Transport Phenomena in Biological Systems

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Lecture - 45
Thermal Energy Flux

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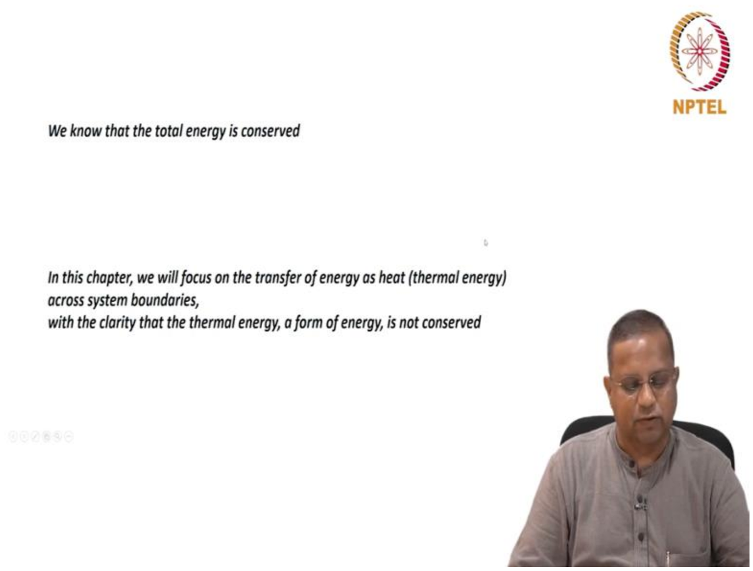

Welcome back, today let us begin our discussion on thermal energy flux. We all know that the total energy is conserved energy conservation principle rate and they can neither be created nor destroyed, but can be converted from one form to another we have all done we have read this from probably sixth standard onwards. In this chapter we will focus on the transfer of energy as heat or thermal energy for short across system boundaries. With the clarity that the thermal energy a form of energy is not conserved.

The total energy is of course conserved; the thermal energy is not conserved. However, we will start with the total energy formulation, because that gives left hand side equals right hand side, then pull out the thermal energy aspect. That is what it is a part of the derivation, which I will direct you to go and see and let us see the clarity is that the thermal energy a form of energy is not conserved. While we will be using the energy conservation equation.

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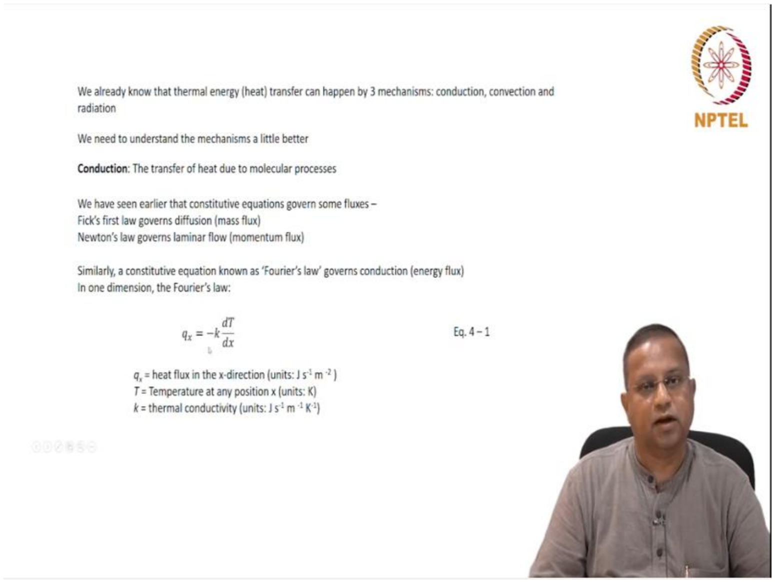

We already know that the thermal energy or heat energy transfer can occur by 3 mechanisms so again we know from high school the mechanisms are conduction in which the heat is transferred by molecular motion, electron motion and so on and so forth. Convection which is the movement of the fluid the bulk motion as it is called and radiation. Radiation is a total different mechanism by which the heat energy gets transferred.

We will look at some of these in a little bit of detail next. So, the mechanisms conduction is transfer of heat due to molecular processes. And we have already seen that constitutive equations, when constitutive equations govern some of the fluxes, we have already seen the Newton's law of viscosity for mass Newton's law of viscosity for momentum flux, the Fick's first law of mass flux and so on.

And those processes were also molecular processes and therefore you expect the heat transfer being a molecular process through conduction alone that something similar would be applicable. So as I mentioned, Fick's first law governs diffusion mass flux, Newtons law governs laminar flow momentum flux. Similarly a constitutive equation known as a Fourier's law governs conduction or energy flux.

In one dimension, Fourier's law is written as

$$q_x = -k\frac{dT}{dx} \tag{4-1}$$

where q_x is heat flux in the x direction (units: J s^{-1} m^{-2}), T is temperature at position x (units: K) and k is thermal conductivity (units: J s^{-1} m^{-1} K).

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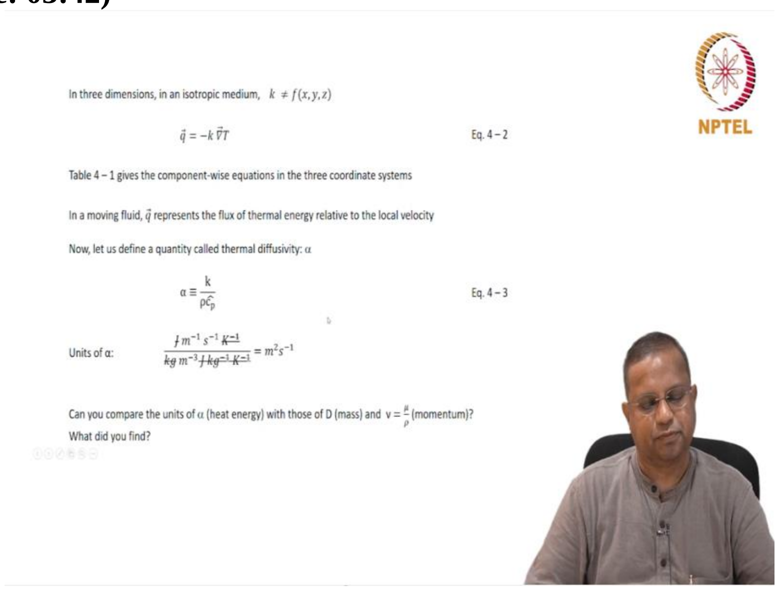

In three dimensions, assuming an isotropic medium where the thermal conductivity is not a function of position, i.e. $k \neq f(x, y, z)$

$$\vec{q} = -k\vec{\nabla}T \tag{4-2}$$

In a moving fluid, $\vec{q}$ represents the flux of thermal energy relative to the local velocity. The equation for thermal energy flux in different coordinate systems is presented in Table 4-1.

And to have the same formulation as the previous ones, let us define a quantity called the thermal diffusivity.

Thermal diffusivity, α can be defined as

$$\alpha \equiv \frac{k}{\rho C_P} \tag{4-3}$$

$$\text{Units of } \alpha = \frac{\cancel{J}\, m^{-1} s^{-1}\, \cancel{K^{-1}}}{\cancel{kg}\, m^{-3}\, \cancel{J\, kg^{-1}} \cancel{K^{-1}}} = m^2 s^{-1}.$$

It is defined as this if you work out the units of this; it turned out to be meter square per second, recall that meter square per second is the same unit as in kinematic viscosity μ/ρ would be this and it is the same unit in diffusivity meter square per second, if you recall that is directly meter square per second. In the case of momentum flux it is μ/ρ that would also be meter square per second.

In this case alpha the thermal diffusivity is k /ρ C_p. If you work out the units that will turn out to be k is joules per meter per second per kelvin, density is kilogram per meter cube, C_p is joules per kilogram per kelvin, and the units are meters square per second. We have already compared the units and we saw all of them to be the same that is of diffusivity and of kinematic viscosity. The units of those are the same as that of the thermal diffusivity

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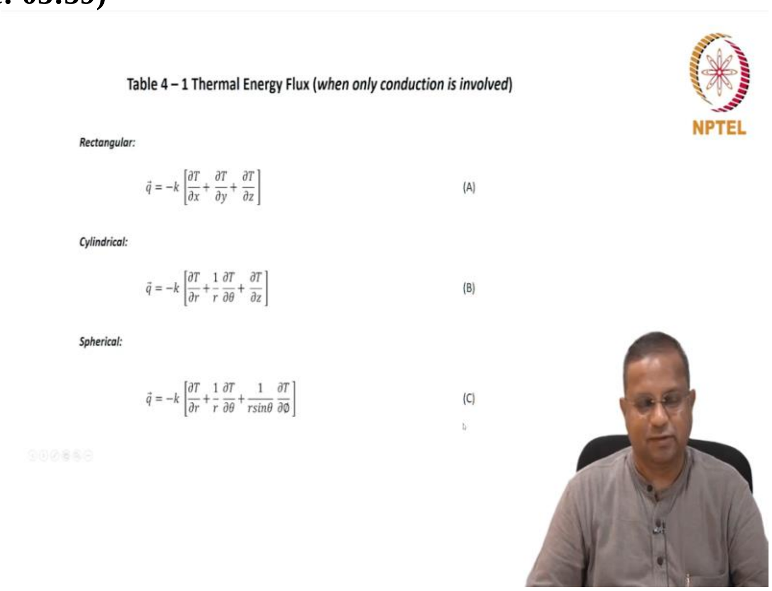

Now, this is the equation that I talked about the thermal energy flux when only conduction is involved. For the rectangular cartesian coordinate system, we have already seen this equation,

Table 4-1 Thermal energy flux (when only conduction is involved)

Rectangular

$$\vec{q} = -k\left[\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z}\right] \qquad (A)$$

Cylindrical

$$\vec{q} = -k\left[\frac{\partial T}{\partial r} + \frac{1}{r}\frac{\partial T}{\partial \theta} + \frac{\partial T}{\partial z}\right] \qquad (B)$$

Spherical

$$\vec{q} = -k\left[\frac{\partial T}{\partial r} + \frac{1}{r}\frac{\partial T}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\right] \qquad (C)$$

Please make a copy of this table also and keep it as a part of your notes soft copy hard copy whatever it is, it must be available for easy reference so that you can just pull out these equations and start working things up. Can you do that please?

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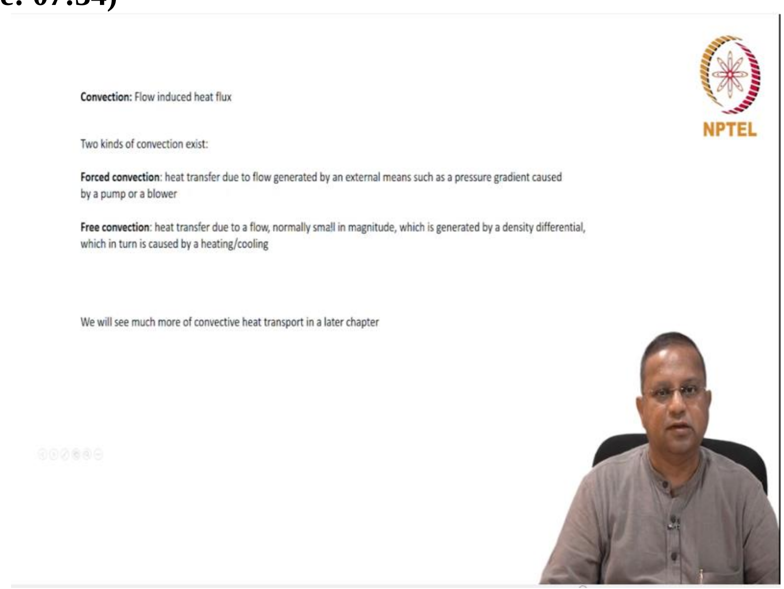

Now that was conduction, there are 2 other mechanisms of heat flux rate or heat transfer. Convection is flow induced heat flux, bulk flow as I said or convective flow. That is where convection comes from 2 kinds of convection exists; one is forced convection where the heat transfer due to which is which refers to the heat transfer due to flow generated by an external means such as a pressure gradient caused by a pump or a blower, You have a pump that moves the liquid or moves the air you have a fan that moves the air that is forcing the convection. So, that is called forced convection. You could also have free convection or natural convection where heat transferred due to flow normally small in magnitude which is generated by a density differential which in turn is caused by a heating or cooling happens. In other words, you heat a part of the liquid not another part, the heated part the density goes down.

Since the density is lower, it starts to move up, the heavier liquid goes down, and that induces a certain motion as a circulatory motion. That is called free convection. There is no forced convection by a pump or a motor or blower; you have free convection that is caused by maybe a temperature gradient. So we will see more of convective heat transfer in the last chapter where we combine the driving forces.

Here again very briefly, you have the flux, the heat flux, heat energy flux and then the primary driving force is the temperature gradient. Recall the primary driving force for mass was the concentration gradient the primary driving force for the momentum was the velocity gradient, moment flux was the velocity gradient. In this case, the primary driving force for heat flux is the temperature gradient that was directly apparent in the conduction case. It is good to see the connection.

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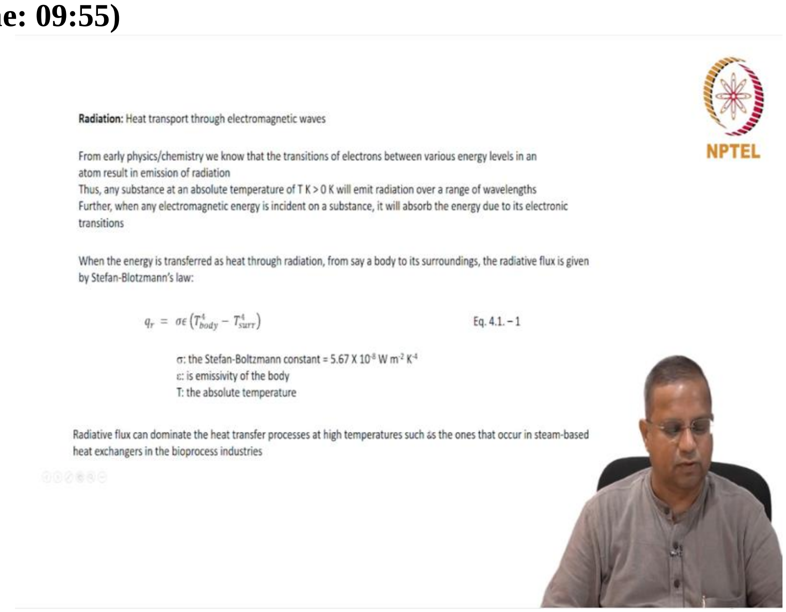

Now radiation is different mechanism altogether. You do not have the equivalent for the other conserved quantities that we are looking at in this case yeah or at least you do not have the equivalent of mass and mass flux and momentum flux as radiation. Radiation is heat transfer through electromagnetic waves. From the early courses in physics and chemistry we know that the transitions of electrons between the various energy levels when it absorbs radiation in an atom result in the emission of radiation.

It absorbs goes to a higher level then when at emits, there is radiation that comes out. That is any substance at an absolute temperature of T which is greater than 0 Kelvin will emit radiation over a range of wavelengths. That is a necessary thing to happen. Further when any electromagnetic energy is incident on a surface it will absorb energy due to its electronic transitions. So that is what I mentioned earlier.

When the energy is transferred as heat through radiation, say from a body to its surroundings, the radiative flux is given by something called a Stefan Boltzmann's law this also you had have done in higher secondary school.

The Stefan-Boltzmann law that governs radiation states that the intensity of radiation is proportional to the fourth power of the temperature in K of the emitting body. When the energy is transferred as heat through radiation, from say, a body to its surroundings, the radiative flux can be expressed as

$$q_r = \sigma \varepsilon \left(T_{\text{body}}^4 - T_{\text{surr}}^4\right) \tag{4.1-1}$$

where σ, the Stefan-Boltzmann constant = 5.67×10^{-8} W m^{-2} K^{-4}, ε is emissivity of the body, and T is the absolute temperature.

T is the absolute temperature, in this expression T is the absolute temperature if you have temperature and centigrade you need to add 273 to get Kelvin the absolute temperature.

Radiative flux can dominate the heat transfer process at high temperatures such as the ones that occur in steam based heat exchangers in bioprocess industries and so on, so it is certainly very relevant, we need to take care of this at appropriate situations, typically high temperatures and so on. That was the introduction to heat flux I think that stopped here. And in the next class, we would take things further.

Again, you know that there are 2 approaches shell balance approach and equation of conservation equation approach. Here, what I am going to do is that I am not going to do shell balances; you have enough exposure to shell balances, so you know how to do shell balances. So you whenever needed if at all needed, you just need to use the same principles, you do balances of heat energy of energy over a shell and then separate out the heat energy.

That is what you need to do. We are not going to do that here at all, we will directly jump to the conservation equation approach and there also let me see what I am going to take, then let us meet in the next class see you again.