f for the 12 inch the first term as this K_f for the 8 the K_f for the 12 inch total is this 2.9, the 90 degree elbows 0.9 the K value, the gate valve is in the 12 inch thing that is 0.2. And then for the 8 inch you have a 40 to 45 degree elbow. So, 2.47 contraction is 0.4 times 1 - A_b / A_a. So, you put all that together you get 1.02 for 8 inch and 2 for 12 inch.

So we have a handle on the K_f values. And if you substitute all the above values into the engineering Bernoulli equation, we have, Substituting the above in the engineering Bernoulli equation, we get

$$\frac{\Delta p}{870}+\frac{1.7^2-0.763^2}{2}+9.8(45-5)+\left(4\times0.0045\times\frac{50}{0.289}\times\frac{0.763^2}{2}\right)$$

$$+\left(4\times0.0045\times\frac{20}{0.194}\times\frac{1.7^2}{2}\right)+\left(2\times\frac{0.763^2}{2}\right)+1.02\times\frac{1.7^2}{2}=0$$

$$\frac{\Delta p}{870}=4.9-1.154-0.91-2.65-0.582-1.474$$

$$\Delta p = -\ 1626.9\ \text{Pa or}\ -\ 1.63\ \text{kPa}$$

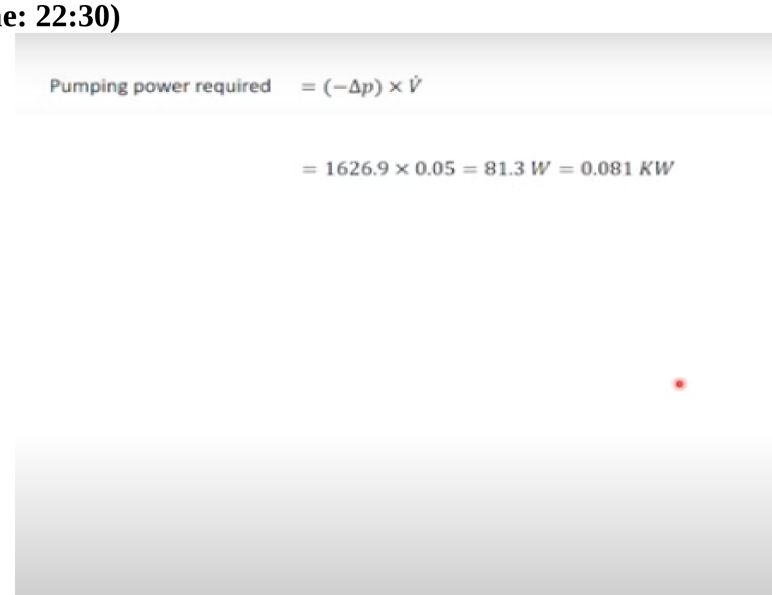
Pumping power required

$$= (-\ \Delta p)\times\dot{V}$$

$$= 1626.9\times 0.05 = 81.3\ \text{W} = 0.081\ \text{kW}$$

So, only the losses need to be considered this is the last term which consists of this the straight section, this is the fitting section loss for the 12 inch pipe both and this is for the, this is straight pipe, yes.

(Refer Slide Time: 22:30)



So, the pumping power required is nothing but the pressure drop times the volumetric flow rate. You work out the units here you will figure out that this indeed turns out to be the power units which is the actual power that is required here. We know the minus Δp which turns out to be 1626.9 times the volumetric flow rate that is given in the problem but 0.05 m^3/s. That turns out to be 81.3

watts 0.081 KW this is the pumping power that was required. So, we found both the Δp the pressure drop and the pumping power that is required in this case.

So, that is an application of the engineering but not the equation to the flow in cylindrical pipes with fittings we saw how to handle the fittings also. And hopefully that gave you an idea as to how to approach these problems the very practical situation here you have a piping network, your piping network, and all kinds of situations and this gives you a way to directly design the piping network. What amount of pressure drop you need to cause a certain amount of flow for a given piping network and what is the pumping power that is required so that you can choose an appropriate pump for the situation.

Let us stop here for this class when we continue and show you another application of the friction factor another situation in which we, in which we can use a friction factor for that situation to calculate relevant aspects. See.