Transport Phenomena in Biological Systems
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Lecture - 51
Charge Flux - Some Fundamentals

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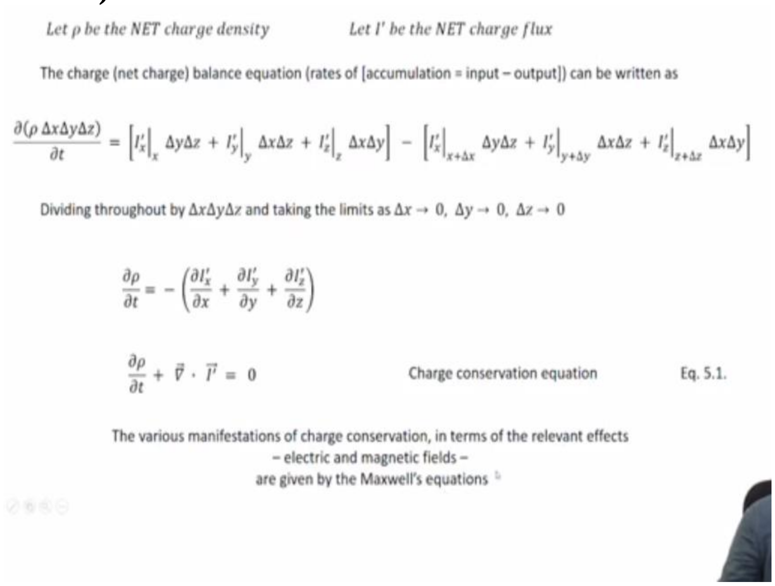

Welcome, let us continue to understand the fundamental aspects of electromagnetism better partly in a review mode partly in new information mode. In the previous class we had derived the charge conservation equation in the few steps, we are considered the control volume, control volume and the rectangular Cartesian coordinate systems. Then we did a balance on the net charge which considered the charge as a quantity.

So, rate of change of charge with time in the system the control volume equals the rate of input minus rate of output because there is no generation and consumption when it comes to net total charge. So, in terms of the net charge density and the net charge flux, we could get $\frac{\partial \rho}{\partial t} + \nabla I' = 0$. So, note that I' is the net charge flux and we had said that we will retain I for the current alone charge per time charge per time per unit area will become I' for our discussions.

So, let us continue with some more important aspects, the various manifestation of charge conservation in terms of the relevant effects, that is electric and magnetic fields are given by what is called the Maxwell's equation. Maxwell's equations are nothing but a collection of various different equations given by various different researchers. And those set of 4 Maxwell's equations, along with the tax conservation equation is all that you need for electromagnetism. Of course, that is a combined in a concentrated way of saying the truth.

Same way as mass conservation equation is a concentrated way of saying the truth, we had to work out different ways to make it useful. We had to write it in, in the form of useful quantities to make it useful. Similar things will happen here. However, we need to first at least review those the Maxwell's equations. You would have already seen this in your first year physics course. But let us review this.

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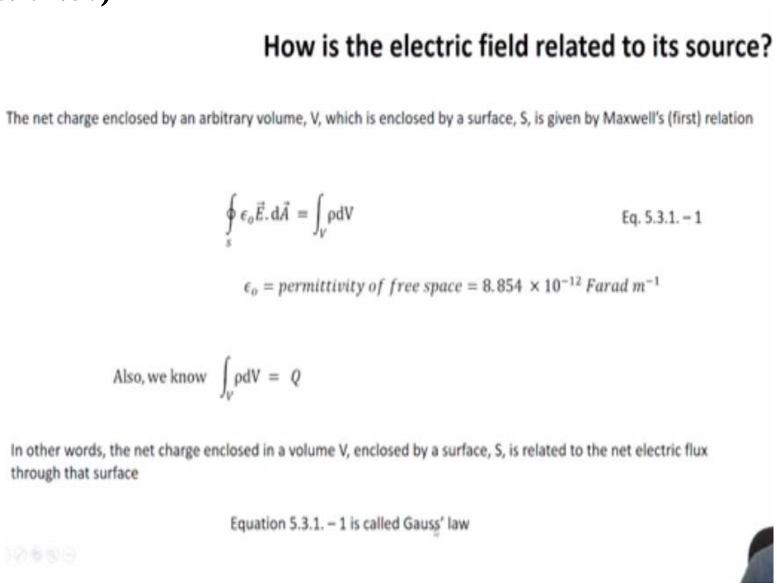

The first Maxwell's equation answers the question, how is the electric field related to its source? How is the influence of the electric field when of the charges it is electric field related to its source the charges the net charge and closed by an arbitrary volume V, which is enclosed by us surface S, arbitrary volume V enclosed by surface S, you know, these are terms that make it a lot more general. That is why some of these you know RK I mean very high φ sounding terms are used, but they make it more general they make it mathematically applicable.

Therefore more general. And once we follow that you do not have to worry about its applicability. That is the whole point. There is a reason why mathematics is written and so complete a form it gives us so much confidence to apply it wherever we want. So, the net charge enclosed by an arbitrary volume V with the same enclosed by a surface S is given by the Maxwell's first equation or relation.

The net charge enclosed by an arbitrary volume V which is enclosed by a surface S is given by Maxwell's (first) relation

$$\oint_S \epsilon_o \vec{E}.d\vec{A} = \int_V \rho dV \qquad (5.3.1\text{-}1)$$

where ϵ_o is permittivity of free space = 8.854×10^{-12} Farad m^{-1}. Further, note that $\int_V \rho dV = Q$.

In other words, the net charge enclosed in a volume V, enclosed by a surface S is related to the net electric flux through that surface. Equation 5.3.1-1 is also called Gauss' law.

So, the charge density net charge per unit volume divided by the differential volume there you get the charge in that volume integrate over the entire volume.

You get the entire charge in that volume. As simple as that says written this appears a little daunting this nothing more to that. In other words, the net charge enclosed in a volume V enclosed by surface S is related to the net electric flux through the surface as simple as that there is a net electric flux. But this how is the electric field related to the source? That is answered by the Maxwell's first equation Gauss' law.

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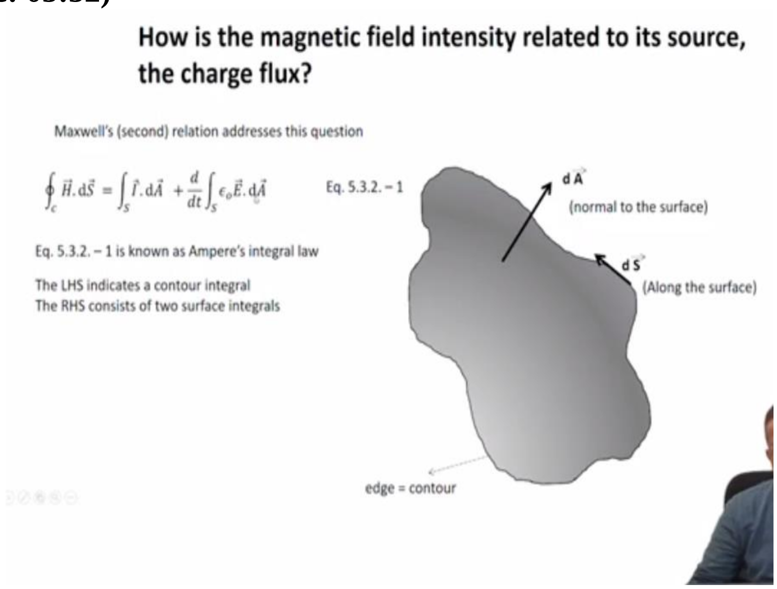

How has magnetic field intensity related to the source the charge flux is that question is answered by the second Maxwell's equation or relation integral

Maxwell's (second) relation shows how the magnetic field intensity is related to its source, as follows

$$\oint_C \vec{H}.d\vec{S} = \int_S \vec{I}\,'.d\vec{A} + \frac{d}{dt}\int_S \epsilon_o \vec{E}.d\vec{A} \qquad (5.3.2\text{-}1)$$

This is also known as Ampere's integral law.

This is known as Ampere's integral law the left hand side is a contour integral over the perimeter contour s is surface integral and it is something like this you need to understand the relationship between the various quantities here.

5.3.2-1 is how the magnetic field intensity is related to its source, the moment of charge. Whenever there is a time varying electric field there will be a magnetic field trying to you know.

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In other words,
the line integral (circulation) of the magnetic field intensity, $\vec{H}$, around a closed contour is equal to the sum of
- the net current passing through the surface spanning the contour and
- the time rate of change of the net displacement flux density (displacement current) through the surface

Eq 5.3.2. – 1 can be written as

$$\oint \vec{H}.d\vec{s} = I + \epsilon_o \frac{d\phi_E}{dt} \qquad \text{Eq. 5.3.2.} - 2$$

ϕ_E: electric 'flux' (historically called flux, and we just use quotes to avoid confusion in our context)

I: current

In other words, an electric current and a time-variant electric 'flux' produce a magnetic field

That is one example. In other words, the line integral or the circulation of the magnetic field and density edge around a closed contour is equal to the sum of the net current passing through the surface panning the contour and the time rate of change of the net displacement flux density, or the displacement current through the surface. Let me go back and show you. So this is the first time that we are talking about this is the second time that we are talking about this is the third time that we are talking about.

In other words, the line integral (circulation) of the magnetic field intensity $\vec{H}$ around a closed contour is equal to the sum of the net current passing through the surface spanning the contour and the time rate of change of the net displacement flux density (displacement current) through the surface.

Alternatively, Eq. 5.3.2-1 can be written as

$$\oint \vec{H} \,.d\vec{S} = I + \epsilon_o \frac{d\phi_E}{dt} \qquad (5.3.2\text{-}2)$$

where ϕ_E is electric 'flux' (historically called flux – we use quotes here to avoid confusion in our context) and I is current.

In other words, an electric current and a time-variant electric 'flux' produce a magnetic field.

So, I hope you are able to form a mental picture of what the Ampere's integral law or the Maxwell second equation. This historically is called flux and we are just going to use quotes to avoid confusion. We will use very little of this term.

You will use very little of this φ_E in our discussions. But this is the relationship that exists and you need to know this relationship. This is historically called flux we will stick to that is current, in other words, an electric current and a time variant electric flux produce a magnetic field. That is all it says in a complete quantitative sense.

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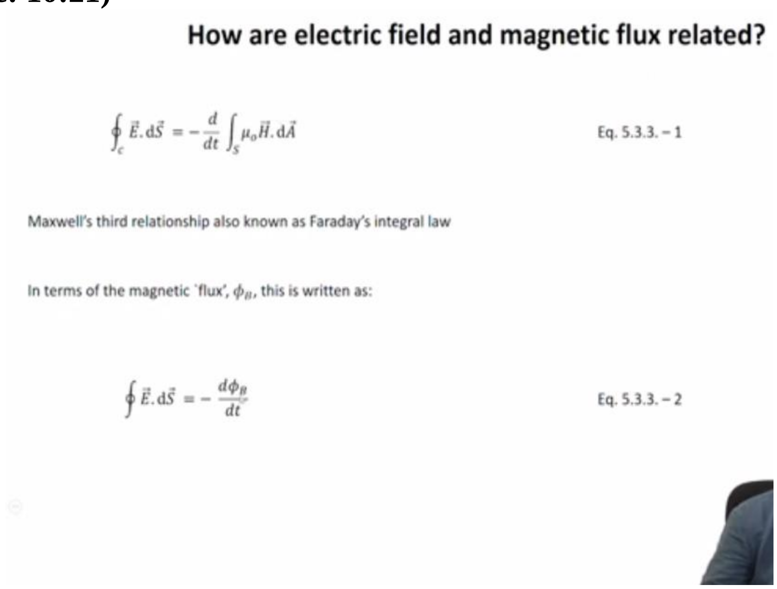

Third Maxwell's relation equation, how our electric field and magnetic flux related our electric field and magnetic flux are related. That is given by this a contour integral of the surface integral over the surface 5.3.3 - 1, this is called the Faraday's integral law and in terms of the magnetic flux again, this is written as integral

In terms of the magnetic 'flux' (the term flux here is again, historical), ϕ_B, this can also be written as

$$\oint \vec{E} . d\vec{S} = -\frac{d\phi_B}{dt} \qquad (5.3.3\text{-}2)$$

So, that is essentially how the electric field is related to the magnetic flux electric field is related to the magnetic flux.

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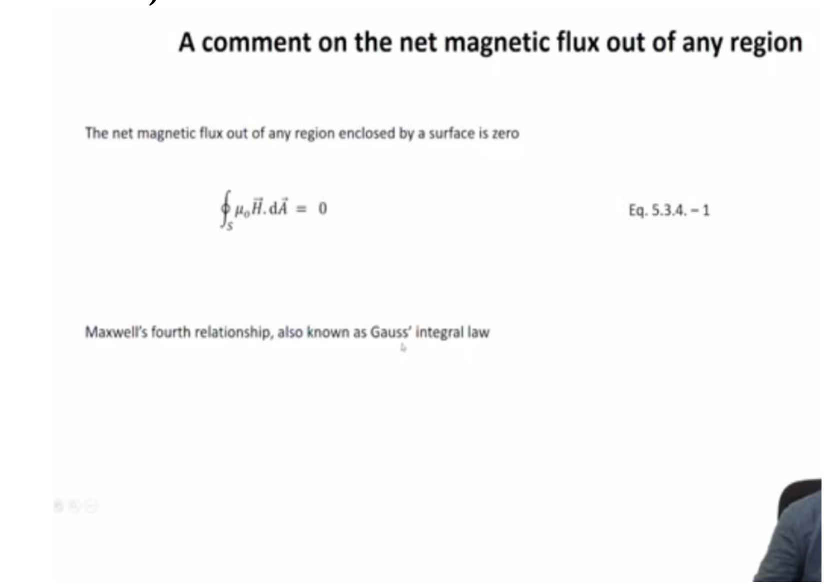

A comment on the net magnetic flux out of any region that is what is given by the maximum fourth relationship. The net magnetic flux out of any region enclosed by surfaces 0. Very simple statement very useful statement.

The net magnetic flux out of any region enclosed by a surface is zero. This is Maxwell's (fourth) relationship and is also known as Gauss' integral law. Mathematically, it can be expressed as

$$\oint_S \mu_o \vec{H} . d\vec{A} = 0 \qquad (5.3.4\text{-}1)$$

It is also known as Gauss' integral law.

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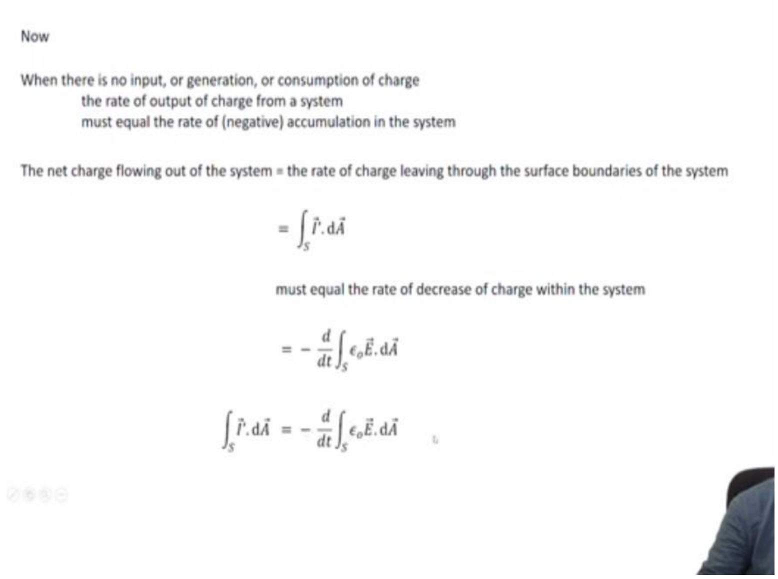

When there is no input no generation or that is those are the 4 Maxwell's relationship. Now, when there is no input no generation or consumption of charge, the rate of output of charge from the system must equal the rate of negative accumulation in the system. That is what that is essentially what this one says. And there is no input no generation or conception of charge, the net rate of output of charge from a system must equal the negative rate of accumulation in the system.

Now, let us look at things further. When there is no input or generation or consumption of change, the net rate the rate of output of charge from a system must equal at the rate of negative accumulation in the system, neither combination is what is going out. So, the net charge flowing out of the system must equal the rate of charge in leaving through the surface boundary to the system.

It is useful to obtain a relationship for charge conservation in terms of charge and charge flux. The principle of charge conservation can be stated as the net charge flowing out of the system, given by the rate of charge leaving through the surface boundaries of the system, $\int_S \vec{I}'.d\vec{A}$, is equal to the rate of decrease of charge within the system, $-\frac{d}{dt}\int_S \epsilon_o \vec{E}.d\vec{A}$. In other words, when there is no input, or generation, or consumption of charge, the rate of output must equal the rate of (negative) accumulation in the system. In the form of an equation, the above principle can be written as

$$\int_S \vec{I}'.d\vec{A} + \frac{d}{dt}\int_S \epsilon_o \vec{E}.d\vec{A} = 0 \qquad (5.4\text{-}1)$$

From Maxwell's (first) relationship, Eq. 5.3.1-1, we can replace the second term on the LHS of Eq. 5.4-1 as

$$\int_S \vec{I}'.d\vec{A} + \frac{d}{dt}\int_V \rho dV = 0 \qquad (5.4\text{-}2)$$

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$$\int_S \vec{I}'.d\vec{A} + \frac{d}{dt}\int_S \epsilon_o \vec{E}.d\vec{A} = 0 \qquad \text{Eq. } 5.4-1$$

From Maxwell's (first) relationship, we can replace the second term on the LHS of the above equation to get:

$$\int_S \vec{I}'.d\vec{A} + \frac{d}{dt}\int_V \rho dV = 0 \qquad \text{Eq. } 5.4-2$$

The charge conservation equation in its integral form
The earlier one was in a differential form

And this is the charge conservation equation in its integral form. Earlier, we had seen it in the differential form by our consideration of a control volume and all that. It is also nice to know the integral form of that this is a way to derive the integral form. Let us stop here for now. And when we meet next, in the next class, we will take things forward we will get to know more about some fundamentals, then extended and so on so forth. We need to spend quite a bit of time because we need to be very clear that we are reasonably comfortable in these electromagnetic aspects. See you bye.