Transport Phenomena in Biological Systems
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Lecture - 41
Friction Factor for Relative Motion between a Solid and a Liquid

Welcome back after a series of reasonably long lectures, I thought we will have a short lecture that will provide a nice balance also. The idea here is to cover a reasonable aspect related to a concept in one class, so we have variable times. So my classes probably are only about 8, 9 minutes some are reach about 20 minutes, some 30 20 it is, 20 to 30 it is very common very rarely we go beyond 30 plus, today is class will hopefully be very short.

I thought I will introduce the friction factor for relative motion between a solid and a liquid. For a relative solid liquid motion as I call it here this will be relevant in many different situations solid moving through a liquid which you may encounter in as a part of a biological system analysis and so on and so forth, if you are going to, if you are looking at the macroscopic way of looking at it mainly for design and operation let us look at this way. We will have to take some part of this later when we look at other more complex systems.

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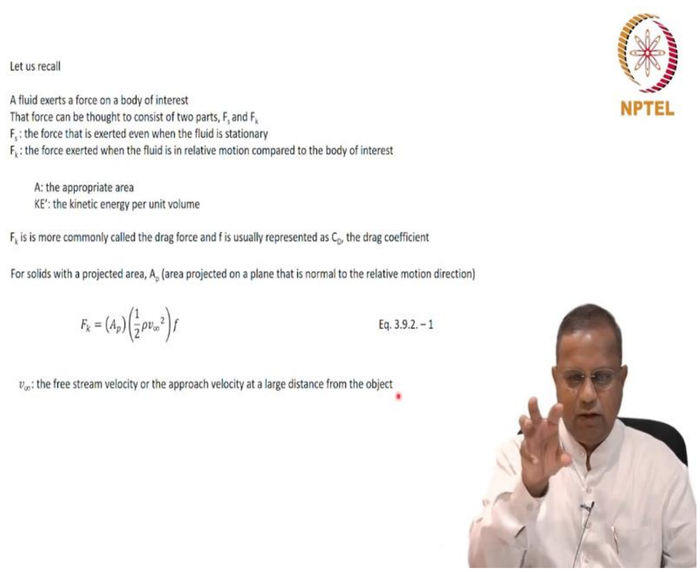

Let us recall that the fluid exerts a force on the body of interest when it flows and that force can be thought to consist of 2 parts static force and kinematic having a static component and kinematic component, a fluid even if it does not flow will exert a force in the body right that you know and a hydrostatic pressure always exerts. So the force due to that hydrostatic part is called the static force and the force that is brought about when because the liquid is moving is called the kinematic part F_k.

F_s is the force that is exerted even when the fluid is stationary and F_k the force exerted when the fluid is in relative motion compared to the body of interest. Is the appropriate area kinetic energy KE' is the kinetic energy per unit volume and F_k is what is more commonly called as the drag force. And it is usually represented through a drag coefficient C_D. So when you have a C_D coming along note that it represents the kinematic component of the force exerted by the fluid on the solid, for solids with a projected area of A_p.

What is projected area? You take a solid it could be a whatever shape okay let us take a sphere for simplicity then if you project the image onto a surface here if it is a sphere your projected area is going to be that of circle yeah this one, one projection is good enough to understand if you shine a light whatever you going to see a circle, so the projected area is going to be that of a circle, this could be a whatever shape and you could figure out a projected area which is appropriate to this shape it could be complex that does not matter.

For solids with a projected area, A_p (area projected on a plane that is normal to the relative motion direction)

$$F_k = (A_p)\left(\frac{1}{2}\rho v_\infty^2\right)f \quad (3.9.2\text{-}1)$$

where v_∞ is the free stream velocity or the approach velocity at a large distance from the object.

F_k is often referred to as the drag force, while f is usually represented as C_D, the drag coefficient. A plot of variation of C_D with Reynolds number, N_{Re}, is available in handbooks, e.g. the one referred to in the earlier section.

When a sphere ($A_p = \pi R^2 = \pi D^2/4$) of density ρ_p falls through a fluid of density ρ, at a terminal velocity of v_t ($= v_\infty$), a simple force balance provides

$$F_k = \left(\frac{4}{3}\pi R^3\right)\rho_p g - \left(\frac{4}{3}\pi R^3\right)\rho g \quad (3.9.2\text{-}2)$$

Using Eq. 3.9.2-1

$$F_k = (\pi R^2)\left(\frac{1}{2}\rho v_t^2\right)f$$

So that projected area is A_p the area projected on a plane that is normal to the relative motion direction. So let me revise that here of the relative motion direction is this way then this becomes as okay this is the plane that is normal to the direction of motion as per rectified. So F_k is nothing but the area times half $\rho\, v_\infty^2$ times the friction factor. So this is the friction factor for this situation.

v_∞ is the free stream velocity this is also called the approach velocity at a large distance from the object, near the object the velocity could be different when the fluid is not under the influence of the solid whatever lost it has that is called the approach velocity or the free stream velocity.

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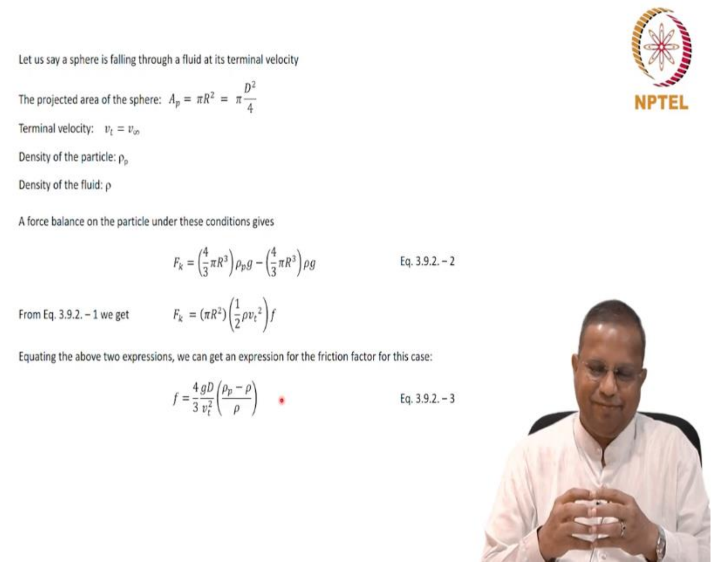

Now let us say that a sphere is falling through a fluid at its terminal velocity. This is something that you are very familiar with all the Stokes' law and things like that situation have been derived using this scenario. The sphere is falling through a fluid at terminal velocity, the projected area of the sphere it is falling the area the direction that is normal is this the projected area is whatever is projected onto that plane that would be a sphere of force sorry that would be a circle of force therefore the A_p is going to be πR^2 or π D^2 / 4.

The terminal velocity you all know are let us represent the terminal velocity as v_t and let us say that it is equal to v_∞ and the density of the particle is ρ_p, the density of the fluid is ρ and if you do a force balance on the particle under these conditions you know the force balance right there is a gravitational force here there is a buoyancy force in this direction and there is a drag force that direction opposite to that of the motion.

All these are give you the F_k as the difference between the gravitational force (4 /3) π R^3 is the volume, volume times the density of the particle as the mass of the particle. Mass of the particle times g is the weight the force of the gravitational force on the particle minus the buoyancy force (4 /3) π R^3 the volume times the density of the fluid times g that will act in the opposite direction. We all know this equation 3.9.2 - 2.

Using Eq. 3.9.2-1

$$F_k = (\pi R^2)\left(\frac{1}{2}\rho v_t^2\right)f$$

Equating the above two expressions, we can get an expression for the friction factor for this case, as

$$f = \frac{4}{3}\frac{gD}{v_t^2}\left(\frac{\rho_p - \rho}{\rho}\right) \qquad (3.9.2\text{-}3)$$

Therefore the friction factor if you these 2 have the left hand sides are the same therefore we can equate the right hand sides. If you equate the right hand sides we get the friction and simplify and like you to do that quite straightforward . So we have a friction factor equation 3.9.2 - 3 for this case of a relative motion between a solid and a fluid. I thought I will this tell you this and make it a short class. So that there is some sense of black balance in terms of time. See you in the next class.