Transport Phenomena in Biological Systems

Prof. G. K. Suraishkumar
Department of Biotechnology Bhupat and Jyoti Mehta School of Biosciences Building
Indian Institute of Technology - Madras

Lecture - 46
Equation of Energy

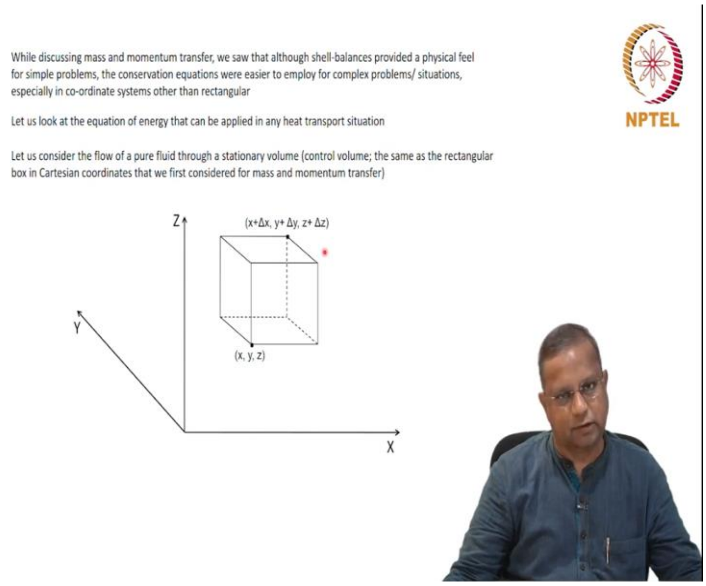
Welcome to this class; today let us begin with the equation of energy we are looking at energy flux with a focus on thermal energy flux only the total energy is conserved. Therefore, for the conservation aspects we need to look at total energy from that we need to extract out the thermal energy and write thermal energy in as a function of the parameters that we measure that we have a handle on and so on and so forth.

That is the whole idea here conservation itself is that of total energy out of which we are extracting out thermal energy because thermal energy happens to be half interest to us and but energy conservation is the principle. Then we said as we have already seen, there are 2 major approaches one the shell balances the other one is conservation equation approach. Here I did not show you the shell balances approach as I showed you in detail for mass flux and momentum flux, energy is anyway scalar.

So, this will be somewhat closer to mass, but there are a lot more complications therefore, the equations would be a lot more complex, that is pretty much it. Then we look when we started out this chapter, on energy flux we looked at the 3 major mechanisms by which a heat energy gets conducted sorry heat energy gets transferred one is by conduction the other one is by convection, the third one is by radiation.

Let us move further in the last class I think we looked at the conduction equations alone if conduction is the only method if molecular means molecular vibrations and motion is the only means by which heat is transferred, then the equations that we looked at are developed earlier, which is nothing but Fourier’s law that we saw in all the 3 coordinate axis and so on; that we could use. However, we need a lot more general, we need a formulation for a lot more general situation, when there is flow and other things happening. And that is what we are going to do in this particular class. So equation of energy let us begin.

(Refer Slide Time: 02:40)



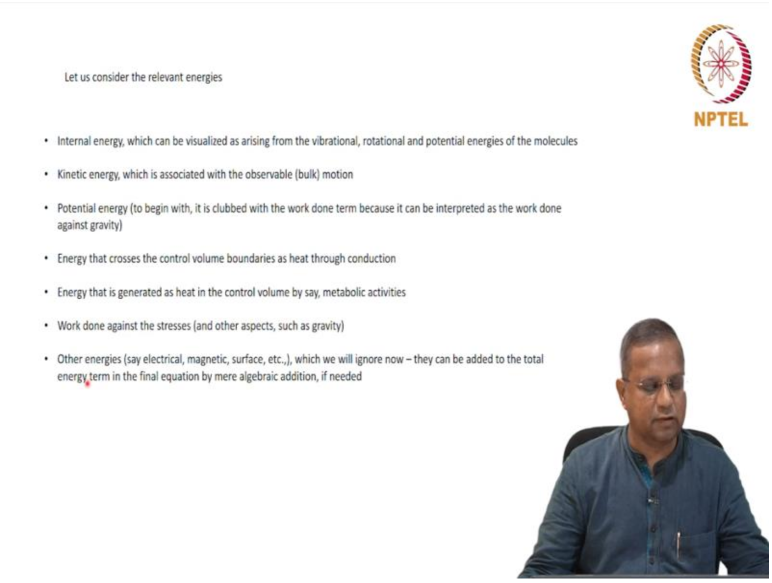
Let me repeat a few of these things that might be helpful while discussing mass and momentum transfer. We saw that through shell balances, or although shell balances provided a physical field for simple problems. The conservation equations were easier to employ for complex problems situations, especially in coordinate systems other than rectangular that we have already seen. Now, let us look at the equation of energy that can be applied in any heat transport situation.

And we are going to begin the same way, we are going to consider the flow of a pure fluid, simple a pure fluid, it does not have been have anything else through a stationary volume which we know as the control volume the same as the rectangular box or the cuboidal box in Cartesian coordinates that we first considered for mass and momentum transfer the same control volume, we are going to consider because it is intuitive.

The rectangular Cartesian coordinate system is intuitive for us. So, this is what it is again the X axis here we have chosen we have chosen the Y axis like this. Therefore, the Z axis by the right handed screw rule as you go from X to Y, whichever direction a right handed screw move will be the direction of the Z axis which is this and then this is the cuboidal box that we are considering x, y, z are the coordinates of this point the dimensions of this box are Δx, Δy and Δz.

Therefore, the time the coordinates of this point are $x + \Delta x$, $y + \Delta y$ and $z + \Delta z$. So, this is the same control volume over which we are going to do our energy balances.

(Refer Slide Time: 04:31)



So, we are going to consider the relevant energies if there are other energies you need to add at this stage. So, let us look at the usual energies that are involved internal energy which can be visualized as arising from the vibrational, rotational and potential energies of the molecules themselves that is what we mean by internal energy. And recall concepts from the course on thermodynamics; the abstract formulation for internal energy that happened to be initially an abstract formulation.

That happened to be very helpful for various different analyses insights, design and operation of biological engineering systems. And then if you look at a statistical thermodynamics aspect then we could relate the internal energy to the molecular motions then, let us consider kinetic energy which is associated with the observable motion or the bulk motion. So, when a fluid moves let us say or a solid moves over that matter.

There is a kinetic energy you know it is $(1/2)mv^2$ or equivalent in fluid systems and that is kinetic energy that is a second energy that we are going to consider. Then we also know that right from school may be sixth standard we know that there is something called a potential energy; potential energy is energy by virtue of the position of the substance with respect to a certain datum level to begin with.

We are going to club it with the work done term for this particular derivation, because it can be interpreted as the work done against gravity. So, we are not going to consider this separately here, we are going to consider it as work done against gravity, energy that crosses the control volume boundaries as heat through conduction energy that is generated as heat in the control volume.

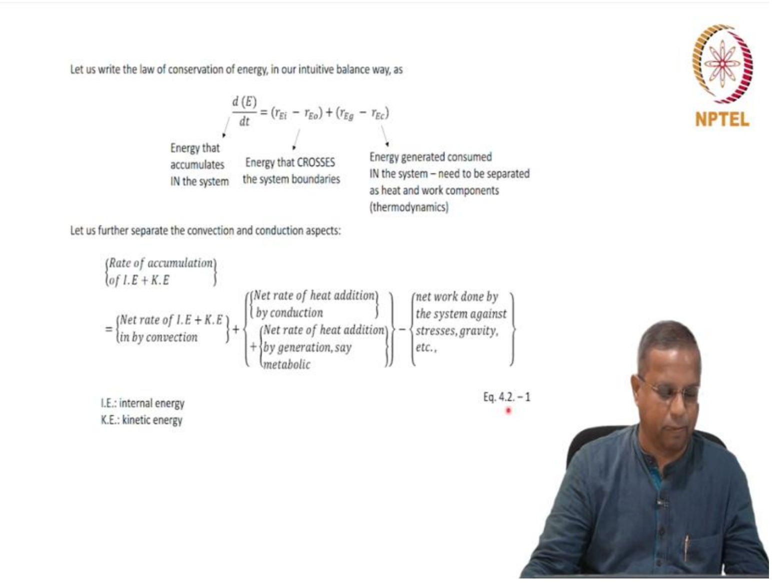
Let us say it could be by metabolic activities or other aspects, metabolic activities because we are interested in biological systems metabolic activities are a significant source of energy generation within a system or a control volume work done against stresses, in fluid flow there is stress and work done against stresses and other aspects of this gravity is taken care of in the work done term. And other energies, say electrical, magnetic, surface energy and so on and so forth.

We are not going to consider them for now. However, if they are important for your system, you need to come back to the derivation at this point, add them here, or if you can see where they get added, you could add them to the final equation. But to be confident it is best to add them here and then do the whole set of manipulations again the whole set of derivation again to come up with the equation or the situation.

Let us say the representation that you would have significant confidence. As I also mentioned, they can be added to the total energy term in the final equation by mere algebraic addition. Here, it would be straightforward it is a scalar. So, the other aspects may not cause too much of a confusion if it is a vector such as momentum, it is not so easy to see unless you have significant insight into the way vectors behave just by looking at the vector equation.

And trying to have a picture of what is happening is calls for certain orientation in those aspects are a lot of experience So we will but here is only energy you could even just add it; you need not come back and re derive the whole thing, so these are the energies that we will be considering internal energy, kinetic energy, potential energy we are going to club it with work done; energy that crosses this control volume boundaries as heat through conduction energy that is generated as heat in the control volume by metabolic activities work done against stresses and other energies.

(Refer Slide Time: 08:51)



Now we are going to write the law of conservation of energy. Law of conservation of energy is energy can neither be created nor destroyed so, for a process, the energy total energy before the process equals the total energy after the process that is simple as that. But we are going to write the left hand side in terms of the parameters of interest to us the right hand side in terms of the parameters of interest to us that is it.

So, the same way that we viewed mass as a certain quantity momentum as a certain quantity, let us view energy as a certain quantity. So the rate of change of energy is the input rate of energy minus the output rate. So the net input rate plus the generation rate minus the consumption rate, so this is the net generation rate. So it is nothing but the same balance the equation that is valid for any conserved quantity, any quantity in this case.

We have taken care of what all can happen to it? Only if it is conserved, can you write left hand side equals right hand side and thereby, this becomes valid. So we are going to consider the same equation for energy now, dE/dt is nothing but the energy that accumulates in the system or control volume, these things are important, please make a note of these and these will become a part of you only when you start doing applications problems and so on so forth.

You will do some of that as a part of the course itself here and you will have an opportunity to do that in the assignments. So, note the IN the system here energy that accumulates in the system. This term as I mentioned, is the energy net energy that CROSSES the system boundaries, input minus output. And this is the energy generated or consumed IN the system or net energy generated IN the system.

$$\begin{Bmatrix} \text{Rate of accumulation of} \\ \text{IE + KE in the system} \\ \text{or control volume} \end{Bmatrix} = \begin{Bmatrix} \text{Net rate of IE + KE} \\ \text{in by convection} \end{Bmatrix}$$
$$+ \begin{Bmatrix} \begin{Bmatrix} \text{Net rate of heat addition} \\ \text{by conduction} \end{Bmatrix} \\ + \begin{Bmatrix} \text{Net rate of heat addition by} \\ \text{generation, say metabolic} \end{Bmatrix} \end{Bmatrix}$$
$$- \begin{Bmatrix} \text{Net work done by the system or} \\ \text{the control volume against} \\ \text{stress, gravity, etc.} \end{Bmatrix} \tag{4.2-1}$$

where IE is internal energy and KE is kinetic energy.

And this if you recall thermodynamics recall the first law it said $\Delta u = q - w$ for a closed system. Why did we need to have q and w? If you understood thermodynamics, you understood that you needed 2 different formulations for heat and work because they are not completely interconvert because of entropy and so on. So, that is the nature of those, they are by nature 2 different aspects and therefore, we need to account for those 2 aspects separately.

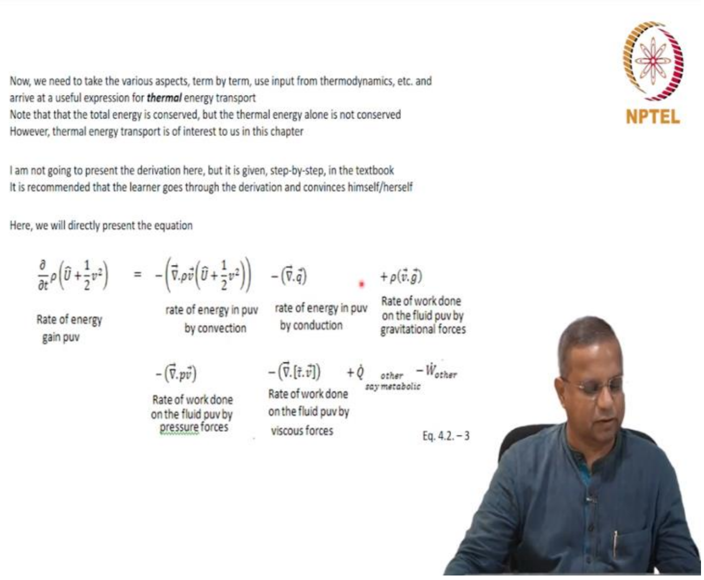
So, the; need to be separated as heat and work components and we know from thermodynamics why we need to do that? Now, let us separate the convection and conduction aspects, all these are to get it of the form or get it in terms of the parameters that are easy to measure or know that is all that so, we are going to write this as a rate of accumulation of internal energy and kinetic energy that is what we are going to consider those are the energies that we considered.

If you go and look at the list the rest we have coupled with others or whatever equals the net rate of internal energy and kinetic energy in by convection. So we have separated out the convection from the conduction we are going to take conduction with something else plus the net rate of heat addition by conduction, net rate by conduction here convection here conduction plus the net rate of heat addition by generation say metabolic minus the net work done by the system against stresses, gravity and so on so forth.

Essentially, we have recast the right hand side in terms of the various separate components that go into determining these that is it. Again, this is the straightforward we are looking at internal energy and kinetic energy, the accumulation of that in the system. And we have separated out the convection and conduction. This is the convection term net rate of internal energy plus kinetic energy in by convection and this is net rate of heat in or heat addition by conduction.

And this is net rate of heat addition by generation and this is the net work negative of that, the net work done by the system against stresses gravity and so on; we have separated out the heat and the work components. Let us call this equation 4.2 - 1.

(Refer Slide Time: 13:46)



Now, we need to take the various term aspects, term by term use input from thermodynamics and so on because there is heat, there is work a lot of interconversions and so on so forth; we have an internal energy sitting there. And then after we do all this, we arrive at a useful expression for thermal energy transport. What I am going to do is? Before that note that, the total energy is conserved but thermal energy alone is not conserved it helps to repeat this.

Some of you might feel bored, you have already gotten it, but it helps to repeat this we are looking at thermal energy alone where the total energy is conserved. The thermal energy transport is of interest to us in this chapter. That is the reason for looking at thermal energy. Let me say that I am not going to present the lengthy derivation here. It is there in the textbook step by step, this derivation is certainly there.

These shell balances are not there even in the book I because we had spent enough time on the mass and momentum aspects, shell balances for those, I did not spend time even in the book for energy transport, here, I am not even going to spend time for the conservation equation derivation, but it is given in the textbook step by step. So, those who are interested can go and take a look at.

It is recommended certainly it is recommended that the learner goes through the derivation and convinces himself or herself that it is indeed the case. Here we are in this course; we are just going to directly present the equation. It will turn out to be something like this we put everything together and go through about I think it is about 7 or 8 pages of textbook pages of derivation with a lot of considerations a lot of thermodynamics and so on so forth.

In vector notation

$$\frac{\partial}{\partial t}\rho\left(\hat{U}+\frac{1}{2}v^2\right) = -\left(\vec{\nabla}.\rho\vec{v}\left(\hat{U}+\frac{1}{2}v^2\right)\right) - (\vec{\nabla}.\vec{q}) + \rho(\vec{v}.\vec{g})$$

Rate of energy gain puv; Rate of energy in, puv by convection; Rate of energy in, puv by conduction; Rate of work done on the fluid puv by gravitational forces

$$-(\vec{\nabla}.p\vec{v}) \quad -(\vec{\nabla}.[\tilde{\tau}.\vec{v}])$$

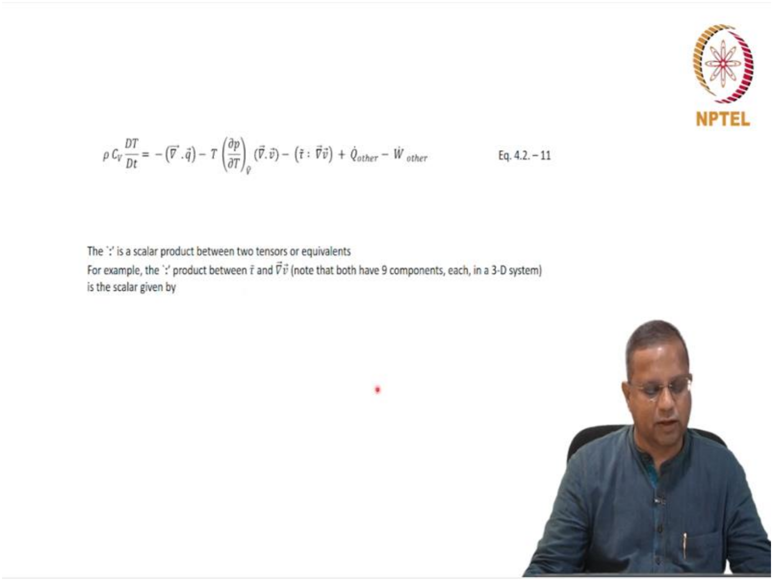
Rate of work done on the fluid puv by pressure forces; Rate of work done on the fluid puv by viscous forces

$$+\dot{Q}_{\text{say, other like metabolic heat}} - \dot{W}_{\text{other}} \qquad (4.2\text{-}3)$$

where puv is per unit volume.

Let us call this equation 4.2 - 3 this is what it will come down to after a lot of manipulation 6 or 7 pages of manipulation or maybe 3 or 4 pages for this and then subsequent manipulation other 3 or 4 pages. If you have any doubt in that derivation, please write to me or write in the forum and then we can discuss.

(Refer Slide Time: 18:11)



After that you bring in some more thermodynamics and some more relationships and so on and so forth. In other Maxwell's equations equivalent and things like that, then this equation can be written as in a nice compact form as a

$$\rho C_V \frac{DT}{Dt} = -(\vec{\nabla}\cdot\vec{q}) - T\left(\frac{\partial p}{\partial T}\right)_{\hat{V}}(\vec{\nabla}\cdot\vec{v}) - (\tilde{\tau}:\vec{\nabla}\vec{v}) + \dot{Q}_{\text{other}} - \dot{W}_{\text{other}}$$

A dyadic product which will result in a second order tensor. This I will talk to you about in a little bit, there is a double dotted colon kind of a thing here; this represents a totally different kind of manipulation between vectorial tensorial quantities. The double dot represents a scalar product between 2 tensors are equals.

$$(\vec{v}\cdot[\vec{\nabla}\cdot\tilde{\tau}]) - (\vec{\nabla}\cdot[\tilde{\tau}\cdot\vec{v}]) = -(\tilde{\tau}:\vec{\nabla}\vec{v})$$

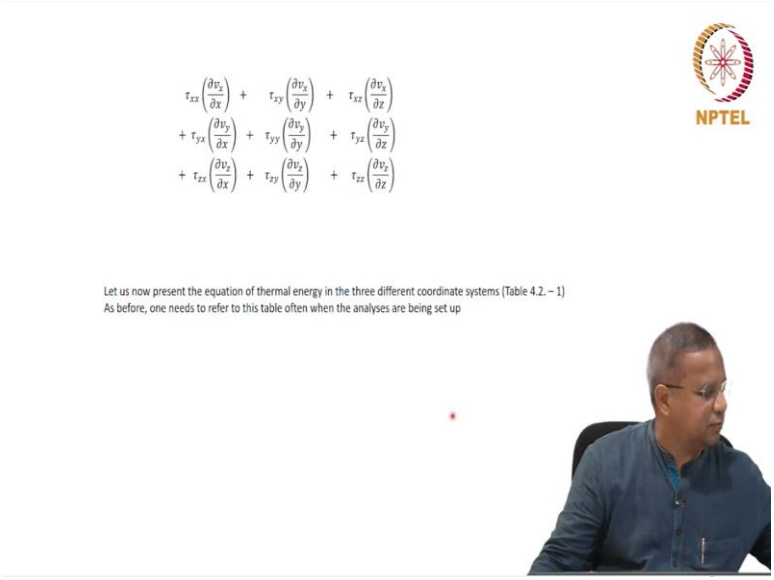
where ':' is a scalar product between two tensors or equivalents. For example, the ':' product between $\tilde{\tau}$ and $\vec{\nabla}\vec{v}$ (note that both have 9 components, each, in a 3-D system) is the scalar given by

$$\tau_{xx}\left(\frac{\partial v_x}{\partial x}\right) + \tau_{xy}\left(\frac{\partial v_x}{\partial y}\right) + \tau_{xz}\left(\frac{\partial v_x}{\partial z}\right)$$

$$+\,\tau_{yx}\left(\frac{\partial v_y}{\partial x}\right) + \tau_{yy}\left(\frac{\partial v_y}{\partial y}\right) + \tau_{yz}\left(\frac{\partial v_y}{\partial z}\right)$$

$$+\,\tau_{zx}\left(\frac{\partial v_z}{\partial x}\right) + \tau_{zy}\left(\frac{\partial v_z}{\partial y}\right) + \tau_{zz}\left(\frac{\partial v_z}{\partial z}\right)$$

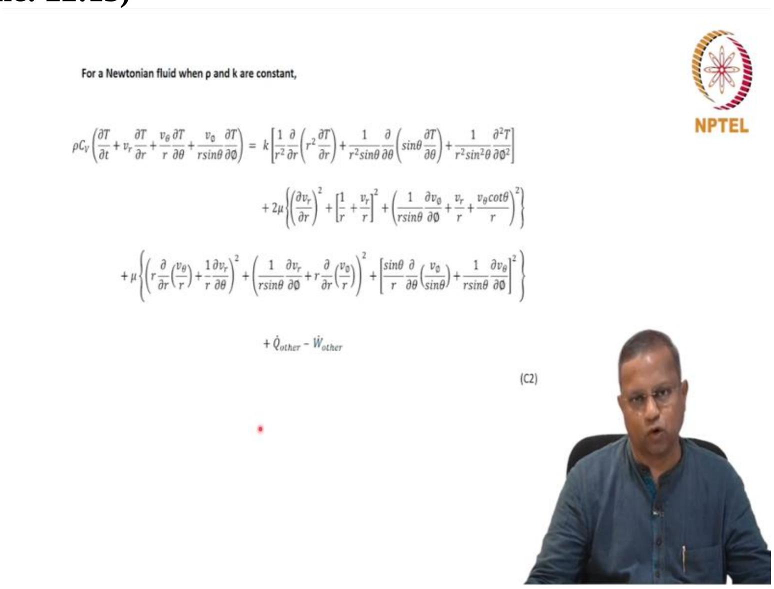
(Refer Slide Time: 20:16)



Therefore, it is just a sum of all these quantities, which in turn out to be a scalar. Let me show you how this equation appears you know.

This is the form that has been expanded into its various components. This is a scalar so, if you do not have components in each direction, and so on and so forth, energy is a scalar, but in 3 different coordinate systems, you have these equations directly presented in table 4.2 - 1, I am going to show you that next, please make a copy of a table 4.2 - 1 and keep it at a readily accessible place for reference whenever it is needed; please do that because we need to refer to this table often when analysis are being set up. And let me show you that set of tables, set of equations in the table.

(Refer Slide Time: 22:13)



So, this is table 4.2 - 1 the equation of thermal energy. So, this is for the rectangular Cartesian coordinate system, you have it of the one, on the same equation written down as individual components because we can directly substitute the individual components, that is the whole point here. So, these are the various terms this is the we call this A1 and when the same way in the earlier case, we have an equation for the simpler situation, this is in terms of shear stresses which are not directly obtainable.

Or you would like to have these in terms of velocities that you could do easily if you if the fluid happens to be Newtonian. And when ρ and k are constant, density and thermal conductivity are constants; then you could write it of this form following pretty much similar arguments that we did for the momentum flux or the momentum balance equations, equations of motion. So this is for the rectangular Cartesian coordinate system case.

You could always convert it into cylindrical coordinates by using the principles given the appendix or using the methodology given in the appendix. These are the equations B1 B2, the first one is for a general situation, the second one is for Newtonian fluid when ρ and k are constant. Then for spherical coordinate systems, please make a copy of this set of tables and keep it as a part of your notes. I think we will stop here for this class been at it for some time. We will take things forward when we meet again see you then.