Transport Phenomena in Biological Systems
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Lecture - 40
Stenosis in an Artery

Welcome in today class will see another application of the engineering Bernoulli equation. Again very relevant to our situation application to stenosis in an artery the narrowing in an artery which can cause diseases you know circulatory diseases heart disease and so on so forth. So we are going to see how we can apply the engineering Bernoulli equation to one such situation. Interesting is not it the we are going to apply it to an artery and a reduction in the diameter due to buildup of plaque or something like that.

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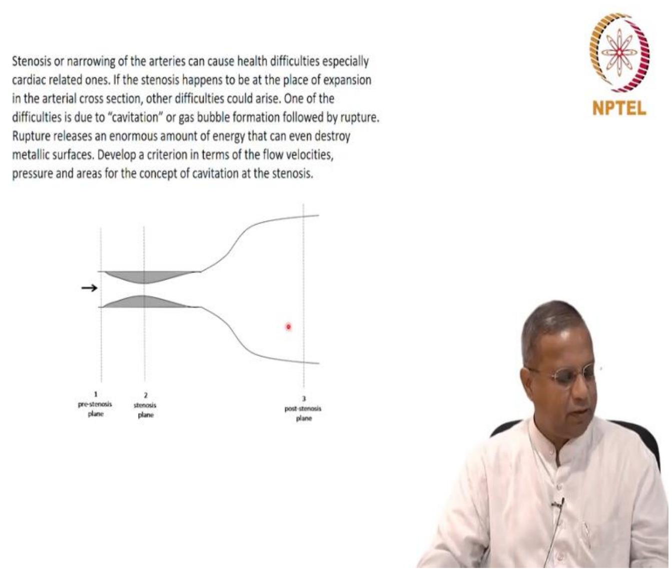

Let us look at again from the point of view or with a problem as a backdrop that gives you a certain framework to look at it. Stenosis or narrowing of the arteries can cause health difficulties especially cardiac related ones. If these stenosis happens to be at the place of expansion in the arterial cross section other difficulties could arise. One of the difficulties is due to cavitation or gas bubble formation followed by rupture cavitation is a very interesting phenomenon you might want to read about that separately.

You know it is very briefly we all know thermodynamics. So if you have atmospheric pressure in this in the system of interest then a liquid will start boiling when you reach its boiling point.

However if the pressure is reduced it will start boiling at temperatures much lower than the boiling point you know this you know this from thermodynamics it is a pressure temperature to volume relationship and so on so forth.

Look at your PV diagram PT diagram that will give you an idea as to why the happening the conversion from the liquid phase to the vapor phase happens over a range of things depending on the pressure temperature conditions. So if the pressure gets reduced to reasonably low value the liquid will start becoming a vapor at even room temperatures even 20,25,30 degrees C it will start turning out to be a vapour.

And it will start becoming a vapour. Vapour in this situation you have bubbles, bubbles of various sizes those bubbles will burst. And when they burst they release a huge amount of energy where you would believe it if I said so that energy can cause pickling our pockets in steel and that amount of high energy high temperature pressure conditions are so high temperature conditions are created when a bubble burst so much of energy is released.

So this process of causing damage even to a metal surfaces when the pressure is low because of the conversion of liquid to a gas and subsequent bursting of gas bubbles. That process is called cavitation. Cavitation is very difficult problem to deal with in many different situations. Any underwater moment needs to take this into account why because we know that pressure is involved there you have the pressure.

And various different equations that you have already seen the pressure will vary with the flow under certain conditions if the pressure falls much below a certain value and so on and so forth this process will start happening vapor gets getting formed in the flow of liquids. So that is a situation here, here that is relevant. One of the difficulties is due to cavitation or gas bubble formation from followed by rupture.

Rupture releases an enormous amount of energy that can even destroy metallic surfaces develop a criterion in terms of flow velocities pressure and areas for the concept of cavitation. At this stenosis that is what our interest is very interesting. I have given you a figure here. So this is the artery the cylindrical part of the artery. Let us say that is joining a larger or expanding it is diameter as it goes along here and the buildup of plaque occurs here narrowing the diameter here.

So this is this stenosis the narrowing of the artery the stenosis is plane. This is the pre stenosis plane where there is no reduction in the diameter. And this is the post stenosis plane in this case and larger diameter. So that is the situation here we are going to develop a criterion in terms of flow velocities pressure and areas for the concept of cavitation at this stenosis.

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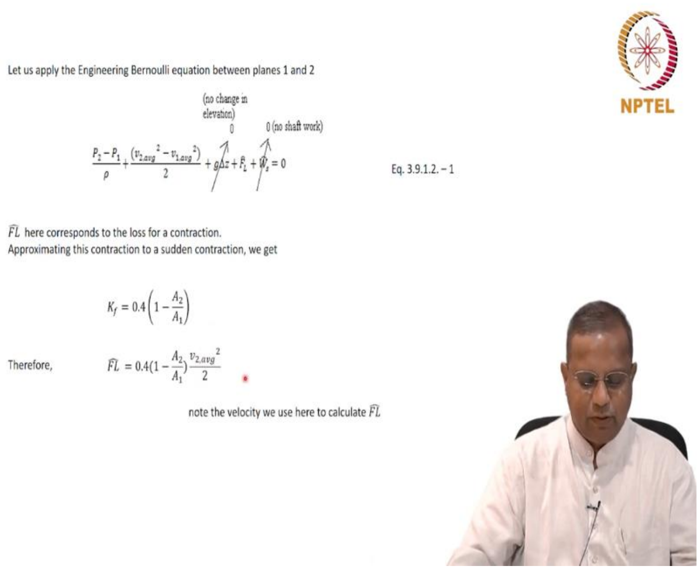

FL here corresponds to the loss due to contraction. Approximating this contraction to a sudden contraction, from Example 3.9.1-1 we get

$$K_f = 0.4\left(1 - \frac{A_2}{A_1}\right)$$

where A is CS area.

Thus

$$\widehat{FL} = 0.4\left(1 - \frac{A_2}{A_1}\right)\frac{v_{2,\text{avg}}^2}{2}$$

Note the velocity we use here to calculate $\widehat{FL}$.

In this case there is no pumping as a part of the system which is whatever we drew there only the piece from the pre stenosis plane to the post stenosis plane with the stenosis somewhere in the middle.

So there, there is no motor and so on so forth pump so there is no shaft work to be taken into account also this is a horizontal pipe therefore no change in elevation Δz is 0. So you have this, this and these terms remaining equation 3.9.1.2 - 1 friction loss here corresponds to a loss for a contraction. See we have applied the engineering Bernoulli equation between planes 1 and 2 what are planes 1 and 2 which are placed in assess and this stenosis plane here between here and here.

Thus

$$\frac{p_2 - p_1}{\rho} + \frac{(v_{2,\text{avg}}^2 - v_{1,\text{avg}}^2)}{2} + 0.4\left(1 - \frac{A_2}{A_1}\right)\frac{v_{2,\text{avg}}^2}{2} = 0 \qquad (3.9.1.2\text{-}2)$$

For cavitation to occur, bubbles of gas need to form or nucleation of gas bubbles need to take place. To understand the conditions for gas bubble formation, consider the case of boiling water. In boiling, bubbles begin to appear when the pressure increases due to temperature increase, and finally equals the saturated vapour pressure (note that this is not an equilibrium situation, and thus we cannot apply the phase diagram to find the relevant temperature-pressure relationship for the vapour and liquid phases). In the case of cavitation, the approach is from the other direction; the pressure decreases with increase in velocity of the fluid, and when the pressure equals or becomes lower than the saturated vapour pressure, bubbles form and cavitation occurs. Let us define the difference between the actual pressure and the saturated vapour pressure as p_g.

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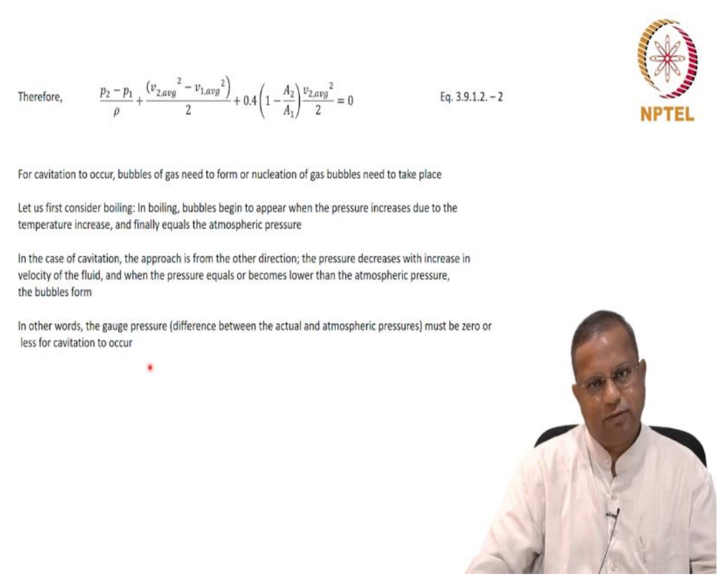

We just substituted this for the refers the last term 3.9.1.2 - 2 for cavitation to occur bubbles of gas need to form or nucleation of gas bubbles need to take place.

Thus

$$p_2 - p_1 = p_{2g} - p_{1g} \qquad (3.9.1.2\text{-}3)$$

Also for continuity

$$A_1 v_1 = A_2 v_2 \quad \text{or} \quad v_2 = \frac{A_1}{A_2} v_1 \qquad (3.9.1.2\text{-}4)$$

Since the pressure and velocity are inversely related, and since $v_2 > v_1$, $p_{g2} = 0$ is the condition for the onset of cavitation.

I am going to explain the same thing again that I told you earlier so that you understand it better. Let us first consider boiling in boiling bubbles begin to appear when the pressure increases due to temperature increase and finally equals the atmospheric pressure. In the case of cavitation the approaches in the other direction from the other direction the pressure decreases with increase in the velocity of the fluid and when the pressure equals or becomes lower than the atmospheric pressure the bubbles begin to form.

In other words the gauge pressure or the difference between the actual and atmospheric pressures must be 0 for the cavitation to occur so we have a criterion in this situation for cavitation to occur.

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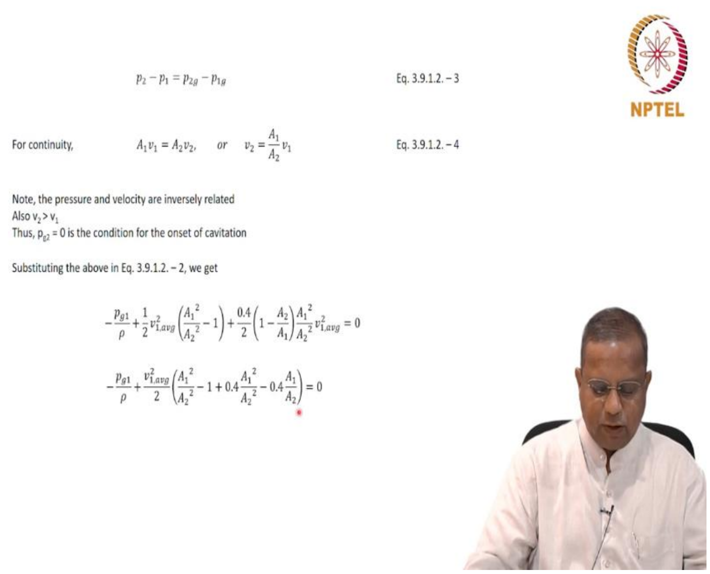

Since the pressure and velocity are inversely related, and since $v_2 > v_1$, $p_{g2} = 0$ is the condition for the onset of cavitation.

Substituting the above in Eq. 3.9.1.2-2, we get

$$-\frac{p_{g1}}{\rho}+\frac{1}{2}v_{1,\text{avg}}^{2}\left(\frac{A_1^2}{A_2^2}-1\right)+\frac{0.4}{2}\left(1-\frac{A_2}{A_1}\right)\frac{A_1^2}{A_2^2}v_{1,\text{avg}}^{2}=0$$

$$-\frac{p_{g1}}{\rho}+\frac{v_{1,\text{avg}}^{2}}{2}\left(\frac{A_1^2}{A_2^2}-1+0.4\frac{A_1^2}{A_2^2}-0.4\frac{A_1}{A_2}\right)=0$$

$$1.4\left(\frac{A_1}{A_2}\right)^2-0.4\left(\frac{A_1}{A_2}\right)-1=\frac{p_{g1}}{\rho}\times\frac{2}{v_{1,\text{avg}}^{2}}$$

$$1.4\left(\frac{A_1}{A_2}\right)^2-0.4\left(\frac{A_1}{A_2}\right)-\left(1+\frac{p_{g1}}{\rho}\frac{2}{v_{1,\text{avg}}^{2}}\right)=0$$

Therefore the velocity at 2 is greater than the velocity at 1. Therefore $p_{g2} = 0$ is the condition for onset of cavitation p_{g2} getting to the gauge for the difference in pressure is what we are looking at the gauge pressure difference is what we are looking at when this equals the atmospheric pressure then the liquid becomes a vapor subsequent bursting of the vapor bubbles of the gas bubbles would happen and that because cavitation.

So we are considering the condition when this would become vapor.

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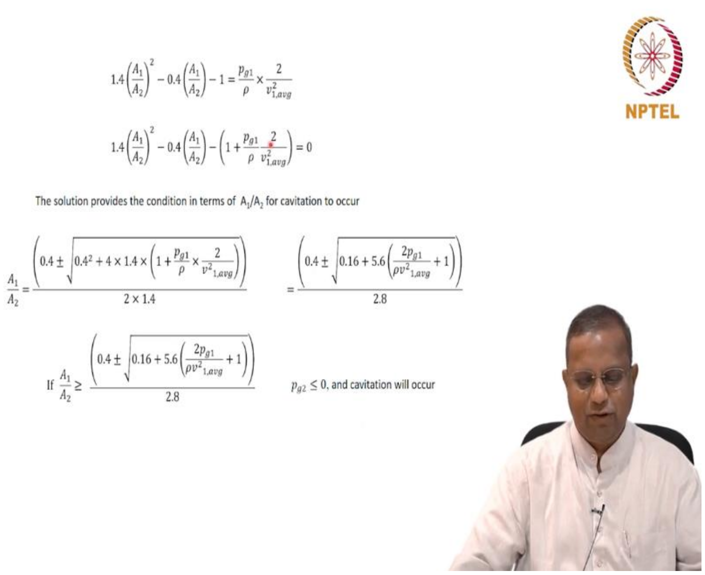

$$\frac{A_1}{A_2} = \frac{\left(0.4 \pm \sqrt{0.4^2 - 4\times 1.4\times\left(-1+\frac{p_{g1}}{\rho}\times\frac{2}{v_{1,avg}^2}\right)}\right)}{2\times 1.4}$$

$$= \frac{\left(0.4 \pm \sqrt{0.16 + 5.6\left(\frac{2p_{g1}}{\rho v_{1,avg}^2} - 1\right)}\right)}{2.8}$$

If $\frac{A_1}{A_2} \geq$ the above RHS, $p_{g2} \leq 0$, and cavitation will occur.

we use the engineering Bernoulli equation to get an insight into the cavitation phenomena in artists. I am sure that must have felt exciting. When we meet next we did look at another application of the engineering Bernoulli equation see you.