Transport Phenomena in Biological Systems

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Lecture – 53
Getting Useful Relationships through Maxwell's Equations

Welcome. In the previous couple of classes, we looked at the very fundamental principles of electrical magnetic aspects, such as Maxwell's equations, charge conservation equation. The electro quasi state, magnet quasi state approximations the modifications of Maxwell's equations to a biological medium, with finite with different permittivity and permeable with different permittivity.

Essentially, permeability remains the same as well as the ohms law as a constitutive equation in this case. What we are going to do in this class is look at some very basic relationships that you have known from high school. How they are actually obtained through Maxwell's equations, they have been derived using Maxwell's equations but you have been presented that equation so far and you have just been using it as a fundamental equation, the basis for that is all Maxwell's equations.

Maxwell's equations all the common sense and whatever comes out must be equal to what is inside and so on and so forth, if you look at it physically. So, it is essentially like material balance, nothing very different from that only thing is the development is in terms of feelings and so on so forth, which is necessary, which is the way the development has happened and therefore you have that. Let us start looking at getting useful relationships, through Maxwell's equations in this class.

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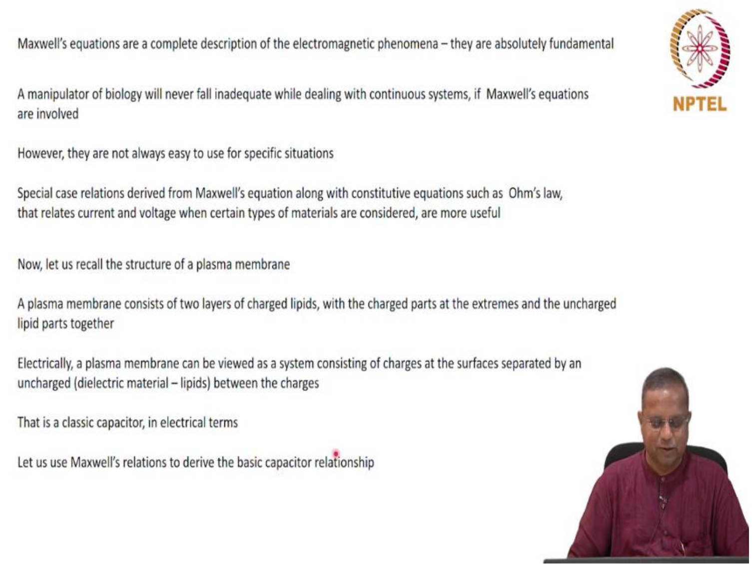

Maxwell's equations are a complete description of the electromagnetic phenomena, and therefore they are absolutely fundamental. A manipulator of biology which is you in a couple of years from now, we will never fall inadequate while dealing with continuous systems, if Maxwell's equations are involved, of course continuum is a given, if you have a continuum and especially in biological systems, Maxwell's equations are certainly the fundamental aspect for electrical aspects.

However, they are always not easy to use for specific situations. The same way as mass conservation equation which is essentially mass can neither be created nor destroyed, that is what mass conservation is. But by the way, it is stated it is not very useful in being modified to input rate plus generation rate minus output rate minus consumption rate equals accumulation rate. But this is essentially saying the same thing, mass can neither be created nor destroyed.

But we write it of the form in terms of the variables that are important to us and so on and so forth with the time involved rates involved time involved and so on then it became very useful to us to apply to processes. Similarly, the Maxwell's equations themselves are directly difficult to win when you try to use them directly they may not be very tractable. Therefore, special case relations derived from Maxwell's equation along with constitutive equations at this ohms law.

That relate current to voltage current is charged but time not charged flux is charged per time charge rate this is current and voltage when certain types of materials are considered are more useful. For use we need to do this. Let us recall what a plasma membrane is the plasma membrane is the membrane of the cell, the first membrane of the cell and lipid by layer and depending on the organism.

You could have a cell wall and another lipid layer on top of it 3 layers maximum mammalian cells have only a plasma membrane they do not have a cell wall plant cells do have a cell wall bacteria have 3 layers and so on so forth. So, this is the plasma membrane that you could recall from your biology courses by chemistry and so on cell biology. A plasma membrane consists of 2 layers of charged lipids bi-layer of lipids with the charge parts at the extremes and the uncharged parts in the middle.

You know this and charges lipids, lipid parts together. Electrically, a plasma membrane can be viewed as a system consisting of charges at the surfaces that are separated by a uncharged or dielectric material lipids. So, you have a lipid and head group you have another lipid and the head group liquid parts come together they have groups charged at separate. So, you have charges separated by this dielectric lipid.

That is one of the ways of looking at the dielectric lipid is in between the charges. Does it remind you of some electrical device basic electrical device? That is a classic capacitor is not it? You have charges separated by a dielectric, charge plates separated by a dielectric is nothing but your classic capacitor. And we will use Maxwell's relations to derive the basic capacitor relationship and thus you could see how the plasma membrane can be considered as a capacitor.

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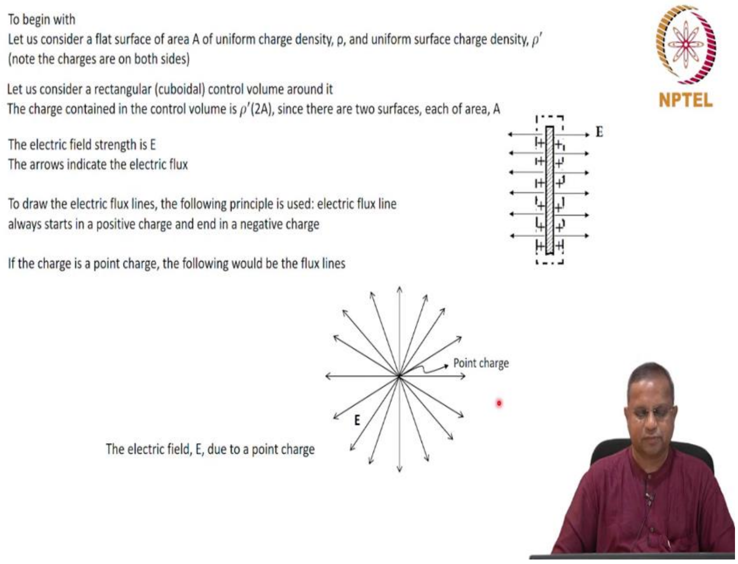

To begin with, let us consider a flat surface area A. Here this is a flat surface. I have just drawn a section of it therefore, it looks like a rectangle. It has another dimension into the screen here. Flat surface of area A of uniform charge density ρ connect charge density is ρ and uniform surface charge density as ρ' surface charge density is density per unit mass density per unit area, in this case, surface charge density therefore ρ' is density per unit area.

Note the charges are on both sides of the plate here. And you know the electric fields how to draw the electric fields, they are drawn from the positive to the negative and in this case, they are all straight lines emanating out of the plate. And we are going to consider this as a system for analysis. So, let us consider the rectangular, cuboid a control volume around it as a system. The charge contained in the control volume is nothing but ρ' which is the surface charge density times to total area.

Because charge by the area is your surface charge density. Therefore the total charge is surface area density times area, in this case

because there are 2 surfaces each of area A would be the charge contain the charge contained in the control volume. The electric field strength is E the arrows indicate the direction of the electric flux from the positive to the negative.

To draw the electric flux lines the following principle is used this story recall from your high school, electric flux line always starts in a positive charge and ends in a negative charge. So, this is the electric flux lines there are drawn for visualization. If the charge is a point charge, the following would be the flux lines. If the point charge then it emanates in all directions from the positive charge here and these are the flux lines it will just to tell you to review how to draw flux lines. This electric field, due to the point charge E.

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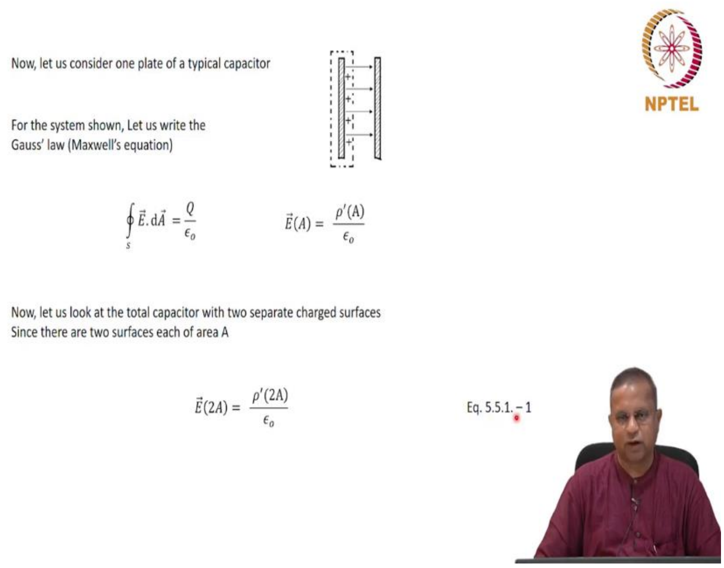

Now, we are going to consider one plate of a typical capacitor which is this. The capacitor has 2 plates. So, there are positive charges on one side of the plate along here in this capacitor for the system shown let us write the Gauss' law which is Maxwell's equation.

Let us now consider a capacitor of plate area A as shown in Fig. 5.5.1-3. From Gauss' law (Maxwell's relation)

$$\oint_S \vec{E}.d\vec{A} = \frac{Q}{\epsilon_o}$$

In this case, since there are two surfaces each of area A

$$\vec{E}(2A) = \frac{\rho'(2A)}{\epsilon_o} \qquad (5.5.1\text{-}1)$$

Therefore

$$\vec{E} = \frac{\rho'}{\epsilon_o} \qquad (5.5.1\text{-}2)$$

The integral of the electric field with respect to displacement gives the potential difference

$$\Delta \mathbf{V} = \int \vec{E}.d\vec{l} \qquad (5.5.1\text{-}3)$$

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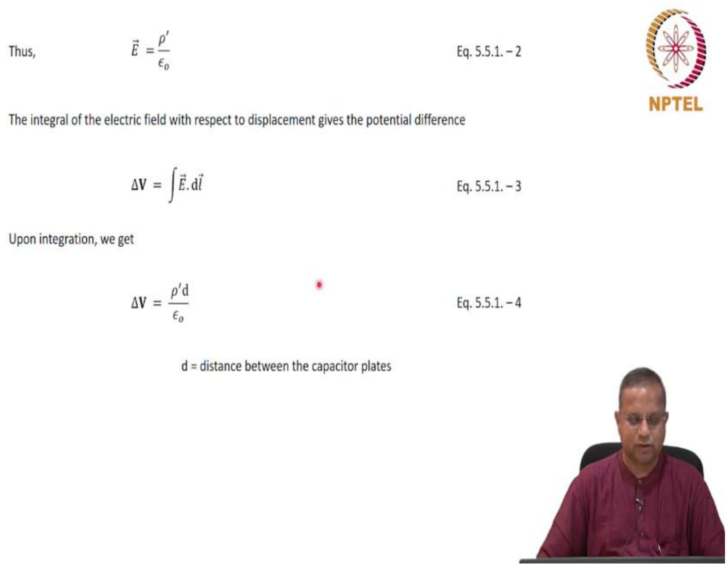

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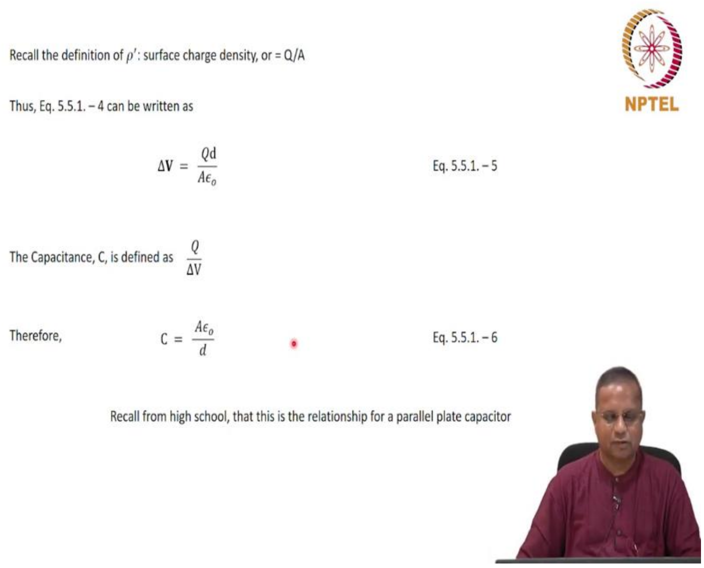

In this case, the integral yields

$$\Delta \mathbf{V} = \frac{\rho' d}{\epsilon_o} \qquad (5.5.1\text{-}4)$$

where d is the distance between the capacitor plates.

Replacing ρ' by Q/A, we get

$$\Delta \mathbf{V} = \frac{Qd}{A\epsilon_o} \qquad (5.5.1\text{-}5)$$

The capacitance C, defined as $\frac{Q}{\Delta V}$ is, therefore

$$C = \frac{A\epsilon_o}{d} \qquad (5.5.1\text{-}6)$$

which is a known relationship of a parallel plate capacitor.

This is the capacitance of parallel plate capacitor that you have seen from high school days. And this we have derived using Gauss' first rather Maxwell's first equation law. Maxwell's first equation, so, fundamental equation applied to the capacitance case gives you this and this can be extended to the lipid by layer membrane of a cell capacitance because it can be viewed as charges separated by a dielectric material.

This is equation 5.5.1- 6. So, we have seen how you could use a fundamental equation the Maxwell's first equation to derive the capacitor equation. Now, I am going to make a big jump, especially electrical engineers would feel very erratic when I do this. Do not worry about it. I am just going to my purpose is not to make you expert electrical engineers experts at use of Maxwell's equations and all that that is not the purpose here.

This is just to tell you what are relevant. And then at a later stage whenever you need it, you can go pick up the necessary additional skills to become experts at this. So I am going to jump now significantly.

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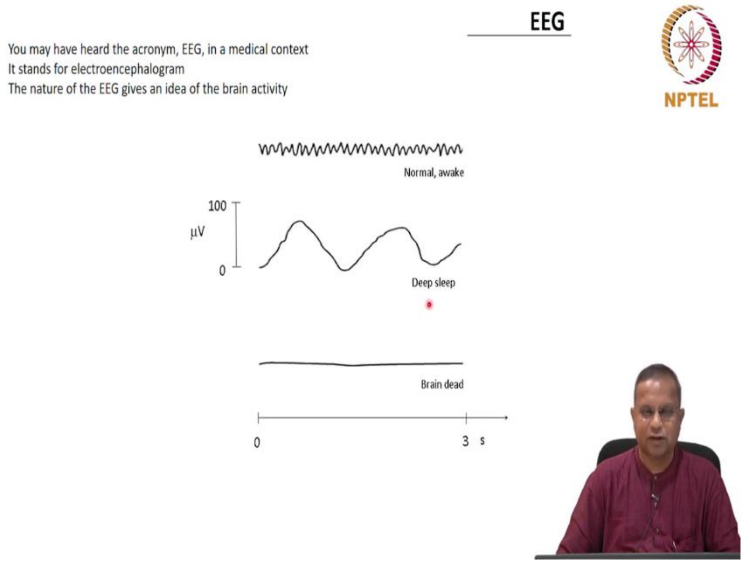

I am going to talk about something called EEG. I am sure you have heard the term EEG electroencephalogram. This is of course a medical tool, electroencephalogram; the nature of EEG gives an idea of the brain activity this you all know. For example, if you are normal awake, then the EEG would look something like this, this, the timescale this 0 to 3 seconds, you will have a lot of squiggles in your EEG if you are awake.

And if you are in deep sleep, if you look at the scale it is 0 to 100 micro volts. It goes through a nice rhythmic pattern in deep sleep and your brain dead, it is flat. There is no electrical activity when one is brain dead and therefore, that is flat. So, EEG gives you an idea of what does this is a very useful tool in medicine, we all know this.

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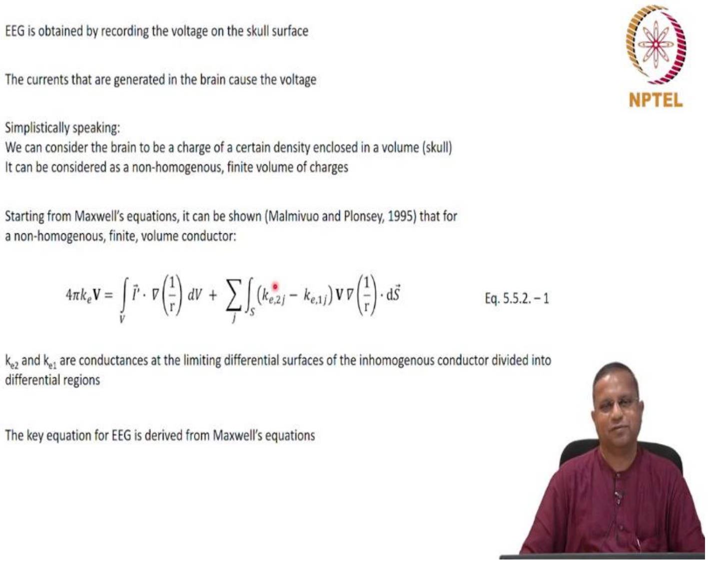

And the basis for design of EEG is the Maxwell's equations. EEG is obtained by recording the voltage on the skull surface you can have all those leads that go out and measure the currents that are generated in the brain caused the voltage which are measured. And simplistically speaking, again, very simplistically speaking, we can consider the brain to be a charge of a certain density enclosed in a volume which is the skull.

So you could look at it that way, some charge that is enclosed in a skull it can be considered as a non-homogeneous changing in all directions, of course, finite volume of charges. So that brain, of course is non-homogeneous, you cannot have the same thing all over the brain that the brain, there is no point in having a brain. So, this is a non-homogeneous finite volume of charges. Starting from the Maxwell's equations, it can be shown, this is a nice book, Malmivuo and Plonsey 1995.

These if you are interested, go read the book. It is a reference book for us. That is it is a textbook for electrical engineers that for a non-homogeneous finite volume conductor, this relationship is derived.

it has been shown (Malmivuo and Plonsey 1995) that for a non-homogenous, finite volume conductor, the following equation holds:

$$4\pi k_e \mathbf{V} = \int_V \vec{I}\,' \cdot \nabla\left(\frac{1}{r}\right) dV + \sum_j \int_S (k_{e,2j} - k_{e,1j}) \mathbf{V} \nabla\left(\frac{1}{r}\right) \cdot d\vec{S} \qquad (5.5.2\text{-}1)$$

where k_{e2} and k_{e1} are conductances at the limiting differential surfaces of the inhomogenous conductor divided into differential regions.

So, they start from the Maxwell's equations and get here or you can you need to start with Maxwell's equations. That is the bedrock bottom line, and then you can build this up. And such a useful devices as EEG is based on Maxwell's equations. Here in this equation, $k_{e,2}$ and $k_{e,1}$ are the conductances at the limiting differential surfaces of the inhomogeneous conductor divided into differential sections.

Do not worry too much about this, I needed to define this. I mean, this cannot give you $k_{e,2}$ and $k_{e,1}$ without mentioning what they are. It does not really matter to us. The main idea is for you to appreciate that Maxwell's equations can lead to very, very useful relationships. Key equation for EEG is derived from Maxwell's equations. This is the key equation for EEG. That is what I have in this class. We will meet in the next class and take things forward. See you then.