Transport Phenomena in Biological Systems

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Lecture - 59
Electrical Circuit Representation of a Membrane

Welcome, let us continue our discussion on simultaneous concentration gradient and electrical potential gradient essentially mass flux under the action of these 2 simultaneous gradients. In the previous classes, we looked at the movement of ions under these conditions in a liquid and then across a membrane, then when we looked at the membrane, of course, it has huge relevance in all that we saw that.

We looked at Nernst equation applied Nernst equation and then looked at the condition of Donnan equilibrium with an approximation to the movement that has described by the Nernst equation. So, when there are multiple ions involved, so, today let us look at the electrical circuit representation of a membrane because this will come in useful to for us to understand the, what happens with the ion movement across the membrane? So, let us take it further.

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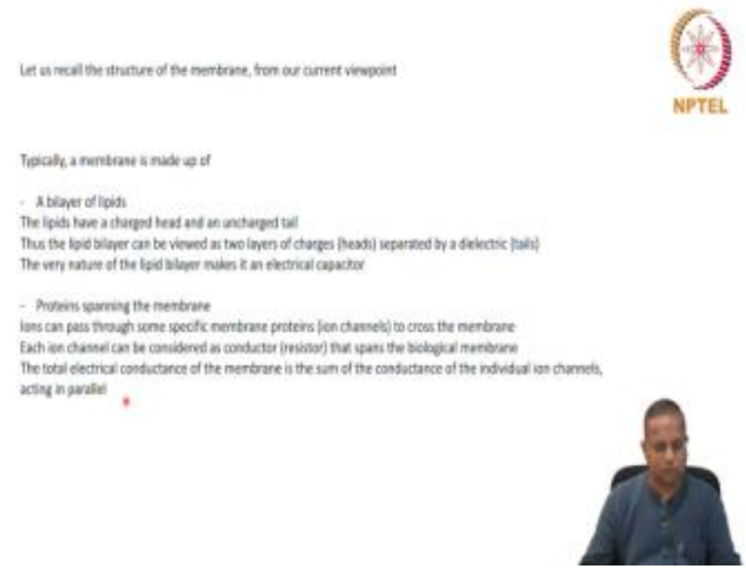

Let us recall the structure of the membrane we all know this from at least a current viewpoint. Sanger Nicholson typically a membrane is made up of a lipid bilayer. These lipids have a charged head and uncharged tail we all know this. Thus the lipid bilayer charged head and an uncharged tail then a charged head and uncharged tail a bilayer. Thus, the lipid bilayer can be viewed as 2 layers of charges or the heads separated by dielectric tails.

Therefore, the very nature of the lipid bilayer makes it an electrical capacitor that is the classical capacitor electrical capacitor charges separated by dielectric. And there are proteins spanning the membrane there are many things but let us look at by layer of lipids and proteins spanning the membrane, ions can pass through some specific membrane proteins to cross the membrane ion channels these are called and this is what is passive transport.

Each ion channel can be considered, therefore, to be a conductor of ions that spans the biological membrane. And when you have a conductance inverse of that is the resistance therefore, we could consider that as a resistor that spans the biological membrane. The total electrical conductance of the membrane is the sum of the conductance of individual ion channels acting in parallel.

So you have various ions channels in parallel each one with its own conductance or resistance, and when you have resistances in parallel, you know how to find out the effective resistance that will result in $1 / R_{total} = 1 / R_1 + 1/R_2$ so until by R_n we know this 1 / R is nothing but conductance therefore, the conductance equals the total conductance equals the sum of the individual conductances that is what this says.

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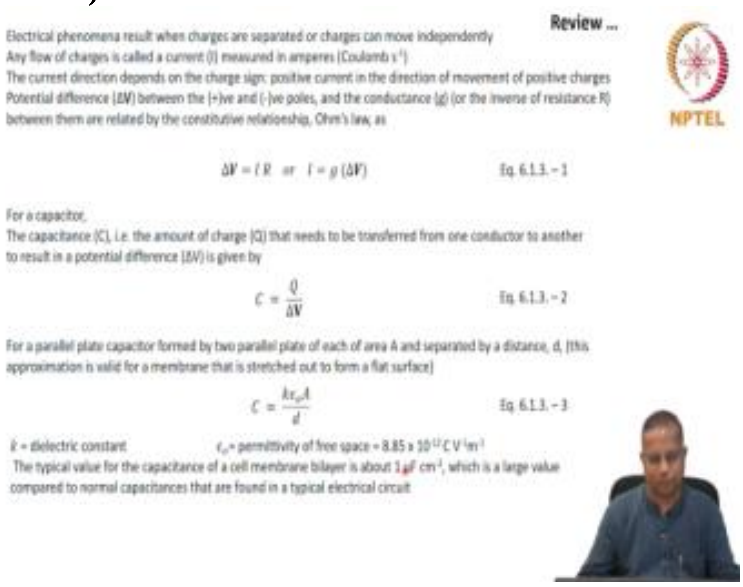

Now, let us review something that we already know we need to make sure that we understand this clearly. Electrical phenomena result when charges are separated or when charges move or can move independently. Any flow of charges is called current I and it is measured in amperes which is coulomb per second. The current direction depends on the charge sign positive direction is the direction of the movement of positive charges.

Potential difference ($\Delta\mathbf{V}$) between the (+)ve and (–)ve poles, and the conductance (g) (or the inverse of resistance R) between them are related by the constitutive relationship, Ohm's law, as

$$\Delta\mathbf{V} = IR \text{ or } I = g(\Delta\mathbf{V}) \qquad (6.1.3\text{-}1)$$

The capacitance C, i.e. the amount of charge Q that needs to be transferred from one conductor to another to result in a potential difference $\Delta\mathbf{V}$ is given by

$$C = \frac{Q}{\Delta\mathbf{V}} \qquad (6.1.3\text{-}2)$$

For a parallel plate capacitor formed by two parallel plates, each of area A and separated by a distance d (this approximation is valid for a membrane that is stretched out to form a flat surface)

$$C = \frac{k\epsilon_o A}{d} \qquad (6.1.3\text{-}3)$$

where k is dielectric constant and ϵ_o is permittivity of free space = 8.85 × 10^{-12} C $V^{-1}m^{-1}$.

The typical numbers typical value of capacitance of a cell membrane is about 1 micro farad per centimeter square, which is actually a very large value compared to the normal capacitances that are found in a typical electrical circuit. So, 1 micro farad per centimeter square is actually very large value.

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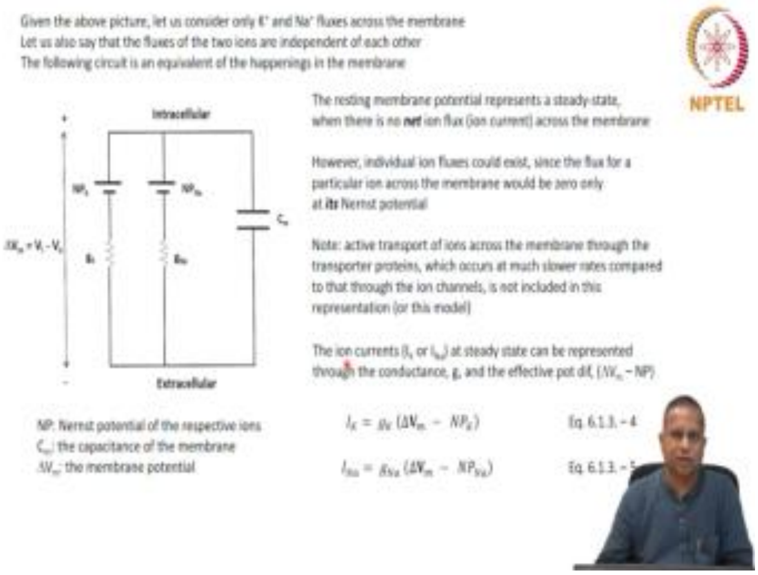

Now, given the picture that we have seen, let us consider only potassium and sodium fluxes across the membrane. Recall that the earlier development were all done in terms of only potassium and sodium in fact, the experimental results match the model results so much that people felt these 2 charges these 2 ions are good enough to for us to understand what happens across the membrane.

Let us also say that the fluxes of these 2 ions are independent of each other Hodgkin and Huxley assumption and if that if these 2 if the above is the case, the following circuit electrical circuit is an equivalent of the happenings in the membrane. Now, once we draw an electrical equivalent we can bring in our electrical engineering principles, techniques to analyze a circuit and then take it back to the membrane case and expect it to be valid.

So, that is the reason why we are doing this. So, if we know that the lipid bilayer can be represented by a capacitor, simple capacitor we know that the ion channels can be represented by conductances resistances whatever it is. So, if you take this to be the intracellular side and this to be the extracellular side, we have first represented the capacitance.

This is the conductance of this sodium and the Nernst potential of this sodium represented like this and this is the conductance of potassium and the Nernst potential of potassium that is represented like this and from our very definition, the intracellular potential minus the extracellular potential is the membrane potential ΔV_m membrane potential difference membrane potential ΔV_m.

So this is the electrical equivalent of the cell membranes or a typical membrane let us say and NP is the Nernst potential of the respective ions and C_m is the capacitance of the membrane ΔV_m is the membrane potential. The resting membrane potential represents a steady state we all know this when there is no net ion flux or ion current across the membrane. However, individual ion fluxes could exist that because at resting membrane potential.

The Nernst potentials may not equal the resting membrane potential the Nernst potential of each ion is different that may not equal the resting membrane potential. So, each ion could move the individual ion fluxes could exist since the flux for a particular ion across the membrane would be 0, only at its Nernst potential. All the Hodgkin and Huxley assumptions said was that the moment is independent of each other it did not say anything about anything else.

Here, if we start looking at it closer at the resting membrane potential, the net ion flux is 0. However, individual ion fluxes could exist because the flux for a particular ion across the membrane would be 0 only at its Nernst potential. And of course, note that the active transport of ions across the membrane through the transporter ions, which occurs at much slower rates compared to that through the ion channels is not included in this representation, or is not a part of this model.

You will have to look at other models to represent active transport we are limiting our discussion only to passive transport. Now, the ion currents I_K due to potassium or I_{Na} due to sodium at steady state can be represented to the conductance and the effect of potential difference we have already seen that ion the current is nothing but the conductance times the potential difference.

The ion currents (I_K or I_{Na}, for example) can be represented by the conductance, g, and the effective potential difference ($\Delta \mathbf{V}_m$ – NP).

$$I_K = g_K (\Delta \mathbf{V}_m - NP_K) \quad (6.1.3\text{-}4)$$

$$I_{Na} = g_{Na} (\Delta \mathbf{V}_m - NP_{Na}) \quad (6.1.3\text{-}5)$$

So, I_K can be written as the conductance of potassium times the effective potential difference, the effective potential difference has the membrane potential minus the potential due to the Nernst potential of potassium. So this is the effective potential difference equation 6.1.3 - 4 and the current ion current due to sodium is the conductance of sodium times the membrane potential, this remains the same minus the Nernst potential of potassium.

Also, let me state this do not worry too much about it now, for now, when you start working on this will make sense working out problems would make sense. Do not worry about the direction here the directions will automatically take care. That is the reason why we have written both these in the same way and so on. We are taking the net the negatives and positives will automatically take care of themselves.

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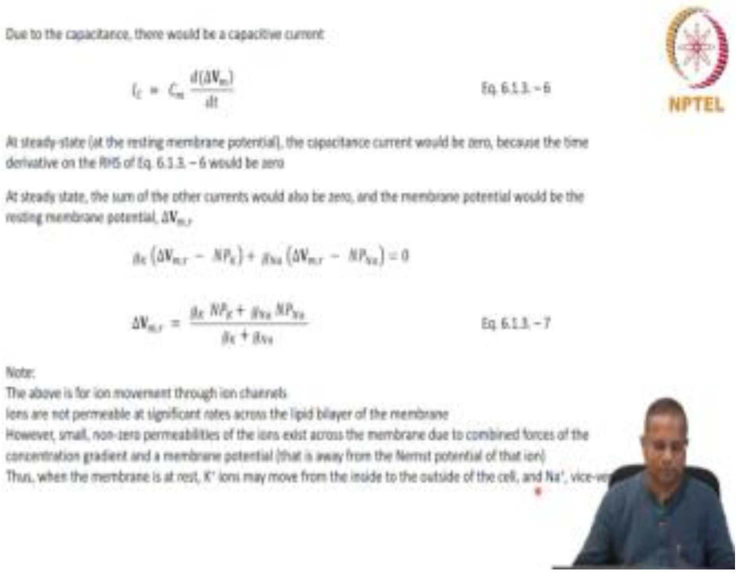

So due to capacitance, they would be a capacitive current of course, and if you recall your basic electrical engineering, the capacitive current is nothing but capacitance times the time derivative of the potential difference ΔV_m in this case, the membrane potential $d(\Delta V_m)/dt$.

Due to the capacitance, there would also be a capacitive current that can be represented by

$$I_C = C_m \frac{d(\Delta \mathbf{V}_m)}{dt} \qquad (6.1.3\text{-}6)$$

So, 6.1.3 - 6 at steady state, you are not going to have any time derivate or at the resting membrane potential, the capacitance current therefore to be 0 because this derivative would be 0.

At steady state, the sum of the other currents would also be 0. There is no net current and the membrane potential would be the resting membrane potential $\Delta V_{m,r}$, you have a membrane potential, you have the resting membrane potential at equilibrium. And at the resting membrane potential which is at steady state, the sum of the other currents would be 0 and the membrane potential would be at the resting membrane potential.

$$g_K(\Delta \mathbf{V}_{m,r} - NP_K) + g_{Na}(\Delta \mathbf{V}_{m,r} - NP_{Na}) = 0$$

Therefore, the resting membrane potential can be calculated if the conductances and Nernst potentials of the ions involved are known, as follows

$$\Delta \mathbf{V}_{m,r} = \frac{g_K NP_K + g_{Na} NP_{Na}}{g_K + g_{Na}} \qquad (6.1.3\text{-}7)$$

And note that the above is for ion moment through ion channels, ions are not permeable at significant rates across a liquid bilayer of the membrane. However small non zero permeabilities of the ions exists across the membrane, due to the combined forces of the concentration gradient and the membrane potential gradient, and that is away from the Nernst potential for that ion.

So that would anyway happened, but the rates are so small, that we even if you do not consider it, it is not a big, it is not going to make a big difference in comparison, with other aspects. Thus, when the membrane potential is at rest, potassium ions may move from the inside to the outside of the cell and sodium vice versa.

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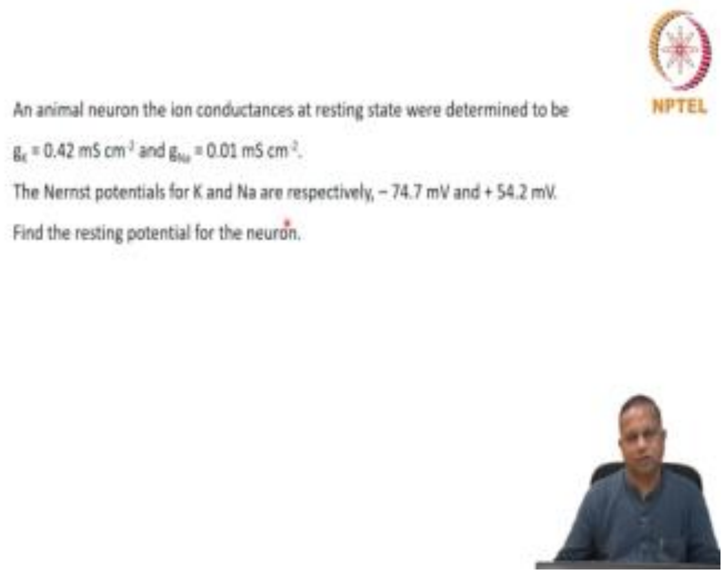

Now, let us work out a quick problem an animal neuron or in an animal neuron, the ion conductances at resting state, were determined to be for potassium 0.42 mS/cm^2 and for sodium 0.01 mS/cm^2. The Nernst potential for sodium and potassium respectively -74.7 millivolts and +54.2 millivolts. This potassium is minus sodium is plus intracellular minus extracellular, find the resting potential for the neuron. Can you go ahead and try this? Pause the video for some time. Please try this and then I will give you the solution, go ahead please pause.

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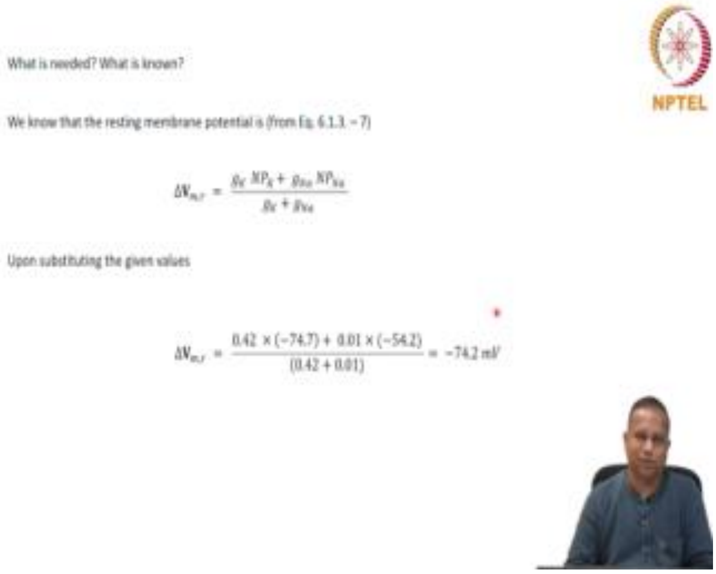

This is a closed problem, closed ended problem. So, what is needed? What is known? We know that the resting membrane potential is this in terms of the conductances and Nernst potentials and we substitute given values. What is needed is in this problem? The resting potential neuron what is given as the conductances as well as the Nernst potentials are given, how do you equate that to this equation? Or how do you relate that to this expression?

From Eq. 6.1.3-7, we know that the resting membrane potential is given by

$$\Delta \mathbf{V}_{m,r} = \frac{g_{\mathrm{K}}\ \mathrm{NP_K} + g_{\mathrm{Na}}\ \mathrm{NP_{Na}}}{g_{\mathrm{K}} + g_{\mathrm{Na}}}$$

Substituting the values given, we get

$$\Delta \mathbf{V}_{m,r} = \frac{0.42 \times (-74.7) + 0.01 \times (+54.2)}{(0.42 + 0.01)} = -71.7\ \mathrm{mV}$$

And the actual membrane potential is very close to the actual resting membrane potential is very close to that in mammalian cells.

So, that is a good thumb rule value to remember. And let us stop here in the class in which we saw an electrical representation of the membrane. Thereby we could use the relationships that are valid for an electrical circuit analysis and then we could get useful insights. The next class we will see directly how the transport of ions across the membrane are related to our the working of our 5 senses. See you there.