

Transport Phenomena in Biological Systems
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Lecture – 55
Charge Flux: Review

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The fundamental biomolecules:

Lipids	Charged
Carbohydrates	Many are charged
Proteins	Charged
Nucleic acid	Charged



Charges and their dynamics are responsible for our ability to sense our environment through sight, smell, taste, touch and hearing
The dynamics of charges are essential for the functioning of our nervous system, our brain and our heart

Charge is a fundamental physical quantity that is conserved

A better understanding of the fundamental relationships related to electrical charges and consequent or parallel aspect – magnetism, is expected to significantly equip the manipulator of biology



Welcome back. In this class, we are going to review the concepts that we learned in the chapter on charge flux. We said there is a huge need to look at the charge flux aspects which are typically not looked at by many different engineering disciplines, except maybe for electrical engineers, not many people look at charge flux. We need we biological engineers need to look at it because our fundamental biomolecules out of which all biology is made of, such as lipids, carbohydrates, proteins and nucleic acids.

You know these 4 fundamental biomolecules. In these lipids are always charged by carbohydrates. Many are charged proteins charged, of course nucleic acids charged. And therefore, we need to look at charges. We need to understand charges if we need to manipulate biological systems. Also charges and dynamics and charges and their dynamics are responsible for our ability to even sense the environment.

Our brain, our nerves and so on so forth, are based on the distribution of charges and the dynamics. And that we are going to start seeing in the next chapter. Charge is a fundamental physical quantity that is conserved. And that is our interest it gives us something and very valuable tool that we can manipulate because the left hand side equals to right hand side.

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Fundamental visualizations



The space between interacting charges can be considered to be influenced by the charges (Faraday)

The forces between (say two) charges are transferred from one charge to the other charge through the space in which they are located

Thus electric and magnetic 'fields' exist at a point in space even in the absence of actual charges at that particular point

Let us now consider the effect of those fields on a charged particle, and the force experienced by the particle

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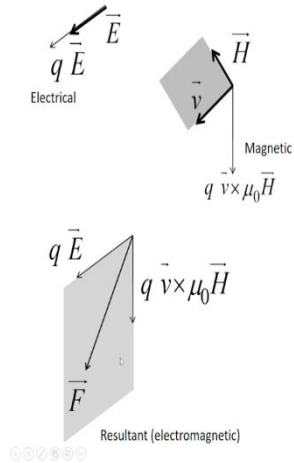


So the some fundamental visualizations, the space between interacting charges can be considered to be influenced by the charges themselves. This is this how Faraday's sort others during that time also and that led to the development of the electromagnetism along those lines of thinking those lines of visualization, the space between the interacting charges can be considered to be influenced by the charges and the forces between two between charges are transferred from one charge to the other through the space in which they are located.

Therefore, electric and magnetic fields exist at a point in space even in the absence of actual charges at that point, there will be charges present elsewhere, but there could be a magnetic field where the actual charge does not exist. Because influence of charges over this place. In other words, there is an electric field that is created by the charges, that is a region of influence. And then we looked at reviewed the some basic aspects that charge on the effect of charges effect of fields electric fields on a charged particle.

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Lorentz force law



The force \vec{F} experienced by a test charge, q , that moves at the velocity \vec{v} in such a field is given by Lorentz force law

$$\vec{F} = q (\vec{E} + \vec{v} \times \mu_0 \vec{H}) \quad \text{Eq. 5.1. -1}$$

\vec{E} = electric field density
 \vec{H} = magnetic field intensity
 $\mu_0 \vec{H}$ = magnetic flux density
 μ_0 = permeability of free space = $4\pi \times 10^{-7}$ Henry m^{-1}
 Henry = Volt-s (amp)⁻¹



The force \vec{F} experienced by a test charge q that moves at a velocity \vec{v} in such a field is given by the Lorentz force law

$$\vec{F} = q(\vec{E} + \vec{v} \times \mu_0 \vec{H}) \quad (5.1-1)$$

where \vec{E} is the electric field density, \vec{H} is the magnetic field intensity, $\mu_0 \vec{H}$ is the magnetic flux density and μ_0 is the permeability of free space = $4\pi \times 10^{-7}$ Henry m^{-1} , Henry = Volts (amp)⁻¹.

This is a visualization of that and how the plane or which the direction in which the forces act, the electrical force acts in the same direction as electric field, the magnetic force acts in the direction that is perpendicular to the plane containing v and H . And this is the electrical force here; there is the magnetic force here. This is the plane that contains the electrical component and the magnetic component and this is the resultant force on the charged particle that is moving in an electromagnetic field.

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Charge density, charge flux



Let us consider a small volume, ΔV with a net charge within it.
The charge density, ρ , is defined as

$$\rho \equiv \frac{\text{net charge in } \Delta V}{\Delta V} \text{ coulomb m}^{-3} \quad \text{Eq. 5.2.-1}$$

ΔV is usually chosen to be much smaller compared to the system dimensions, but large enough to contain many charges to ensure continuum conditions

If a charge density, ρ , moves with a velocity, \vec{v} , the charge flux, \vec{J}

$$\vec{J} = \rho \vec{v} \text{ coulomb m}^{-2}\text{s}^{-1} \quad \text{Eq. 5.2.-2}$$

We are more familiar with the term 'current'
Current is charge transport and is a measure of the rate of change of charge with time



So, some basic quantities we needed to define based on which our whole development is going to be charged density is nothing but the net charge in a certain small volume divided by that volume coulomb per meter cubed. And the charge flux is nothing but a charge density that is moving with a velocity V . It is coulomb per second per meter square amount coulomb of charge per time.

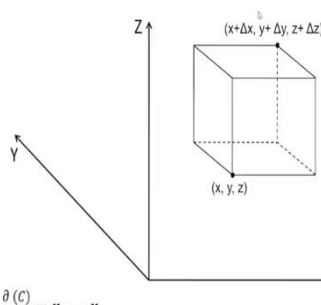
It is the rate of charge divided by the area that is perpendicular to the direction of motion. And that is given by the charge density times of velocity. The same way mass density times of velocity gave mass flux. Then we said we are going to use I' for the charge flux, because I we have reserved for charge per time current.

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Charge conservation equation



Let us now derive the charge conservation equation
Let us consider the intuitive Cartesian coordinate system that we have earlier considered
Let us say that charges are moving through this control volume



(Total) charge conservation: $\frac{\partial(C)}{\partial t} = r_i - r_o$



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Let ρ be the NET charge density Let I' be the NET charge flux

The charge (net charge) balance equation (rates of [accumulation = input - output]) can be written as

$$\frac{\partial(\rho \Delta x \Delta y \Delta z)}{\partial t} = \left[I'_x \Delta y \Delta z + I'_y \Delta x \Delta z + I'_z \Delta x \Delta y \right] - \left[I'_x \Delta y \Delta z + I'_y \Delta x \Delta z + I'_z \Delta x \Delta y \right]$$

Dividing throughout by $\Delta x \Delta y \Delta z$ and taking the limits as $\Delta x \rightarrow 0$, $\Delta y \rightarrow 0$, $\Delta z \rightarrow 0$

$$\frac{\partial \rho}{\partial t} = - \left(\frac{\partial I'_x}{\partial x} + \frac{\partial I'_y}{\partial y} + \frac{\partial I'_z}{\partial z} \right)$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{I}' = 0 \quad \text{Charge conservation equation} \quad \text{Eq. 5.1.}$$

The various manifestations of charge conservation, in terms of the relevant effects
- electric and magnetic fields -
are given by the Maxwell's equations



Then we derived the charge conservation equation from a simple concentrations and the differential form we directly obtained as $\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{I}' = 0$. The various manifestations of charge conservation in terms of the relevant effects, electric and magnetic fields are given by the Maxwell's equations.

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How is the electric field related to its source?

The net charge enclosed by an arbitrary volume, V , which is enclosed by a surface, S , is given by Maxwell's (first) relation

$$\oint_S \epsilon_0 \vec{E} \cdot d\vec{A} = \int_V \rho dV \quad \text{Eq. 5.3.1.-1}$$

$\epsilon_0 = \text{permittivity of free space} = 8.854 \times 10^{-12} \text{ Farad m}^{-1}$

Also, we know $\int_V \rho dV = Q$

In other words, the net charge enclosed in a volume V , enclosed by a surface, S , is related to the net electric flux through that surface

Equation 5.3.1.-1 is called Gauss' law



And we saw the Maxwell's equations, the first Maxwell's equation address the question how is electric field related to the source that is in this fashion

$$\oint_S \epsilon_0 \cdot E \, dA = \int_V \rho dV$$

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How is the magnetic field intensity related to its source, the charge flux?

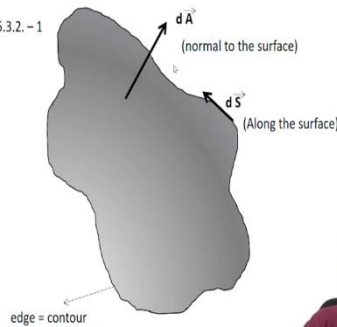


Maxwell's (second) relation addresses this question

$$\oint_C \vec{H} \cdot d\vec{S} = \int_S \vec{I}' \cdot d\vec{A} + \frac{d}{dt} \int_S \epsilon_0 \vec{E} \cdot d\vec{A} \quad \text{Eq. 5.3.2.-1}$$

Eq. 5.3.2.-1 is known as Ampere's integral law

The LHS indicates a contour integral
The RHS consists of two surface integrals



And then the second Maxwell's equation address the question how is the magnetic flux intensity related to source the charge flux into the related in this fashion and which is also known as amperes integral law. So, these separate laws were put together because they were fundamental laws into these 4 Maxwell's equations.

Maxwell's (second) relation shows how the magnetic field intensity is related to its source, as follows

$$\oint_C \vec{H} \cdot d\vec{S} = \int_S \vec{I}' \cdot d\vec{A} + \frac{d}{dt} \int_S \epsilon_0 \vec{E} \cdot d\vec{A} \quad (5.3.2-1)$$

This is also known as Ampere's integral law.

$d\vec{A}$ is the unit vector in the surface in the direction perpendicular to the surface which represents the surface of course.

$$\oint_C \vec{H} \cdot d\vec{S} = I + \epsilon_0 \frac{d\phi_E}{dt} \quad (5.3.2-2)$$

where ϕ_E is electric 'flux' (historically called flux – we use quotes here to avoid confusion in our context) and I is current.

In other words, an electric current and a time-variant electric 'flux' produce a magnetic field.

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In other words,
the line integral (circulation) of the magnetic field intensity, \vec{H} , around a closed contour
is equal to the sum of
the net current passing through the surface spanning the contour and
the time rate of change of the net displacement flux density (displacement current) through the surface



Eq 5.3.2 - 1 can be written as

$$\oint \vec{H} \cdot d\vec{s} = I + \epsilon_0 \frac{d\phi_E}{dt} \quad \text{Eq. 5.3.2 - 2}$$

ϕ_E : electric 'flux' (historically called flux, and we just use quotes to avoid confusion in our context)
I: current

In other words, an electric current and a time-variant electric 'flux' produce a magnetic field



Differential vector validated vector differential vector along the contour and on the surface, and then we interpreted the various aspects. We also said that there is another term called electric flux, it has a different connotation, and we would not use much of this flux in this course. However, note that they could be a source of confusion. Such as this, this flux is different from the way we define our flux in this course.

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How are electric field and magnetic flux related?



$$\oint_c \vec{E} \cdot d\vec{s} = - \frac{d}{dt} \int_S \vec{H} \cdot d\vec{A} \quad \text{Eq. 5.3.3 - 1}$$

Maxwell's third relationship also known as Faraday's integral law

In terms of the magnetic 'flux', ϕ_B , this is written as:

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\phi_B}{dt}$$



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A comment on the net magnetic flux out of any region



The net magnetic flux out of any region enclosed by a surface is zero

$$\oint_S \mu_0 \vec{H} \cdot d\vec{A} = 0 \quad \text{Eq. 5.3.4.-1}$$

Maxwell's fourth relationship, also known as Gauss' integral law



Then how our electric and magnetic field related is governed by the next Maxwell's equation, third one. And here you also run into a term called the magnetic flux. Again, this flux is using a different form than the normal form that we use in this course. Then finally, the 4th Maxwell's equation gives us a comment on the net magnetic flux out of any region is equal to 0. The net magnetic flux out of any region enclosed by surfaces, 0 and that is what is given by this.

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$$\int_S \vec{r} \cdot d\vec{A} + \frac{d}{dt} \int_S \epsilon_0 \vec{E} \cdot d\vec{A} = 0 \quad \text{Eq. 5.4-1}$$

From Maxwell's (first) relationship, we can replace the second term on the LHS of the above equation to get:

$$\int_S \vec{r} \cdot d\vec{A} + \frac{d}{dt} \int_V \rho dV = 0 \quad \text{Eq. 5.4-2}$$

The charge conservation equation in its integral form



The relationship between an electric field and the magnetic flux is given by Maxwell's (third) relationship

$$\oint_C \vec{E} \cdot d\vec{S} = -\frac{d}{dt} \int_S \mu_0 \vec{H} \cdot d\vec{A} \quad (5.3.3-1)$$

This is also known as Faraday's integral law.

In terms of the magnetic 'flux' (the term flux here is again, historical), ϕ_B , this can also be written as

$$\oint \vec{E} \cdot d\vec{S} = -\frac{d\phi_B}{dt} \quad (5.3.3-2)$$

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The Maxwell's equations in differential form are usually useful

To convert the integral Maxwell equations into their differential forms, we used two theorems in mathematics



Gauss' theorem states: $\oint_S \vec{D} \cdot d\vec{A} = \int_V (\vec{\nabla} \cdot \vec{D}) dV$

Relationship between relevant surface and volume integrals

Stokes' theorem states: $\oint_C \vec{D} \cdot d\vec{S} = \int_S (\vec{\nabla} \times \vec{D}) \cdot d\vec{A}$

Relationship between relevant contour and surface integrals



The net magnetic flux out of any region enclosed by a surface is zero. This is Maxwell's (fourth) relationship and is also known as Gauss' integral law. Mathematically, it can be expressed as

$$\oint_S \mu_o \vec{H} \cdot d\vec{A} = 0 \quad (5.3.4-1)$$

Then, we looked at some more fundamentals. We first said that the differential forms of Maxwell's equations are more useful usually, and therefore, we converted the integral forms of Maxwell's equations into their differential forms. To do that, we use 2 theorems from mathematics, the Gauss' theorem and the Stokes theorem. The Gauss theorem gives the equivalence between a surface integral and the volume integral.

And the Stokes theorem gives us the equivalence between a contour integral and the relevant surface integral. So, surface integral if you want to convert it into a volume integral, you need to do a del dot of the integrals. Whereas, if you want to convert a contour integral into a surface integral, you have to do a del cross of the integral. That is essentially what it is.

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If we apply Gauss' theorem to Maxwell's first equation $\oint_S \epsilon_0 \vec{E} \cdot d\vec{A} = \int_V \rho dV$

$$\begin{aligned} \text{LHS: } \oint_S \epsilon_0 \vec{E} \cdot d\vec{A} &= \int_V (\vec{\nabla} \cdot \epsilon_0 \vec{E}) dV \\ &= \int_V \rho dV \end{aligned}$$

Since dV is arbitrary,

$$(\vec{\nabla} \cdot \epsilon_0 \vec{E}) = \rho \quad \text{Differential form of Maxwell's first equation}$$



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Now, let us consider Maxwell's second equation $\oint_C \vec{H} \cdot d\vec{s} = \int_S \vec{j} \cdot d\vec{A} + \frac{d}{dt} \int_S \epsilon_0 \vec{E} \cdot d\vec{A}$

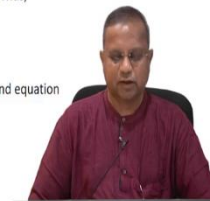
From Stokes' theorem, we know $\oint_C \vec{D} \cdot d\vec{s} = \int_S (\vec{\nabla} \times \vec{D}) \cdot d\vec{A}$

Thus, the LHS of Maxwell's second equation becomes $\int_S (\vec{\nabla} \times \vec{H}) \cdot d\vec{A}$

Therefore, $\int_S (\vec{\nabla} \times \vec{H}) \cdot d\vec{A} = \int_S \vec{j} \cdot d\vec{A} + \frac{d}{dt} \int_S \epsilon_0 \vec{E} \cdot d\vec{A}$

The surface S is fixed in time. Thus, the derivative can be taken inside the integral. Also, S is arbitrary. Thus,

$$(\vec{\nabla} \times \vec{H}) = \vec{j} + \frac{d}{dt} (\epsilon_0 \vec{E}) \quad \text{Differential form of Maxwell's second equation}$$



And we applied it to this and got the differential form of the first Maxwell's equation, then we apply the Stokes' theorem to get the differential form of the second Maxwell's equation.

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On similar lines, the other two equations can also be converted to their differential forms
 Also, we have a differential form of the charge conservation equation, from an earlier derivation
 Let us list all of them here



$$(\vec{\nabla} \cdot \epsilon_0 \vec{E}) = \rho \quad \text{Eq. 5.5-1}$$

$$(\vec{\nabla} \times \vec{H}) = \vec{I} + \frac{d}{dt}(\epsilon_0 \vec{E}) \quad \text{Eq. 5.5-2}$$

$$(\vec{\nabla} \times \vec{E}) = -\frac{d}{dt}(\mu_0 \vec{H}) \quad \text{Eq. 5.5-3}$$

$$(\vec{\nabla} \cdot \mu_0 \vec{H}) = 0 \quad \text{Eq. 5.5-4}$$

$$\frac{d\rho}{dt} + \vec{\nabla} \cdot \vec{I} = 0 \quad \text{Eq. 5.5-5}$$



This is and then you did an exercise to convert the other 2 to their differential forms and this is the list of all the differential forms of the Maxwell's relationship along Maxwell's equations, along with the charge conservation equation in differential form, which we asked to derived right in the beginning of this chapter.

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When a medium is present



The equations that we have seen thus far are valid in free space (vacuum). Recall that the electrical properties of free space, ϵ_0 and μ_0 were used
 Or, they are valid when no medium is present
 However, when we deal with biological systems, almost always a medium is present

When electromagnetic fields interact with the medium (or any material), the fields induce effects

- polarization
- magnetization

in the medium



And then we said, when a medium is present the effects of the medium come into the picture. So far the development was in free space, the permittivity of free space was used ϵ_0 and the magnetic magnet, the permeability of free space μ_0 was used. Whereas in a biological system, a medium is usually present and therefore, we need to consider the effect of the medium on the Maxwell's equations.

Then we saw that the medium causes bring in effects such as polarization and magnetization. Polarization is because of the dipoles that are present in the molecules comprising the medium

and magnetization is a similar thing that happens in the medium in the presence of a magnetic field.

$$\int_S \vec{I}' \cdot d\vec{A} + \frac{d}{dt} \int_S \epsilon_o \vec{E} \cdot d\vec{A} = 0 \quad (5.4-1)$$

From Maxwell's (first) relationship, Eq. 5.3.1-1, we can replace the second term on the LHS of Eq. 5.4-1 as

$$\int_S \vec{I}' \cdot d\vec{A} + \frac{d}{dt} \int_V \rho dV = 0 \quad (5.4-2)$$

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In a medium in the presence of an electric field, there could be free charges and polarization charges

Let the charge density due to free charges be ρ_{fc}

Let the charge density due to polarization charges be ρ_{pc}

The Gauss' law for this system can be written as

$$\vec{\nabla} \cdot \epsilon \vec{E} = \rho_{fc} + \rho_{pc}$$

The form of Maxwell's equations for *isotropic* media remain the same with the replacement of the free space permittivity, ϵ_o , by the medium permittivity, ϵ
 Isotropic medium is a uniform medium, or the medium in which its properties do not change with space/position

Interestingly, the permeability of most biological materials such as cells and tissues, can be approximated very well to μ_o



And what we saw was that you could take the same Magnus Maxwell's equations 1 2 3 4 replace the permittivity of free space ϵ_0 with the medium permittivity ϵ and everything becomes is valid. So, that way it is nice only thing is that you also had to replace the charge density with the sum of free charges and polarized charges that of course, it had to be done and along with it you change $\epsilon_0 \mu_0$ actually remains pretty much the same for a medium as that of free space.

Therefore, we do not need too much about here not. So, that is what happens when you consider medium which is relevant to us. Then we saw how we could further simplify the Maxwell's equation to get the electrophoresis state and magnetic was a state equations that involved the concept of pseudo steady state the same that we saw that we have seen earlier many times.

And if you apply the concept of pseudo steady state to the rates of interaction between the waves and the biological material, and the rate of variation, the waves themselves the rate of variation, the waves which are described by all these equations are much slower compared to

the rates of interaction of the waves with the particles, and therefore, all the time derivatives can be ignored, neglected and thereby we could get much simpler equations that we could work out.

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$$(\vec{\nabla} \cdot \epsilon \vec{E}) = \rho \quad \text{Eq. 5.5-6}$$

$$(\vec{\nabla} \times \vec{H}) = \vec{I} \quad \text{Eq. 5.5-7}$$

$$(\vec{\nabla} \times \vec{E}) = 0 \quad \text{Eq. 5.5-8}$$

$$(\vec{\nabla} \cdot \mu_0 \vec{H}) = 0 \quad \text{Eq. 5.5-9}$$



And those equations were this here there is no difference. This is a much simpler equation to work with this is a much simpler equation to work with. And here it is pretty much the same. So, these are the electrophoresis state and the magnetophoresis state approximated equations of Maxwell's equations.

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An electrical potential, V , is related to the electric field as:

$$\vec{E} = -\vec{\nabla}V \quad \text{Eq. 5.5-10}$$

Therefore, $\vec{\nabla} \cdot \epsilon \vec{E} = \vec{\nabla} \cdot (-\epsilon \vec{\nabla}V)$

Substituting this in Eq. 5.5-6, $\vec{\nabla} \cdot (-\epsilon \vec{\nabla}V) = \rho$

$$\text{Thus, } \vec{\nabla}^2 V = -\frac{\rho}{\epsilon} \quad \text{Eq. 5.5-11}$$

Poisson equation

In the region where no charges are present ($\rho = 0$)

$$\vec{\nabla}^2 V = 0 \quad \text{Eq. 5.5-12}$$

Laplace equation

These equations are useful in the analysis of biological systems, e.g. certain marine organisms such as electric eels can be considered to be electric dipoles that satisfy the Laplace equation



Then we looked at the concept of an electric potential which in your earlier we looked at it in the context of an electric field and so on, and developed some useful relationships from Maxwell's first equation, the Poisson equation and the Laplace equation.

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Constitutive equation



Let us recall that Fick's law was a constitutive equation

It related diffusive flux and concentration gradient, and is valid for a class of materials

For certain materials, the charge flux is proportional to the potential gradient

$$\vec{J}' = -k_e \vec{\nabla} V = k_e \vec{E} \quad \text{Eq. 5.6.-1}$$

k_e is the electrical conductivity of the medium (typical unit: Siemens cm^{-1})

Eq. 5.6.-1 is a constitutive equation, which is valid for a class of materials

Ohm's law



And then we looked at a constitutive equation in electrical aspects. The ohms law actually turns out to be the constitutive equation one of the constitutive equations that is very useful in electrical aspects. Constitutive equation as you recall, is valid for a class of substances but not universally valid. It depends on the constitution of the material, and therefore it is called the constitutive equation.

$$\vec{J}' = -k_e \vec{\nabla} V = k_e \vec{E} \quad (5.6-1)$$

where k_e is the electrical conductivity of the medium (typical unit: Siemens cm^{-1}).

This is a constitutive equation, which is valid for a class of materials, and is known as the Ohm's law.

written in terms of the variables that we have been talking about is nothing but current equals to by voltage by potential difference by resistance, and that is written in this form for our systems.

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Maxwell's equations are a complete description of the electromagnetic phenomena – they are absolutely fundamental

A manipulator of biology will never fall inadequate while dealing with continuous systems, if Maxwell's equations are involved

However, they are not always easy to use for specific situations

Special case relations derived from Maxwell's equation along with constitutive equations such as Ohm's law, that relates current and voltage when certain types of materials are considered, are more useful

Now, let us recall the structure of a plasma membrane

A plasma membrane consists of two layers of charged lipids, with the charged parts at the extremes and the uncharged lipid parts together

Electrically, a plasma membrane can be viewed as a system consisting of charges at the surfaces separated by an uncharged (dielectric material – lipids) between the charges

That is a classic capacitor, in electrical terms

Let us use Maxwell's relations to derive the basic capacitor relationship



Then, we looked at the application of Maxwell's equations, to get some useful relationships and we saw how things are written when we go about doing that how field lines are drawn and so on, so forth. We said that, membrane lipid bilayer membrane can be considered as a capacitor because there are charges on both ends that are separated by a dielectric material which is a classic capacitor.

And therefore, we derive the equation for a classic capacitor using Maxwell's equations to illustrate how you are going to use the fundamental equations to derive useful relationships.

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Recall the definition of ρ' : surface charge density, or = Q/A

Thus, Eq. 5.5.1 – 4 can be written as

$$\Delta V = \frac{Qd}{A\epsilon_0} \quad \text{Eq. 5.5.1 – 5}$$

The Capacitance, C, is defined as $\frac{Q}{\Delta V}$

Therefore, $C = \frac{A\epsilon_0}{d}$ Eq. 5.5.1 – 6

Recall from high school, that this is the relationship for a parallel plate capacitor



And we all know the basic capacitor equation from your high school, which is nothing but $C = (A \epsilon_0 / d)$. This is a classic capacitor equation which can directly be applied even to the lipid bilayer membrane. Then, we said that even the basis of the equation that forms the bedrock of EEG design is based on Maxwell's equations or is derived from Maxwell's equations.

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EEG is obtained by recording the voltage on the skull surface

The currents that are generated in the brain cause the voltage

Simplistically speaking:

We can consider the brain to be a charge of a certain density enclosed in a volume (skull)
It can be considered as a non-homogenous, finite volume of charges

Starting from Maxwell's equations, it can be shown (Malmivuo and Plonsey, 1995) that for a non-homogenous, finite, volume conductor:

$$4\pi k_e \mathbf{V} = \int_V \vec{\mathbf{j}} \cdot \nabla \left(\frac{1}{r} \right) dV + \sum_j \int_S (k_{e,2j} - k_{e,1j}) \mathbf{V} \nabla \left(\frac{1}{r} \right) \cdot d\vec{\mathbf{S}} \quad \text{Eq. 5.5.2. - 1}$$

$k_{e,2j}$ and $k_{e,1j}$ are conductances at the limiting differential surfaces of the inhomogeneous conductor divided into differential regions

The key equation for EEG is derived from Maxwell's equations



EEG we saw what EEG was and then this was the equation without getting into any of the details. We just saw this as just a possible application, you would not be able to do that at this stage, you need a lot more understanding and so on so forth. That is probably not required at this stage. Whenever you need to get into that, of course you go and pick up the necessary knowledge, skills and so on and so forth to get to this thing.

And finally, we saw the effects of charges in solution that contains charges highly relevant for biological situations, and how to approach this is what we saw, we saw 3 concepts.

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Most cells are in an aqueous environment
Their contents are fluid

Thus, aspects related to charges/ions in solution are important to consider
We will discuss three key concepts in this context

1. Electroneutrality

Oppositely charged ions could be present in a solution

There are strong forces of attraction between the opposite charges

As long as the number of positive charges equals the number of negative charges, due to the strong force of attraction between opposite charges, the net charge in that system is zero (electroneutrality)

Thus, an electrolytic solution cannot set up an electric field although it contains charges because the number of positive charges equals the number of negative charges



And those concepts were electroneutrality. Let me go through this once more, is an important thing, which leads to a lot of confusion in when people start out so pay attention. Closer to each one of these oppositely charged ions could be present in a solution. There are strong forces of

attraction between the opposite charges. As long as the number of positive charges equals the number of negative charges, due to the strong force of attraction between the opposite charges, the net charge in that system is 0.

And this is the concept of electro neutrality. A consequence of this is something like this, that is an electromagnetic solution cannot set up an electric field, although it contains charges, because of because the number of positive charges equals the number of negative charges.

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From Gauss' law $(\vec{\nabla} \cdot \epsilon_0 \vec{E}) = \rho$ Eq. 5.7.2.-3
 $\rho =$ charge density at any time, t



Substituting 5.7.2-3 into 5.7.2-2

$$\vec{\nabla} \cdot \vec{I} = \frac{k_e}{\epsilon} \rho$$
 Eq. 5.7.2.-4

If we assume that upon application of ρ_0 , the increase in charge density in the solution is uniform, we can use the charge conservation (charge continuity) equation

$$\vec{\nabla} \cdot \vec{I} = - \frac{\partial \rho}{\partial t}$$
 Eq. 5.5.-5

Substituting Eq. 5.5.-5 in 5.7.2-4

$$\frac{\partial \rho}{\partial t} + \frac{k_e}{\epsilon} \rho = 0$$



The second concept is that of charge relaxation time, if a unit charge or a certain charge density is uniformly added to a conducting solution, how quickly does it dissipates to give you a new equilibrium was what we saw as a charge relaxation time. We calculated the charge relaxation time based on very fundamental principle that is ohms law as well as charge balance equation, charge continuity equation based on these we derived an expression for the variation of the charge density with time in a conducting liquid conducting electrolyte.

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The solution

$$\rho = \rho_0 \exp\left(\frac{-t}{\tau_r}\right)$$

Eq. 5.7.2. - 6



$$\tau_r = \frac{\epsilon}{k_e}$$
 is the charge relaxation time

For water:

$$k_e = 0.01 \text{ S cm}^{-1}$$
$$\epsilon = 80\epsilon_0 = 80 \times 8.85 \times 10^{-12} \text{ F cm}^{-1}$$

Then, $\tau_r = 0.7 \text{ ns}$

Charges can be relaxed rapidly

Charge relaxation needs to be considered while designing systems in which the biological materials – cells, biomolecules, etc., interact with electrical fields such as in electrophoresis



Then the solution was this and the typical $\rho = \rho_0 \exp(-t / \tau_r)$, τ_r is ϵ / k_e and the charge relaxation time is τ_r which is ϵ / k_e for water turns out to be 0.7 nanoseconds, charges can be relaxed very rapidly. And this has major significance whenever we go about designing things using solutions. For example, charge relaxation needs to be considered, while designing systems in which biological materials such as cells biomolecules interact with electrical fields such as in electrophoresis.

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3. Debye length

Say, we have a specially made lipid layer with uniform surface charge density stretched on a flat plate at $x = 0$

This lipid layer is bathed by an electrolytic solution with cations and anions

Near the charged surface, mobile ions whose charge is opposite to that of fixed charge (counter ions) will be attracted. In other words, near the charged surface, there is a region in the solution where electro neutrality does not hold. The region will be charged with the charge of the counter-ions

Debye length: the length of the region in the solution near the charged surface where electro-neutrality does not hold

Weiss (Weiss TF. 1996. Cellular Biophysics. I: Transport. MIT Press) has derived an expression for the Debye length. Let us just state it here:

$$\lambda_D = \sqrt{\frac{\epsilon RT}{2Z^2 F^2 c}}$$

Eq. 5.7.3. - 1

ϵ = Permittivity
 Z = Charge
 F = Faraday's constant
 c = Concentration of positive or negative ions at 'infinite' distance in the solution where electro neutrality holds

Typical Debye length: 10 Å



And the third concept resolves in Debye length, if you have a charge surface and electromagnetic solution, there is a region closer the charge surface where electroneutrality is not valid in the solution. That is because the charges whose sign is opposite to that of charges in the plate, this plate of charges here that is kept in the solution, only the opposite charges will be aligned close to this layer of charges are the plate of charges.

And therefore, there is a certain length there is a certain region or with electroneutrality is not valid and that region is called the Debye length and I gave you an expression to calculate Debye length. There is also something more which I like to mention here regarding all this. The typical Debye length is about 10 angstrom very, very small 10\AA .

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In addition, It is good to realize:

Charge carrying biomolecules in a system do not generate an electric field because they are shielded by counter ions (as a "double-layer counter ion cloud")

However, when an electric field is applied, the "double-layer counter ion cloud" surrounding the charged bio molecule gets disturbed

Then, the charged biomolecule experiences the presence of the field and moves in response to it



And also it is good to realize that charge carrying biomolecules in a system do not generate an electric field because they are shielded by counter ions, this is called the double layered counter ion cloud. So, if we repeat charge carrying biomolecules in a system do not generate an electric field because they are shielded by counter ions. This is important, when you think of designing things.

However, when electric field is applied, the double layer counter ion cloud surrounding the charged biomolecule gets disturbed. Then, the charged biomolecule experiences the presence of the field and moves in response to it. Otherwise it does. That completes the review of the chapter on charge flux. When we begin the next class, we will begin a new chapter, the final chapter of this course, which considers the simultaneous application of multiple driving forces resulting in either one or more fluxes. That has very many significances. We will start looking at that from the next class onwards. See you then.