Transport Phenomena in Biological Systems
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Lecture – 56
Fluxes under simultaneous multiple drive forces

Welcome back. Today, we begin the final chapter of this course, that deals with fluxes under simultaneous multiple driving forces. So far, we have seen the flux associated with its primary driving force. Now, we are going to throw it open, and thereby, this is whatever we are going to see in this chapter is closer to reality. Typically, you have multiple driving forces at the same time.

We already seen that the maximum may not arise from our most likely will not arise from the primary driving force. It is just that it is firstly associated with the flux. That is all. Now we are going to put everything together. And I think this approach of looking at that separately understanding that clearly and then bringing things together would help you in obtaining a certain flow and thereby that will improve your understanding. Let us start looking at this in this chapter, just an introduction today. And then we can take things forward as we move along in the course.

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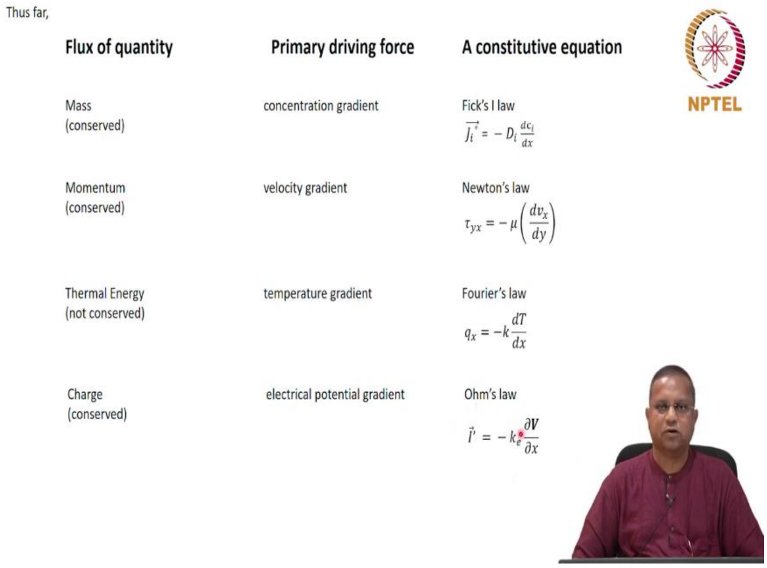

Thus far, if you look at the flux of quantity, the primary driving force or constitutive equation, if we when we were looking at mass, the primary driving force was the concentration gradient. And the constitutive equation was the Fick's first law, $J_i^* = -D_i (dc_i/dx)$. I am just going to write 1 dimension. So that you can better appreciate it, it is more intuitive. So mass is of course is conserved.

Momentum was also conserved; the primary driving force for momentum flux was velocity gradient. And Newton's law was a concert of equation, $\tau_{yx} = -\mu (dv_x/dy)$ is also a direction of motion and a direction of action and so on so forth, there could be combinations in 3 dimensions. Then we looked at thermal energy, which of course is not conserved, starting with the energy balance equation that gave us left hand side equals right hand side.

We nicely pulled out the thermal energy. Or if you look at the book, it will be clear how the thermal energy was pulled out. The primary driving force was the temperature gradient and a constitutive equation was Fourier's law $q_x = - k (dT/dx)$. And then, before this chapter, we looked at charge, which is of course a conserved quantity. The primary driving force was electrical potential gradient and a constitutive equation was the ohms law $I' = -k_e \frac{\partial V}{\partial x}$. Potential gradient is the primary driving force associated with charge flux and conductivity is the constant of proportionality.

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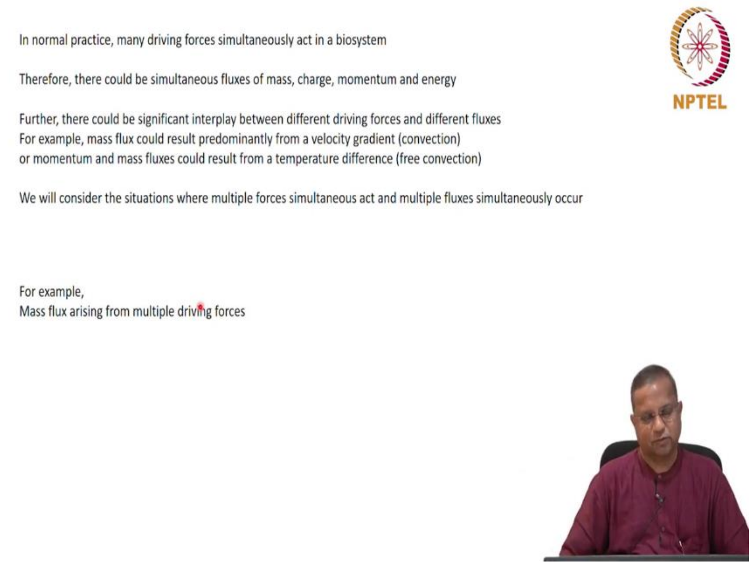

In normal practice many driving forces simultaneously act in a system the bio system that we are looking at. Therefore, there could be simultaneous fluxes of mass, energy, momentum charge and so on so forth. Further there could be significant interplay between the different driving forces, for the same fluxes or different driving forces and different fluxes okay whatever it is.

For example, mass flux could predominantly result from a velocity gradient we all know what convection is right related to motion there can be a huge mass flux because of the motion of the liquid itself, which is many orders of magnitude higher than the flux that is caused by the concentration gradient. So, so far we have carefully avoided the situation when we looked at mass flux, we had seen situations where there was no motion of the fluid where there was no convection, no bulk flow.

Therefore, we could put those v_x or $v_z = 0$ and so on are v_r v_θ = to 0 and so on so forth. That is what we have seen so far. Now, we are going to remove all those constraints. Or momentum and mass fluxes could result from a temperature gradient, which is nothing but free convection. We will consider the situations where multiple forces simultaneously act or multiple driving forces simultaneously act and multiple fluxes simultaneously occur.

Example mass arising from multiple driving forces could be that expression comes later mass flux could arise from multiple driving forces as I just mentioned, of course, the concentration gradient one would always exist then there could be convection, they could be temperature gradient, they could even be a charge gradient, we will see which causes mass flux.

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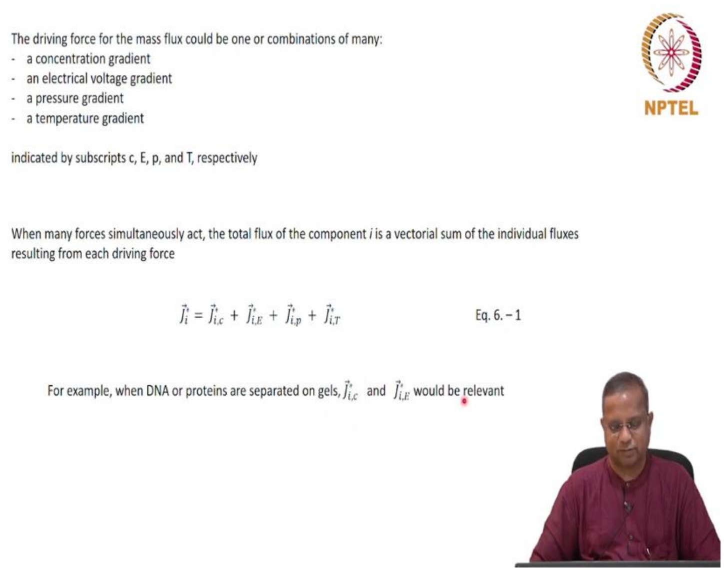

The driving force for the mass flux could be one of the combinations, one or combinations of many. You could have a concentration gradient passing a mass flux as I just mentioned, you will have an electrical voltage gradient, which is causing a mass flux recall electrophoresis

their electron electrical voltage gradient potential gradient cause mass flux, a pressure gradient which results in convection bulk flow that causes mass flux significantly.
A temperature gradient as in the case of free convection could and we are going to represent these aspects concentration aspects by a c electrical voltage aspects by E electrical aspects by E pressure aspects by P and temperature aspects by T. If we do this, when many forces simultaneously act, the total flux of component i, because many different driving forces are causing mass flux here.

So, the total flux or total mass flux of the component i is nothing but a vectorial sum of the individual fluxes remember that flux is a vector, it has direction, therefore, it is a vectorial sum of the individual fluxes resulting from each driving force that is quite straightforward. Therefore, the net molar flux in this case j_i* equals the molar flux due to the concentration gradient.

The molar flux due to the electrical potential gradient the molar flux due to the pressure gradient plus and plus the molar flux due to the temperature gradient of course all these vector sums. This is equation 6 -1. For example, when DNA and proteins are separated on electrophoresis gels, you certainly have a concentration gradient okay there is difference in concentration of protein from one end to the other.

And of course, there is an electrical potential gradient. In fact, the flux due to electrical potential gradient will be much larger than the flux due to concentration gradient in the electrophoresis.

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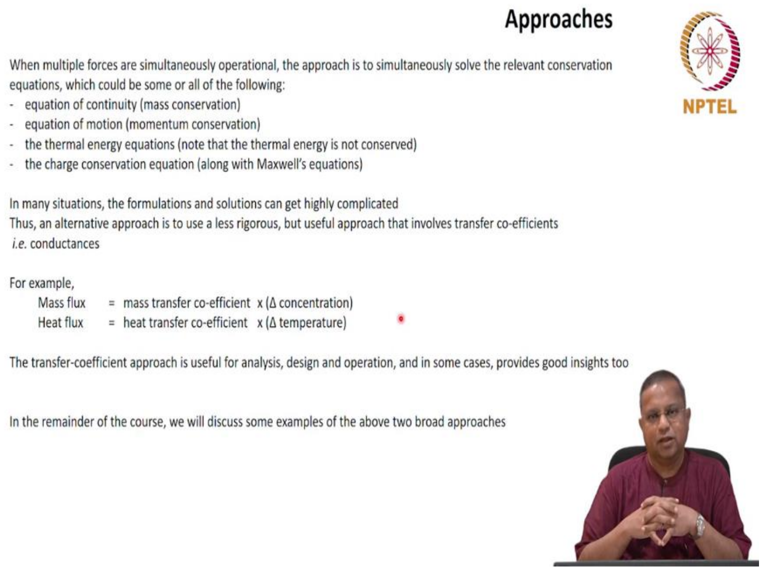

Now, let us look at the approaches that we are going to take in this study of multiple driving forces multiple fluxes. When multiple forces and simultaneously operational multiple driving forces are simultaneously operation the approach is to simultaneously solve the relevant conservation equations, we have a conservation equation for mass continuity equation, we have a conservation equation for momentum, the equation of motion.

Conservation equation of energy the thermal energy equation, this recast around and of course, the electrical continuity equation all these could be solved simultaneously to get the solution. This to list equation of continuity for mass conservation equation of moment motion for momentum conservation, the thermal energy equations note that thermally energy is not conserved, but we got this from energy conservation and the charge conservation equation along with the Maxwell's equations.

Of course, because the development has happened in terms of the Maxwell's equations and in many situations, the formulations and solutions, of course, could get highly complicated. Do you have a set of couple partial differential equations highly complex nature each one it is going to get very messy mathematically. In principle, yes, it is, it is possible to express this. But the mathematical effort is going to be so significant that you might want to take a second look at that.

That is an alternative approach is to use a less rigorous but useful approach that involves something called transfer coefficients. Transfer coefficients are nothing but equivalent of conductances, whatever mass conductance, momentum conductance and so on, so forth. For example, mass flux is represented as a product of a certain mass transfer coefficient. Times the change in concentration, this is no longer a gradient. This is just a change.

Of course the change occurs already. So that gradient aspect is inherent. But we are not going to take that into account in our formulation, we are just going to say that the mass flux equals a certain mass transfer co-efficient times in difference of concentration. That is it. So this is the less rigorous approach, you cannot rely on molecular aspects, as you could show completely when you did diffusion.

When we did diffusion, you could do that, but here you cannot do that and heat flux is expressed as a heat transfer coefficient times the difference in temperature. So there is a difference, not a gradient. The transfer-coefficient approach is useful for analysis, design and operation and in some cases provides good insights. In the remainder of the course, I think what will happen starting from the next class, we will discuss some examples of the about 2 broad approaches.

To repeat the first approach was to simultaneously solve the complex partial differential equations that correspond to all these conservation equations, equation of continuity equation of motion thermal energy equation, the charge conservation equation this resulting from the energy conservation equation and the charge conservation equation. So, you write all those equations, you solve them in principle, you must get the solution.

But the mathematical complexity is going to be so great. And therefore, we look at a different approach that is the transfer coefficient approach which is highly useful. There is a lot of examples with respect to this, which we directly use in the bioreactor context and various analysis context, which I will show you. So here the formulation is something like this, the flux is given as a product of a transfer coefficient times the change in a certain quantity, which is equivalent to the driving force quantity.

It is no longer a gradient, but that is just a change. For example, mass flux is a mass transfer coefficient times the change in concentration. Heat flux is heat transfer coefficients times the change in temperature, and so on and so forth. I think that is all I have for this class. It is a good time to stop this is the introduction class. Some sort of an orientation as to how we are going to approach this complex realistic situation. When we meet next, we will start with 2 driving forces causing a mass flux. See you.